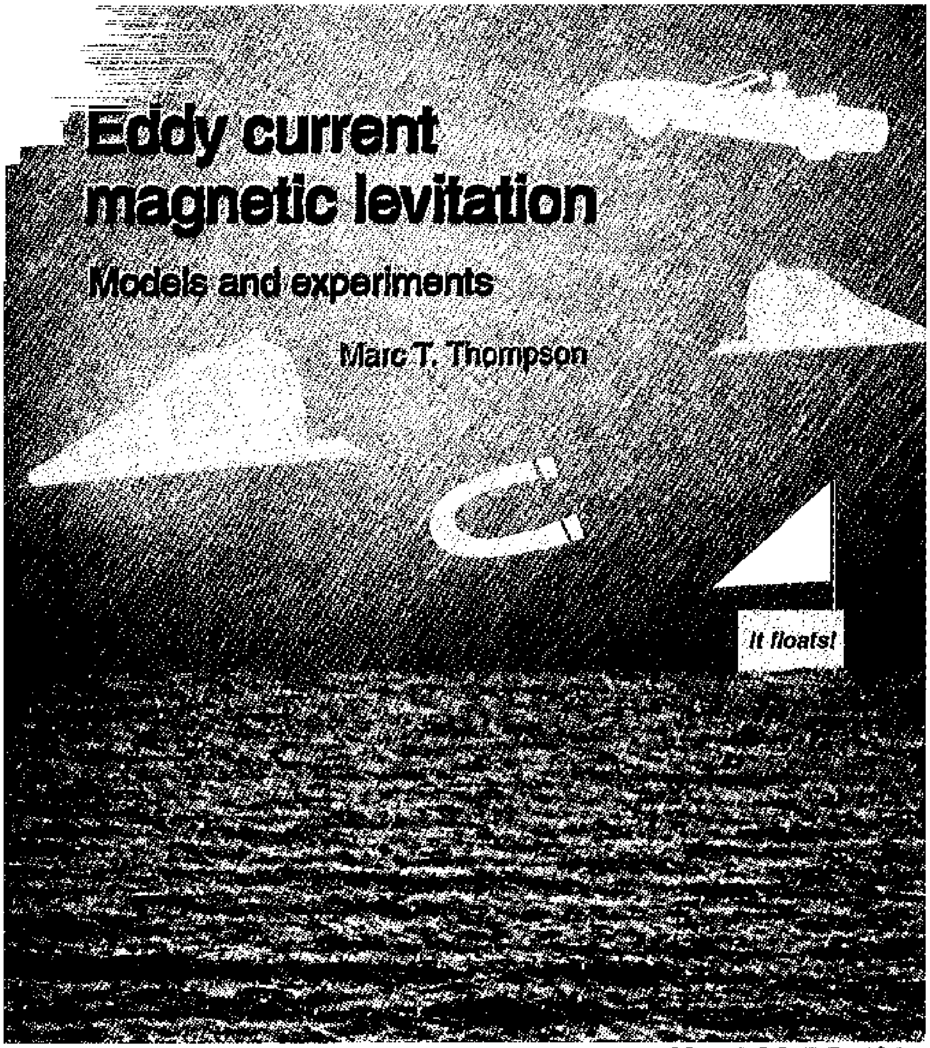
Eddy current magnetic levitation

Models and experiments

Marc T. Thompson



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Magnetic levitation has numerous practical applications in research and in industry where friction must be reduced or eliminated. Some of the more promising applications are transportation (low and high speed Maglev), low friction bearings for gyroscopes and flywheel energy storage. Other applications have been proposed, such as levitation melting of conductive metals. Applications—such as eddy-current braking and induction heating—that involve similar physical processes as magnetic levitation can be analyzed using similar or slightly modified solution techniques.

There are two "flavors" of magnetic levitation: attractive and repulsive. In "attractive" levitation, a ferromagnetic body is attracted to a source of magnetic flux, as a piece of steel is attracted to a permanent magnet. Levitation forces can be created with a DC magnetic field created by DC currents, superconducting coils or permanent magnets.

