Power Electronics Notes 30C
Notes on Permanent Magnets

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Brief History of Permanent Magnets

• c. 1000 BC: Chinese compasses using lodestone
  – Later used to cross the Gobi desert

Fig. 2  Man-shaped compass mounted on a chariot, after Kartsev [2]

Fig. 3  Chinese spoon compass

Brief History of Permanent Magnets (cont.)

Figure 1.1 Magnetic needles for compasses are being made by craftsmen in this print of 1637. Good steel was manufactured in China from 500 A.D. onwards.

Brief History of Permanent Magnets (cont.)

- c. 200 BC: Lodestone (magnetite) known to the Greeks
  - Touching iron needles to magnetite magnetized them
- 1200 AD: French troubadour de Provins describes use of a primitive compass to magnetize needles
- 1600: William Gilbert publishes first journal article on permanent magnets
- 1819: Oersted reports that an electric current moves compass needle

References:
Brief History of Permanent Magnets (cont.)

- c. 1825: Sturgeon invents the electromagnet, resulting in a way to artificially magnetize materials
- 7-ounce magnet was able to lift 9 pounds

References:
2. Britannica Online

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Brief History of Permanent Magnets (cont.)

- c. 1830: Joseph Henry (U.S.) constructs electromagnets

Reference: Smithsonian Institute archives

Joseph Henry

Reference: Smithsonian Institute archives
Brief History of Permanent Magnets (cont.)

• 1917: Cobalt magnet steels developed by Honda and Takagi in Japan
• 1940: Alnico --- first “modern” material still in use
  – Good for high temperatures
• 1960: SmCo (samarium cobalt) rare earth magnets
  – Good thermal stability
• 1983: GE (later Magnequench) and Sumitomo develop neodymium iron boron (NdFeB) rare earth magnet
  – Highest energy product, but limited temperature range

References:

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Brief History of Permanent Magnets (cont.)

• Late 1990s --- Hybrid car utilizing high strength permanent magnets
Prius and Next-Generation Hybrid

Table 1. THS motor specifications.

<table>
<thead>
<tr>
<th>System</th>
<th>THS</th>
<th>THS II</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vehicle</td>
<td>Prius</td>
<td>SUV</td>
</tr>
<tr>
<td>E/G</td>
<td>1.5L</td>
<td>3.3L</td>
</tr>
<tr>
<td>Launch</td>
<td>1997</td>
<td>2000</td>
</tr>
<tr>
<td>DC Bus Voltage</td>
<td>About 274V</td>
<td>500V</td>
</tr>
<tr>
<td>Max. Power</td>
<td>30 kW</td>
<td>35 kW</td>
</tr>
<tr>
<td>Max. Torque</td>
<td>305 Nm</td>
<td>350 Nm</td>
</tr>
<tr>
<td>Max. Speed</td>
<td>6000 rpm</td>
<td>6700 rpm</td>
</tr>
</tbody>
</table>

Reference: M. Kamiya, “Development of Traction Drive Motors for the Toyota Hybrid System”

Fig. 2. Toyota Hybrid System.

Fig. 3. High voltage motors drive system employing a boost DC/DC converter.
Maglev System #1

- Magnemotion (Acton, MA)

Fig. 2.1. A vehicle’s magnet pod attracted upwards to a suspension rail.

Maglev System #2

- General Atomics (San Diego, CA)

**Figure 2-9**
Cross-section of Magnet System Assembly

Eddy Current Brake

- Magnetar Corp. (Seal Beach, CA)
Magnetizing Permanent Magnets

- Material is placed inside magnetizing fixture
- Magnetizing coil is energized with a current producing sufficient field to magnetize the PM material

FIGURE 1.33 Magnetization of a bar magnet using a solenoid

FIGURE 1.34 Conventional magnetizing fixture.

Pictorial View of Magnetization Process

Figure 3.6 Pictorial explanation of magnetization curve in a ferromagnetic bar.

