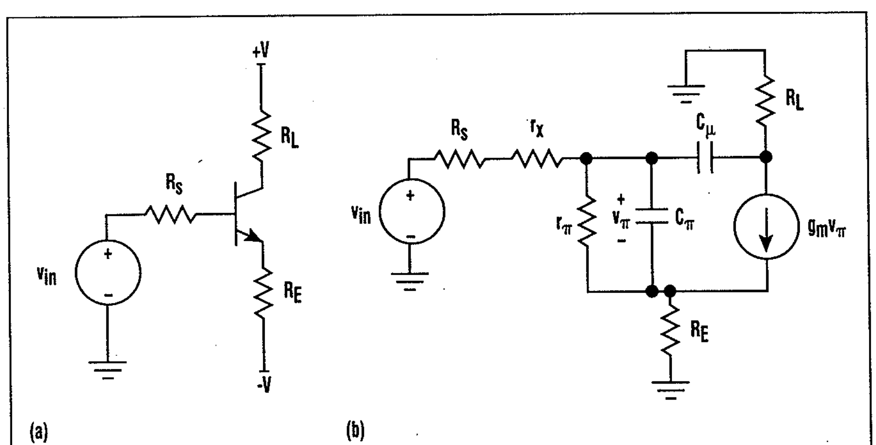
USEFUL APPROXIMATIONS AND TECHNIQUES HELP ENGINEERS USE OPEN CIRCUIT TIME CONSTANTS TO EXPLORE A CIRCUIT'S BANDWIDTH.

# NETWORK TRICKS AID IN OCTC CIRCUIT ANALYSIS

The main function of the open circuit time constants (OCTC) analysis method is to estimate the high-frequency –3-dB point of an amplifier. A previous article (ELECTRONIC DESIGN SPECIAL ANALOG ISSUE, *June 24, p. 41)* introduced the technique, and illustrated that the major benefit of OCTC analysis was the information it provided on the circuit element most affecting the bandwidth. This article discusses the nitty-gritty of how to calculate the time constants for a transistor amplifier, and suggests circuit-analysis techniques and approximations.

First, it's useful to go through the derivation of the open-circuit resistances of the most general transistor amplifier *(Fig. 1a)*. This transistor amplifier has a source resistance ($R_s$), an emitter resistance ($R_E$), and a collector load resistance ($R_L$). Of course, you can calculate open-circuit resistance by writing lots of node equations and grunging through pages and pages of math. But with a little thought and judicious application of intuitive models, the problem becomes almost reasonable. It's important to stop, take a deep breath, and interpret the physical significance of results as you go along.

You can determine the resistances using the circuit's small-signal model.




**1. THE MOST GENERAL TRANSISTOR AMPLIFIER** has a source resistance, an emitter resistance, and a collector load resistance (a). The circuit's small-signal model substitutes resistors and capacitors for the transistor (b).

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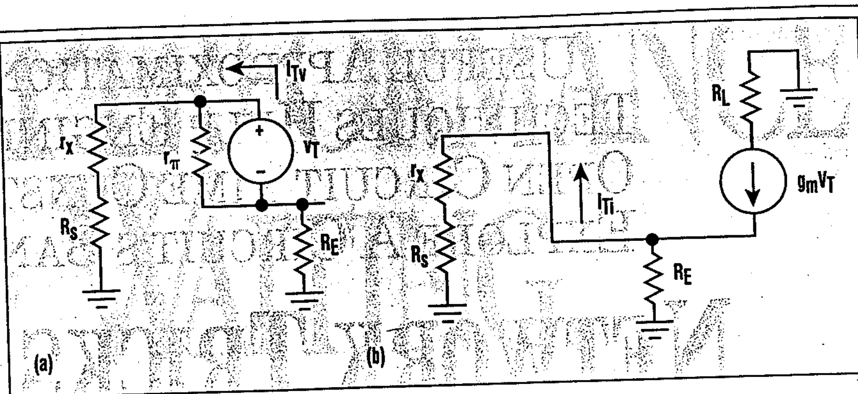
For the most difficult case, calculate $R_{1o}$ (the resistance facing $C_\pi$ with $C_\mu$ open-circuited), and $R_{2o}$ (the resistance facing $C_\mu$ with $C_\pi$ open-circuited).

To calculate $R_{1o}$, apply a test-voltage source ($V_T$) to the $C_\pi$ circuit terminals, and compute the resulting current ($I_T$). The resistance $R_{1o}$ is the ratio of voltage to current. The first trick to pull out of the engineering toolbox is the principle of superposition. Engineering school teaches you not to use superposition with dependent sources. But you can in this case, because the test-voltage source constrains $v_\pi$ to be a constant $V_T$. Therefore, you can compute two components of the resulting current separately—one due to the voltage source $V_T$ and the other due to the dependent current source $g_mV_T$. The total test current is the sum of these two currents. In other words, you're breaking a difficult problem into two easier pieces.