This type of levitation is unstable without feedback control (by Earnshaw's theorem), but numerous analog and digital control techniques are available. A full-scale electromagnetic suspension (EMS) Maglev system using copper coils for generation of magnetic is currently being tested in Germany. The projected revenue-producing train service would begin in 2005 from Berlin to Hamburg.

In "repulsive" levitation or electrodynamic or "EDS" levitation, eddy currents are generated in a conducting body when the body is subjected to a time-varying magnetic flux. The interaction of the eddy currents with the magnetic flux generates forces levitates the body. In EDS Maglev, the changing magnetic flux is produced by a superconducting magnet on the moving train. This changing magnetic flux generates circulating currents in stationary conducting loops (or sheets) over which the train levitates. This changing magnetic flux generates circulating currents in stationary conducting loops (or sheets) over which the train levitates. The interaction of the induced currents with the magnetic field creates the forces. There is active research in superconducting EDS Maglev. These include efforts at the Massachusetts Institute of Technology and in Japan where full-scale tests are being done.

Another way to test magnetic levitation principles is a stationary coil carrying a time-varying current, levitated above a conducting sheet. A coil may be levitated in a stable, but underdamped equilibrium without feedback control. This experiment demonstrates, in a simple way, eddy current levitation, a phenomenon that is not well understood. The approach we will show here is more intuitive and less mathematical than others, and the results are confirmed by a very simple experiment. A simple method is shown for calculating the levitation height, suspension resonant frequency and lift-off power. These scaling laws are very important to understand for the design of electrical machinery.

Analyzing the levitation experiment

The geometry of the experimental levitation system is shown in Fig. 1. A circular copper coil was built by winding #16 gauge insulated copper wire on a non-conductive winding form. The resultant coil was impregnated with epoxy. After curing, the coil was placed on top of a conducting plate that is much wider than the coil. The copper coil was energized with 60 Hz AC with adjustable voltage amplitude controlled by a variable transformer. When the voltage was of sufficient amplitude, the coil achieved "lift-off" and levitated in a stable equilibrium at height h. By adjusting the voltage amplitude, the levitation height can be adjusted.

The mechanisms involved in electrodynamic (EDS) levitation are identified through the use of Maxwell's equations. Simplifying assumptions are identified and used to generate models for evaluating the levitation force and lift-off power.

Elementary theory

The "quasistatic" or low frequency forms of Maxwell's equations are used to analyze this structure. This means that magnetic energy storage is dominant (as compared to energy stored in the electric field) and wave phenomena

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are small enough to be ignored. The first law that is useful is Ampere's Law, which simply states that a flowing current creates a magnetic field, or:

$$\oint_c \vec{H}\cdot d\vec{l} = \int_s \vec{J}\cdot d\vec{A}$$

where $\vec{H}$ is the magnetic field (Amps/meter) and $\vec{J}$ is current density (Amps/meter^2). In words, Ampere's Law means that the magnetic flux density integrated around a closed contour equals the net current through the surface bounded by the contour. Faraday's law shows the mechanisms by which a changing magnetic flux generates circulating (or "eddy") currents. The relationship in a conducting "Ohmic" material relating the current density and electric field is $\vec{J} = \sigma\vec{E}$ and we can derive:

$$\frac{1}{\sigma}\oint_c \vec{J}\cdot d\vec{l} = -\frac{d}{dt}\int_s \vec{B}\cdot d\vec{A}$$

The term on the right is the negative of the time rate of change of the magnetic flux through the surface bounded by the contour dl. This is the mechanism by which a changing magnetic flux impinging on a conductor creates eddy currents.