Permanent Magnets

- External effects of PMs can be modeled as surface current

Figure 1.4. Uniformly magnetized magnet (a), which may be modeled by a current density over its boundary (b).

Permanent Magnets

- After magnetization, magnetization vector $\mathbf{M}$ has value of either $+M_{\text{sat}}$ or $-M_{\text{sat}}$. $H_{ci}$ is the temperature-dependent field strength (A/m) which causes the $\mathbf{M}$ vector to flip direction.

![Diagram of magnetization characteristic]

Figure 1.10. Intrinsic magnetization characteristic for an elemental volume of a magnet.

Permanent Magnets with High $H_{ci}$

- Constitutive relationship: $B = \mu_0(H+M)$
- Since $M$ has values of either $+M_{sat}$ or $-M_{sat}$, it follows that the slope of the BH curve for the permanent magnet is $\sim \mu_0$; this holds for NdFeB at moderate temperature

![Diagram of BH curve](image)

Figure 1.11. $B$ versus $H$ characteristic for a magnet. ($H_{ci} > M_{sat}$)

Permanent Magnets with Lower $H_{ci}$

- Note that $H_{ci}$ is temperature-dependent.
- At higher temperature, the “knee” in the 2nd quadrant shows up. Operation with $H$ inside the magnet less than $H_{ci}$ results in irreversible demagnetization.

Demagnetization Curves of Ceramic 8

- Typical sintered ceramic magnet

Figure 3.3. Demagnetization curves of Ceramic 8 at various temperatures.

Demagnetization Curves of NdFeB

- Strong neodymium-iron-boron

Figure 3.14. Demagnetization curves of (Nd, Dy)–Fe–B at various temperatures.

Curie Temperature

• The “Curie temperature” is the temperature at which the magnetization is totally destroyed.
• The practical maximum operating temperature for a permanent magnet is well below the Curie temperature.

Maximum Working Temperature

• The practical maximum operating temperature for a permanent magnet is well below the Curie temperature.

Magnetic materials have a wide range of working temperatures. The following chart lists the various materials and their maximum working temperature. NdFeB material comes in many different heat tolerances but as the heat tolerance increases the maximum available flux density decreases:

<table>
<thead>
<tr>
<th>Material</th>
<th>Maximum Working Temperature</th>
<th>°C</th>
<th>°F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ceramic</td>
<td>400</td>
<td></td>
<td>752</td>
</tr>
<tr>
<td>Alnico</td>
<td>540</td>
<td></td>
<td>1004</td>
</tr>
<tr>
<td>SmCo 1,5</td>
<td>260</td>
<td></td>
<td>500</td>
</tr>
<tr>
<td>SmCo 2, 17</td>
<td>350</td>
<td></td>
<td>662</td>
</tr>
<tr>
<td>NdFeB N</td>
<td>80</td>
<td></td>
<td>176</td>
</tr>
<tr>
<td>NdFeB M</td>
<td>100</td>
<td></td>
<td>212</td>
</tr>
<tr>
<td>NdFeB H</td>
<td>120</td>
<td></td>
<td>248</td>
</tr>
<tr>
<td>NdFeB SH</td>
<td>150</td>
<td></td>
<td>302</td>
</tr>
<tr>
<td>NdFeB UH</td>
<td>180</td>
<td></td>
<td>356</td>
</tr>
<tr>
<td>NdFeB EH</td>
<td>200</td>
<td></td>
<td>392</td>
</tr>
</tbody>
</table>

Table 3 - Working Temperatures

Reference: Magcraft, “Permanent Magnet Selection and Design Handbook”
Permanent Magnets vs. Steel

• Note that PM has much higher coercive force

Permanent magnet: Alnico 5

M-5 steel
B-H Loop for M-5 Grain-Oriented Steel

- Only the top half of the loops shown for steel 0.012” thick
Hysteresis Loop

- Hysteresis loss proportional to shaded area
Permanent Magnets

- “Soft” magnetic materials such as magnetic steel can behave as very weak permanent magnets.
- Permanent magnets, or “hard” magnetic materials, have a high coercive force $H_c$ and can produce significant flux in an airgap.