To determine the current $I_{Tv}$ due to $V_T$ only, disable the $g_mV_T$ current source by open-circuiting it *(Fig. 2a)*. This results in:

$$I_{Tv} = \frac{V_T}{r_\pi} + \frac{V_T}{R_s + r_x + R_E} \quad (1)$$

To find the current $I_{Ti}$ due to the dependent current source only, shut off the $V_T$ voltage source by shorting it *(Fig. 2b)*. The resulting test current is found by calculating a current divider with the current-source value acting as if $V_T$ were still active. This results in:




**2. TO BEGIN CALCULATING** **the circuit's open circuit time constants, first find the resistance $R_{1o}$ facing $C_\pi$, with $C_\mu$ open-circuited. $R_{1o}$ is computed with the ratio of voltage to current. In this circuit, the current must be calculated in two components—one due to the voltage source $V_T$ (a), and the other due to the dependent current source $g_mV_T$ (b).**

$$I_{Ti} = g_mV_T\frac{R_E}{R_s + r_x + R_E} \quad (2)$$

The total test current is the sum of the two components:

$$I_T = V_T\left[\frac{1}{r_\pi} + \frac{1 + g_mR_E}{R_s + r_x + R_E}\right] \quad (3)$$

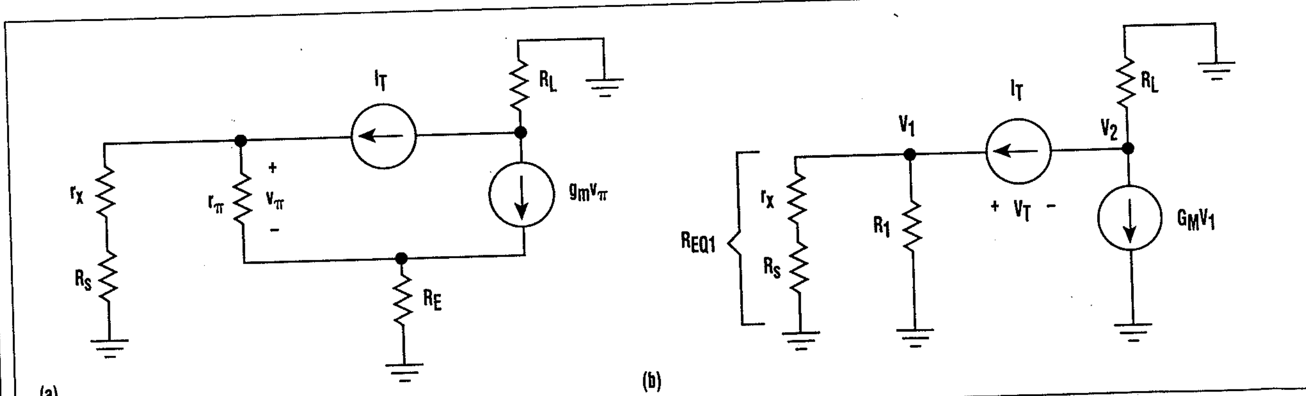
If you think about Equation 3 in reverse, it's clear that this equation form is for resistors in parallel. It follows, then, that the resistance facing $C_\pi$ is:

$$R_{1o} = r_\pi \left\| \frac{R_s + r_x + R_E}{1 + g_mR_E} \right. \quad (4)$$

It makes sense that $R_{1o}$ is equal to $r_\pi$ in parallel with a bunch of other circuit elements, because $r_\pi$ in the transistor circuit is measured directly across $C_\pi$. Also, the time constant decreases as $R_E$ increases because less and less voltage will appear across $C_\pi$, decreasing the relative importance of $C_\pi$.

For $R_{2o}$, the circuit is a bit more difficult, but not impossible. The circuit has a test-current source shown across the $C_\mu$ terminals *(Fig. 3a)*. Now, if you blindly forge ahead and write node equations, you'll arrive at a horrendously complicated result that gives no insight into the circuit's operation. One time-honored technique in electrical engineering is to transform a difficult problem into a form that's easier to solve. Then, the results of the transformed circuit are applied to the original circuit.

Keeping this philosophy in mind, replace the equivalent resistance at node $V_1$ by a resistance $R_1$, which takes into account the input imped-




**3. THE RESISTANCE FACING** **$C_\mu$ with $C_\pi$ open-circuited is even more difficult to calculate. Applying a test current source across the $C_\mu$ terminals yields a complicated result (a). It's better to replace the equivalent resistance at node $V_1$ by a resistance ($R_1$) that accounts for the transistor's input impedance, and to substitute an equivalent dependent source ($G_MV_1$) for the $g_mv_\pi$ generator.**

ance of the transistor *(Fig. 3b)*. Also, replace the $g_m v_\pi$ generator with an equivalent dependent source of value $G_M V_1$. Remember that the input resistance of an emitter follower is:

$$R_{in} = r_x + r_\pi + (1 + h_{fe})R_E \quad (5)$$

Given this result, the resistance $R_1$ at node $V_1$ is:

$$R_1 = r_\pi + (1 + h_{fe})R_E \quad (6)$$

To project this result back to the original circuit, you need to solve for $v_\pi$ by solving the current divider:

$$v_\pi = I_T \frac{r_\pi(R_s + r_x)}{R_s + r_x + r_\pi + (1 + h_{fe})R_E} \quad (7)$$