Furthermore, the Lorentz force law states that a magnetic force results if there is a current flow in a region where there is magnetic flux, by $\vec{F}=\vec{J}\times\vec{B}$. Using these three principles, we can identify the mechanism for creating +z lift force in an electrodynamic levitation system:

- By Ampere's law, the ϕ-directed current in the coils generates a time-varying magnetic flux. This flux has both axial (z) and radial (r) components. Some of this flux impinges on the conducting plate below the coil.
- By Faraday's law, the changing magnetic flux impinging on the plate induces an electric field (and hence, current flow) in the plate.
- The dominant current component in the plate is in the ϕ-direction. This current interacts with the r component of the magnetic field to generate +z lift (by the Lorentz force law).

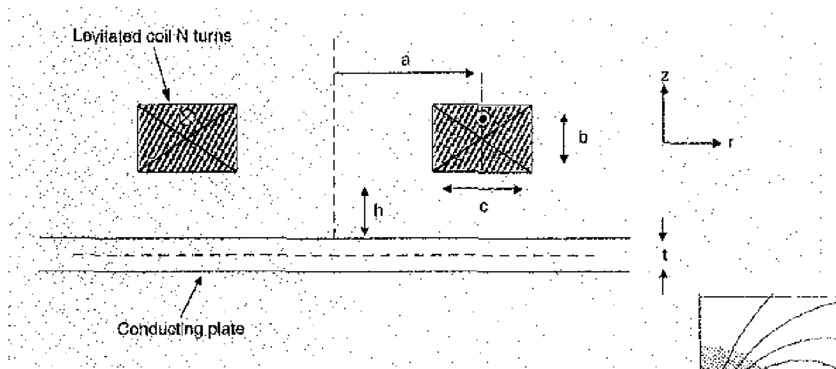

Fig. 1 Levitation experiment showing coil levitated electrodynamically above a conducting plate. The convention for labeling magnetic windings is as follows: The cross indicates current into the paper; the dot indicates current out of the paper. By the right-hand rule, for this coil (for DC current excitation) the axial magnetic field at r=0 is the -z direction.

The circulating currents in the plate create a magnetic field that opposes the incident field. Hence, the field is shielded the field from the inside of the plate. Shown in Figures 2 are simulation results for flux lines and magnetic flux density from finite-element analysis (FEA). In the plots, the red areas are higher magnetic field, and the field intensity decreases the further away from the coil you are. Note that since the system is symmetric, only one half of the problem has been simulated.

In Fig. 2a are the magnetic flux lines for the case of low-frequency excitation. "Low frequency" means that the operating frequency is sufficiently low so that the induced magnetic field (due to induced currents) is small compared to the incident field. Thus, the field passes through the plate as if it weren't there at all. Since there is minimal induced current, there is minimal lift force.

In Fig. 2b, we see the case of high frequency excitation where the incident magnetic field is shielded from passing through the plate. This is due to large induced circulating currents in the plate. Note that the flux lines are "squished" beneath the coil. The resultant induced currents may be used to generate a lift force (as in Maglev) or may be used to heat the conducting plate (as in induction heating). A simple calculation (shown shortly) finds this minimum plate thickness at a given operating frequency to reach the "high frequency" limit.

Critical frequencies

For an applied magnetic field *tangential* to the surface of a wide flat plate, the characteristic length over which the field decays in the plate is the so-called skin depth δ. This is given by:

$$\delta = \sqrt{\frac{1}{\pi f \mu \sigma}}$$

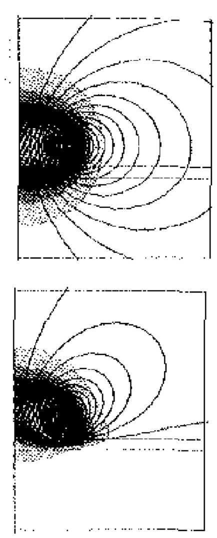

Fig. 2 Flux lines, in low frequency a) and high frequency, b) limits

where f is the operating frequency, μ is the magnetic permeability of the plate and σ is the electrical conductivity of the plate. The magnetic field inside the plate decays with this characteristic length δ. For AC line frequency (f = 60 Hz) and an aluminum plate with $\mu = 4\pi \times 10^{-7}$ H/m and $\sigma = 3.54 \times 10^7$ $(\Omega\text{-m})^{-1}$, the skin depth $\delta \approx 10.9$ millimeters. This means that an aluminum plate with a thickness greater than 10.9 millimeters or so provides effective shielding of a 60 Hz magnetic field.