![Diagram showing B-H loops for soft and hard magnetic materials.]

**FIGURE 1.25** The $B$-$H$ loops for soft and hard magnetic materials.

Cast Alnico

Example 1: Permanent Magnet in a Magnetic Ckt

- This example uses NdFeB which has linear B/H curve in the 3rd quadrant
Example 1: Permanent Magnet in a Magnetic Ckt

Using Ampere’s law (and noting that our approximation of infinite $\mu$ assures that $H = 0$ in the steel) we note:

$$H_m l_m + H_g g = 0$$

where $H_g$ is the magnetic field in the airgap. Next we use Gauss’ magnetic law which says that flux is continuous around a loop, to get:

$$B_m A_m = B_g A_g$$

where $A_m$ is the cross-sectional area of the permanent magnet and $B_g$ is the cross-sectional area of the airgap. Noting that $B_g = \mu_o H_g$, we next solve for $B_m$, as a function of $H_m$, resulting in the load line equation:

$$B_m = -\mu_o \left( \frac{l_m A_g}{g A_m} \right) H_m$$
Example 1: Permanent Magnet in a Magnetic Ckt

![Diagram of B-H curve with labels](image)

- **B_m**: Magnetic field intensity
- **B_r**: Remanent flux density
- **H_m**: Maximum magnetic field
- **H_c**: Critical magnetic field
- **I_p**: Primary current
- **I_m**: Magnetizing current
- **μ → ∞**: Permeability approaching infinity
- **Airgap length**: Distance between the poles of the magnet
- **Load line**: Line indicating the operating point
- **Operating point**: Point on the load line where the system operates

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Permanent Magnets
Example 2: Permanent Magnet in a Magnetic Ckt

Example:

\[ g = 0.2 \text{cm} \]
\[ l_m = 1.0 \text{cm} \]
\[ A_m = A_g = 4 \text{cm}^2 \]

Find flux in airgap \( B_g \) for magnetic material
(a) Alnico 5
(b) M-5 steel
Example 2: With Alnico Permanent Magnet

\[ N I = 0, \therefore \]

\[ H_m l_m + H_g g = 0 \quad \text{(Ampere's Law)} \]

1. \[ H_g = -H_m \left( \frac{g}{l_m} \right) \]

Flux is continuous:

2. \[ A_g B_g = A_m B_m \rightarrow B_g = B_m \left( \frac{A_m}{A_g} \right) \]

Solve for \( B_m \) as a function of \( H_m \):

\[ B_m = B_g \frac{A_g}{A_m} = \mu_0 H_g \frac{A_g}{A_m} \]

\[ B_m = \mu_0 \left( -H_m \frac{g}{l_m} \right) \frac{A_g}{A_m} = -6.28 \times 10^4 H_m \]

Plot this load line on Alnico B-H curve
Example 2: With Alnico Permanent Magnet

- Result: \( B_g = 0.3 \) Tesla

Example 2: Load Line Solution with M-5 Steel

- Use same load line; $B_g = 0.38$ Gauss (much lower than with Alnico)
- Note: Earth’s magnetic field $\sim 0.5$ Gauss
Some Common Permanent Magnet Materials

![Graph showing the magnetic properties of various permanent magnet materials. The graph plots the magnetic field (H, kA/m) against the magnetic induction (B, T). Different lines represent different materials: neodymium-iron-boron, Alnico 5, samarium-cobalt, Alnico 8, and Ceramic 7.]
Typical NdFeB B-H Curve

- Neodymium-iron-boron (NdFeB) is the highest strength permanent magnet material in common use today.
- Good material for applications with temperature less than approximately 80 - 150°C.
- Cost per pound has reduced greatly in the past few years.
- Curve below for “grade 40” or 40 MGOe material.
Typical NdFeB B-H Curve --- Elevated Temp.