Similarly, the voltage at $V_1$ is equal to the current $I_T$ multiplied by the total resistance at node $V_1$. This resistance is $R_s + r_x$ in parallel with $R_1$:

$$V_1 = I_T \frac{(R_s + r_x)(r_\pi + (1 + h_{fe})R_E)}{R_s + r_x + r_\pi + (1 + h_{fe})R_E} \quad (8)$$

The new circuit has replaced the dependent current source $g_m v_\pi$ by an equivalent generator $G_M V_1$. However, the currents of the two dependent current sources must be equal. Given the previously found results, this means that you can solve for the new $G_M$:

$$G_M = g_m \frac{v_\pi}{V_1} = g_m \frac{r_\pi(R_s + r_x)}{(R_s + r_x)(r_\pi + (1 + h_{fe})R_E)} \quad (9)$$

Because $h_{fe}$ is much greater than 1 (and $g_m$ is much greater than $1/v_\pi$), this reduces to:

$$G_M \approx \frac{g_m}{1 + g_m R_E} \quad (10)$$

Now that you've figured out $G_M$ and $V_1$ for the simpler circuit, you need to solve for that circuit with the usual techniques. $R_{EQ1}$ is the equivalent resistance seen between the $V_1$ node and ground:

$$R_{EQ1} = (R_s + r_x) \| (r_\pi + (1 + h_{fe})R_E) \quad (11)$$

You need to solve for $V_1$ and $V_2$, because the resulting test voltage $V_T$ is equal to $V_1 - V_2$. The node voltage $V_1$ is easily found:

$$V_1 = I_T R_{EQ1} \quad (12)$$

Node $V_2$ is not so difficult:

$$V_2 = -(I_T + G_M V_1)R_L \quad (13)$$

$$V_2 = -I_T(1 + G_M R_{EQ1})R_L \quad (14)$$

The open circuit facing $C_\mu$ is found by:

$$R_{2o} = \frac{V_1 - V_2}{I_T} \quad (15)$$

yielding the *final* result:

$$R_{2o} = R_{EQ1} + R_L + G_M R_L R_{EQ1} \quad (16)$$

$$G_M \approx \frac{g_m}{1 + g_m R_E} \quad (17)$$

$$R_{EQ1} = (R_s + r_x) \| (r_\pi + (1 + h_{fe})R_E) \quad (18)$$

In Equation 16, you can see the Miller effect reflected in the $G_M R_L R_{EQ1}$ term. For a high-gain amplifier ($G_M R_L$ large), this term will dominate. The effects of the emitter-degeneration resistor $R_E$ are shown in Equation 17, where the effective $g_m$ of the transistor is reduced.

The results take on physical significance when you make some important approximations based on the specific circuit topology. For the emitter follower, there's no collector load resistor ($R_L = 0$). Furthermore, the emitter load resistor is usually large compared to $r_x$ and $r_\pi$. In the limit where the emitter follower is biased with a current source (in other words, $R_E$ becomes infinite, or very large compared to $r_x$, $r_\pi$, and the source resistance $R_s$), the time constant for $C_\pi$ reduces to:

$$R_{1o} \approx \frac{1}{g_m} \quad (19)$$

The $R_{1o}$ time constant is very small because of the bootstrapping effect of the relatively large emitter resistor $R_E$.

For $C_\mu$, no Miller-effect effective resistance exists, and the open circuit resistance is:

$$R_{2o} = (R_s + r_x) \| (r_\pi + (1 + h_{fe})R_E) \quad (20)$$

Again, if the emitter follower has a large emitter resistor (if $h_{fe}R_E$ is much greater than $R_s$, $r_x$, and $r_\pi$), this may be approximated as:

$$R_{2o} \approx R_s + r_x \quad (21)$$

For a high-gain common-emitter amplifier, the Miller effect is reflected in the open-circuit time constant. Given $R_E = 0$, the $C_\pi$ open-circuit resistance reduces to:

$$R_{1o} = r_\pi \| (R_s + r_x) \quad (22)$$

If $R_s + r_x$ is small compared to $r_\pi$, the dominant term in Equation 22 is the effective source resistance. The $C_\mu$ time constant is:

$$R_{2o} = R_L + (1 + g_m R_L)\left[(R_s + r_x) \| r_\pi\right] \quad (23)$$

Equation 23 again shows the familiar Miller effect term.

These equations should be used as a guideline when making approximations. Every circuit is different, so be prepared to make your own calculations. To summarize, useful techniques when faced with a complicated circuit are:

- Solve it by thinking, not by working too hard.
- Break a difficult problem into several easier ones, and then solve the various parts.
- Convert the circuit to an already solved form.
- Make reasonable approximations.□

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| HOW VALUABLE? | CIRCLE |
|---|---|
| HIGHLY | 530 |
| MODERATELY | 531 |
| SLIGHTLY | 532 |