- Note that the B/H curve degrades at elevated temperatures.
- If you operate the magnet below the “knee” irreversible demagnetization may result.

<table>
<thead>
<tr>
<th>Item</th>
<th>Grade 40</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B_r$</td>
<td>1.27 T</td>
</tr>
<tr>
<td>$H_c$</td>
<td>905 kA/m</td>
</tr>
<tr>
<td>$H_{ci}$</td>
<td>955 kA/m</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>-0.0012</td>
</tr>
<tr>
<td>$\beta$</td>
<td>-0.0065</td>
</tr>
</tbody>
</table>

Max. working temperature $T_{max}$ as reported by manufacturer: 80 °C.
NdFeB B-H Curves for Different Grades

Example 3: Force Between 2” NdFeB Magnet Cubes vs. Airgap

Attraction

Repulsion

Force between 2” cubic magnets

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Permanent Magnets
Example 4: Stray Fields from 2” Grade 40 Magnet Cube

• At distances $r = 6”$, $12”$ and $24”$
Example 5: Magnetic Circuit With Steel

- Estimate airgap field $B_g$ assuming grade 42 NdFeB
- Airgap $g$, magnet thickness $t_m$

Assume no saturation in steel
Example 5: Magnetic Circuit With Steel

- This analysis also ignores leakage

Assume no saturation in steel

\[ 2 H_m \tau_m + H_g g = 0 \]  
(Ampere’s law)

\[ H_g = \frac{B_g}{\mu_0} = \frac{B_m}{\mu_0} \]  
(constitutive relation)

\[ 2 H_m \tau_m + \frac{B_m g}{\mu_0} = 0 \]

\[ \frac{B_m g}{\mu_0} = -2 H_m \tau_m \]

\[ \frac{B_m}{H_m} = -\frac{2 \tau_m \mu_0}{g} \]
Example 5: Magnetic Circuit With Steel --- Operating Point vs. Magnet Thickness $t_m$

\[
\begin{align*}
\mathcal{F}_m = \frac{q}{2} & \rightarrow B_g \sim 0.65 \, T \\
\mathcal{F}_m = q & \rightarrow B_g \sim 0.85 \, T \\
\mathcal{F}_m = 2q & \rightarrow B_g \sim 1.05 \, T
\end{align*}
\]
Example 5: 2D FEA, Magnet Thickness $t_m = g/2$
Example 5: 2D FEA, Magnet Thickness $t_m = g$
Example 5: 2D FEA, Magnet Thickness $t_m = 2g$
Example 5: Comparison of Different Magnet Thicknesses

- $tm = g/2$
  - Bpk, 2D analytic: 0.65T
  - Bpk, 2D FEA: 0.58T
- $tm = g$
  - Bpk, 2D analytic: 0.85T
  - Bpk, 2D FEA: 0.73T
- $tm = 2g$
  - Bpk, 2D analytic: 1.05T
  - Bpk, 2D FEA: 0.82T
Circuit Modeling of Permanent Magnets

Some magnets (i.e., Nd-Fe-B) have linear 2nd quadrant demag curves.

1. $B_m = B_R + M_m H_m$
2. $M_m = \frac{B_R}{H_c}$
3. $H_m = \frac{B_m}{M_m} - H_c$

Circuit Modeling of Permanent Magnets

Let's use this in a magnetic circuit with airgap:

\[ H_m l_m + H_g g = 0 \]

AMPERE'S LAW

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Circuit Modeling of Permanent Magnets

\[ \left( \frac{B_m}{\mu_m} - H_c \right) l_m + H_g g = 0 \]

\[ \Rightarrow \]

\[ H_g g + \frac{B_m}{\mu_m} l_m = H_c l_m \]

or

\[ \frac{B_g g}{\mu_0} + \frac{B_m l_m}{\mu_0} = H_c l_m \]

TOTAL FLUX:

\[ \Phi = B_g A_g = B_m A_m \]

so, \( \mathbf{5} \) REDUCES TO:

\[ \Phi (R_g + R_m) = H_c l_m \]
Circuit Modeling of Permanent Magnets

WE CAN MODEL THIS AS

FOLLOWS:

\[ P_m \]

\[ H_c l_m \pm \]

\[ R_g \]

PM MODEL:

\[ P_m \]

\[ H_c l_m \pm \]
Circuit Modeling of Permanent Magnets

Magnetic circuit model of a magnet with linear demagnetization curve

\[ R_m = \frac{l_m}{\mu_m A_m} \]

\[ F_m = H_c l_m \]
Example 6: Circuit Modeling of Permanent Magnets

**Example:**

Grade 37 NdFeB

\[ H_c = 950,000 \, \text{A/m} \]

\[ \mu_m = 1.048 \mu_0 \]

\[ l_m = 2 \, \text{cm} \]

\[ g = 1 \, \text{cm} \]

\[ A_m = A_g = 25 \, \text{cm}^2 \]

Find \( B_g \)
Example 6: Circuit Modeling of Permanent Magnets

This example is particularly simple because $A_m = A_g$.

Solution:

$$H_c l_m = (950,000)(0.02) = 19,000$$

$$R_m = \frac{l_m}{M_r A_n} = \frac{0.02}{(1.048)(4\pi \times 10^{-7})(25 \times 10^{-9})}$$

$$= 6.07 \times 10^6$$

$$R_g = \frac{g}{M_o A_g} = \frac{0.01}{(4\pi \times 10^{-7})(25 \times 10^{-9})}$$

$$= 3.2 \times 10^6$$

$$\Phi = \frac{H_c l_m}{R_m + R_g} = 2.04 \times 10^{-3} \text{ Wb}$$

$$B_g = \frac{\Phi}{A_g} = \frac{2.04 \times 10^{-3}}{25 \times 10^{-4}}$$

$$\therefore B_g = 0.82 \text{ Wb/m}^2 = 0.82 \text{ Tesla}$$
Example 6: Circuit Modeling of Permanent Magnets--FEA

Permanent Magnets © M. T. Thompson, 2009
Method of Images

- These two are equivalent in the upper half-plane
Another Example: Magnetic Circuit With Steel

- Which scenario has the lowest leakage flux?

Reference: www.mmpa.org
Maximum Energy Product

- BH has units of Joules per unit volume
Why is maximum energy product important?

1. \[ B_g = B_m \left( \frac{A_m}{A_g} \right) \]
2. \[ \frac{H_m l_m}{H_g g} = -1 \]

Let's find \( B_g^2 \)

\[
B_g^2 = B_m \left( \frac{A_m}{A_g} \right) \times \mu_0 H_g
\]

\[
= -B_m \left( \frac{A_m}{A_g} \right) \mu_0 \frac{H_m l_m}{g}
\]

\[
= \mu_0 \left( \frac{Vol_{mag}}{Vol_{gap}} \right) (-B_m H_m)
\]

Solve for \( Vol_{mag} \)

\[
Vol_{mag} = B_g^2 \frac{Vol_{gap}}{(-B_m H_m)}
\]

To use minimum volume of magnetic material at a given \( B_g \), operate magnet at \((BH)_{max}\) point.
Progress in PM Specs

- Maximum \((BH)_{\text{max}}\) product has gone up a lot in the past 20 years

Example 7: Use of Maximum Energy Product

- Find magnet dimensions for desired $B_g = 0.8$ Tesla operating at $(BH)_{\text{max}}$

\[ B_{\text{max}} = 1.0 \text{T and } H_{\text{m}} = -40 \text{ kA/m} \]

\[ A_m = A_g \left( \frac{B_g}{B_{\text{m}}} \right) = \left( 2 \text{ cm}^2 \right) \left( \frac{0.8}{1.0} \right) = 1.6 \text{ cm}^2 \]

\[ l_m = -g \left( \frac{H_g}{H_{\text{m}}} \right) = -g \left( \frac{B_g}{\mu_0 H_{\text{m}}} \right) \]

\[ = \left( 0.2 \text{ cm} \right) \left( \frac{0.8}{4\pi \times 10^{-7}} \right) \left( -40,000 \right) \]

\[ = 3.18 \text{ cm} \]

So, magnet is 3.18 cm long and 1.6 cm$^2$ in area.
Comparison of Different PM Types

Table 2.1 Magnet Material Comparisons

<table>
<thead>
<tr>
<th>Material</th>
<th>Grade</th>
<th>Br</th>
<th>Hc</th>
<th>Hci</th>
<th>BHmax</th>
<th>$T_{\text{max}}$ (Deg C)*</th>
</tr>
</thead>
<tbody>
<tr>
<td>NdFeB</td>
<td>39H</td>
<td>12,800</td>
<td>12,300</td>
<td>21,000</td>
<td>40</td>
<td>150</td>
</tr>
<tr>
<td>SmCo</td>
<td>26</td>
<td>10,500</td>
<td>9,200</td>
<td>10,000</td>
<td>26</td>
<td>300</td>
</tr>
<tr>
<td>NdFeB</td>
<td>B10N</td>
<td>6,800</td>
<td>5,780</td>
<td>10,300</td>
<td>10</td>
<td>150</td>
</tr>
<tr>
<td>Alnico</td>
<td>5</td>
<td>12,500</td>
<td>640</td>
<td>640</td>
<td>5.5</td>
<td>540</td>
</tr>
<tr>
<td>Ceramic</td>
<td>8</td>
<td>3,900</td>
<td>3,200</td>
<td>3,250</td>
<td>3.5</td>
<td>300</td>
</tr>
<tr>
<td>Flexible</td>
<td>1</td>
<td>1,600</td>
<td>1,370</td>
<td>1,380</td>
<td>0.6</td>
<td>100</td>
</tr>
</tbody>
</table>

* $T_{\text{max}}$ (maximum practical operating temperature) is for reference only. The maximum practical operating temperature of any magnet is dependent on the circuit the magnet is operating in.

Reference: www.magnetsales.com
# Magnetic Conversion Factors

**Table 3.2 Conversion Factors**

<table>
<thead>
<tr>
<th>Multiply</th>
<th>By</th>
<th>To obtain</th>
</tr>
</thead>
<tbody>
<tr>
<td>inches</td>
<td>2.54</td>
<td>centimeters</td>
</tr>
<tr>
<td>lines/in(^2)</td>
<td>0.155</td>
<td>Gauss</td>
</tr>
<tr>
<td>lines/in(^2)</td>
<td>(1.55 \times 10^-5)</td>
<td>Tesla</td>
</tr>
<tr>
<td>Gauss</td>
<td>6.45</td>
<td>lines/in(^2)</td>
</tr>
<tr>
<td>Gauss</td>
<td>(0^{-4})</td>
<td>Tesla</td>
</tr>
<tr>
<td>Gilberts</td>
<td>0.79577</td>
<td>ampere turns</td>
</tr>
<tr>
<td>Oersteds</td>
<td>79.577</td>
<td>ampere turns /m</td>
</tr>
<tr>
<td>ampere turns</td>
<td>0.4(\pi)</td>
<td>Gilberts</td>
</tr>
<tr>
<td>ampere turns/in</td>
<td>0.495</td>
<td>Oersteds</td>
</tr>
<tr>
<td>ampere turns/in</td>
<td>39.37</td>
<td>ampere turns/m</td>
</tr>
</tbody>
</table>

Reference: [www.magnetsales.com](http://www.magnetsales.com)
Magnetic Field Estimates

a. Cylindrical Magnets

\[ B_1 = \frac{B_0}{2} \left( \frac{(L + X)}{\sqrt{R^2 + (L + X)^2}} - \frac{X}{\sqrt{R^2 + X^2}} \right) \]

Equation 4

Table 4.1 Flux Density vs. Material

<table>
<thead>
<tr>
<th>Material and Grade</th>
<th>Residual Flux Density, Br</th>
<th>Flux at distance of 0.050&quot; from surface of magnet</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ceramic 1</td>
<td>2,200</td>
<td>629</td>
</tr>
<tr>
<td>Ceramic 8</td>
<td>3,950</td>
<td>1,150</td>
</tr>
<tr>
<td>SmCo 18</td>
<td>8,600</td>
<td>2,400</td>
</tr>
<tr>
<td>SmCo 26</td>
<td>10,500</td>
<td>3,004</td>
</tr>
<tr>
<td>NdFeB 35</td>
<td>12,300</td>
<td>3,518</td>
</tr>
<tr>
<td>NdFeB 42H</td>
<td>13,300</td>
<td>3,804</td>
</tr>
</tbody>
</table>

Table 4.1 shows flux density calculations for a magnet 0.500" in diameter by 0.250" long at a distance of 0.050" from the pole surface, for various materials. Note that you may use any unit of measure for dimensions; since the equation is a ratio of dimensions, the result is the same using any unit system. The resultant flux density is in units of gauss.

Reference: [www.magnetsales.com](http://www.magnetsales.com)
Magnetic Field Estimates

b. Rectangular Magnets

\[ B_x = \frac{B_r}{\pi} \left( \tan^{-1} \left( \frac{AB}{2X\sqrt{4X^2 + A^2 + B^2}} \right) - \tan^{-1} \left( \frac{AB}{2(L + X)\sqrt{4(L + X)^2 + A^2 + B^2}} \right) \right) \]

Equation 5

(where all angles are in radians)

Reference: www.magnetsales.com
Open Circuited Permanent Magnet

Short-Circuited Permanent Magnet

• Find B inside core, ignoring any leakage and assuming infinite permeability in core
Short Circuited Permanent Magnet

- For infinite permeability, load line is vertical
- Intersection of load lines occurs at $B \approx B_r$

PM and a Winding

- Many motors have permanent magnets, steel and windings

Analysis of PM in closed core, with excitation

\[ H_m l_m = NI \quad (\text{Amperes' Law}) \]

\[ : \quad H_m = \frac{NI}{l_m} \]
PM and a Winding --- Load Line

• Note that demagnetization can occur if current is sufficiently high
• At point (a), steel is initially unmagnetized
• As current increases, B and H follow the locus from (a) to (b)
• From (b) to (c), current reduces to zero, and flux density reduces to $B_r$ at $i = 0$
• As current goes negative from (c) to (d), curve traces hysteresis loop. Note that operating point (d) is the same operating point we’d get if there was zero current and an airgap.
• If current goes further negative, locus traces from (d) to (e)
• But, if current is reduced to zero at point (d), locus traces minor loop from (d) to (f). The “recoil line” is an approximation to this minor loop
Another Example --- Excitation and Airgap

1. \( H_m I_m + H_g I_g = NI \) (Amperes' Law)
2. \( B_g = \mu_0 H_g \) (Constitutive)
3. \( B_m A_m = B_g A_g \) (Gauss' Magnetic Law)

Solve for \( B_m - H_m \) Load Line

\[
B_m = -\mu_0 \left( \frac{A_g I_m}{A_m} \right) \left( H_m - \frac{NI}{I_m} \right)
\]
Another Example --- Excitation and Airgap --- Load Line

\[ \text{slope} = -\frac{\mu_0 A g l_m}{R_m g} \]

Operating point

\[ \frac{N I}{l_m} \]
Another Example

- Let’s figure out how to magnetize
Another Example

(a) Find magnet length \( l_m \) so magnet operates on recoil line intersecting \( BH_{\text{max}} \) at \( A_g = 2 \text{ cm}^2 \)

SOLN:

\[
B_{\text{max}} = 1.0 \text{T}, \quad H_{\text{max}} = 10 \text{ kA/m}
\]

From Gauss’ Law

\[
B_g = \frac{A_m}{A_g} B_m \quad (1)
\]

From Ampere’s Law

\[
\frac{H_{\text{max}} l_m}{H_g g} = -1 \quad (2)
\]

From (2)

\[
B_g = -\frac{M_0 H_{\text{max}} l_m}{g}
\]

Put this into (1):

\[
-\frac{M_0 H_{\text{max}} l_m}{g} = \frac{A_m B_m}{A_g}
\]

\[
l_m = g \left( \frac{A_m}{A_g} \right) \left( \frac{B_m}{-\mu_0 H_{\text{max}}} \right)
\]

\[
l_m = 0.2 \text{ cm} \left( \frac{2 \text{ cm}^2}{2 \text{ cm}^2} \right) \left( \frac{1.0 \text{T}}{(4\pi \times 10^{-7})(4\times10^9)} \right) = 4 \text{ cm}
\]
Another Example

(b) How TO MAGNETIZE FULLY?

1. \( NI = H_m I_m + H_g g \)

2. \( B_m A_m = B_g A_g = \mu_0 H_g A_g \)

Solve for \( B_m \) as a function of \( H_m \) and \( I \)

\[
B_m = -\mu_0 \left( \frac{A_g}{A_m} \right) \left( \frac{H_m g}{g} \right) H_m + \frac{\mu_0 N (A_g)}{g} I
\]

\[
= -2.5 \times 10^{-5} H_m + 6.28 \times 10^{-3} I
\]

ESTIMATE \( B_{max} \) and \( H_{max} \)

\[
B_{max} \approx 2.1 T
\]

\[
H_{max} \approx 200 kA/m
\]

\( I = 45 A \)
## What Can You Buy?

### NdFeB Rounds

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Reference: [www.dextermag.com](http://www.dextermag.com)
### What Can You Buy?

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Reference: [www.dextermag.com](http://www.dextermag.com)

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What Can You Buy?

Rectangular Magnets - Finished Sizes sorted in ascending order of Length, Width, and then Thickness.

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The T* dimension represents the orientation direction.

Tolerances on 'machined' blocks are the greater of ±1.5% of the dimension or ±0.015" on cross sectional dimensions, and ±0.005" on the orientation direction.

Reference: www.magnetsales.com

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Magnetization Patterns

Different magnetization patterns give different results.

Except for the regular flexible and ceramic 1 materials, all magnet materials are "pre-oriented" and can only be magnetized in a particular direction.

Standard (or "conventional") magnetization is straight through the orientation direction, and produces one North pole and one South pole. The rare earth magnets are extremely difficult to magnetize in non-standard ways. However, the flexible and ceramic types can be magnetized in many non-standard ways to give special results.

Reference: www.magnetsales.com
Comparison

Reference: www.magnetsales.com
Comparison of Maximum Operating Temperatures

Reference:  http://www.electronenergy.com/media/Magnetics%202005.pdf
Comparison of Maximum Operating Temperatures

Reference: http://www.electronenergy.com/media/Magnetics%202005.pdf
Quotes

It is well to observe the force and virtue and consequence of discoveries, and these are to be seen nowhere more conspicuously than in printing, gunpowder, and the magnet. 
--- Sir Francis Bacon

The mystery of magnetism, explain that to me! No greater mystery, except love and hate.
--- John Wolfgang von Goethe
References

• Dexter Magnetics, [www.dextermag.com](http://www.dextermag.com)
• Magnet Sales, Inc., [www.magnetsales.com](http://www.magnetsales.com)
• Magnetic Materials Producers Association, Std. PMG-88, “Permanent Magnet Guidelines”