Stability of levitation

Experimentation shows that the dynamics of the vertical suspension for electrodynamically levitated bodies are underdamped. By a thought experiment, stability of the suspension can be inferred. We assume a functional dependence of the terminal inductance and use energy methods to calculate the levitation force.

One possible electrical model for

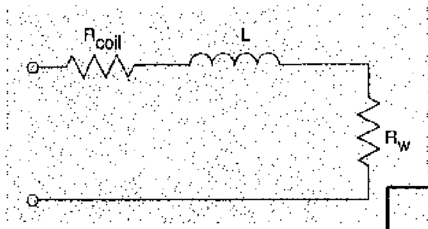

Fig. 3 Electrical model of magnetic levitation system

evaluating the driving impedance at the terminals of the coil is shown in Fig. 3. R_{coil} is the resistance of the coil in free-space due to the finite resistance of the wire. L is the frequency and geometry dependent inductance seen at the coil terminals. R_w is the resistance due to eddy-current losses in the conducting plate. When the coil is brought near the plate, R_w increases and the terminal inductance of the coil decreases. This is because the field beneath the coil is modified due to induced currents.

At high frequencies, the space beneath the coil has the flux concentrated between the coil and the plate, and hence the measured coil inductance is reduced from the free-space value. One possible functional dependence for the measured inductance is:

$$L(z) \approx L_o - L_r e^{-\frac{z}{\gamma}} \quad .$$

The term L_o is the terminal inductance of the coil when it is well away from the plate. The term L_r accounts for the fact that the terminal inductance decreases when the coil is near the plate, due to the eddy currents induced in the plate. Therefore, the inductance is a function of the z height of the coil above the conducting plate. The decays have the characteristic length scale γ. (Note: This is not the only possible mathematical description, but it yields particularly simple and surprisingly accurate results.)

In our thought experiment, let's assume that the coil is driven by an AC current source. The magnetic energy stored in the inductor is:

$$E_m = \frac{1}{2} L(z) I^2 \quad .$$

Note that even though the current is alternating, there is average energy storage in the coil (due to the I^2 term). Since this energy varies as the coil position varies (from the L(z) term), there is a resultant force acting on the coil. Using the energy method, the force due to an energy field can be easily calculated. Also, the force acting on the coil equals the gradient (or spatial derivative) of the energy field. In this case, the stored magnetic energy

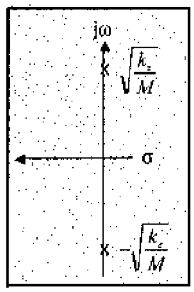

Fig. 4 Approximate pole locations for vertical motion of eddy-current magnetic suspension

changes as the z position of the coil varies, and the magnetic force pulls the coil towards a position of higher stored energy. The force acting on the coil in the +z direction is:

$$f_z = \frac{d}{dz} E_m = \frac{I^2}{2} \frac{dL(z)}{dz} \quad .$$

To determine the stability of the levitated coil, we assume that the total force acting on the coil is an equilibrium value plus a perturbed value, or $f_z = F_Z + \tilde{f}_z$. Furthermore, we assume that the position of the coil is given by an equilibrium position, plus a small perturbation, or $z = Z_o + \tilde{z}$. The equilibrium force F_Z must equal the weight of the coil, or Mg if the coil is to be stably levitated. By using our assumed coil inductance given above and by assuming that perturbations in z are small ($\tilde{z} << \gamma$), the force acting on the coil is:

$$F_z + \tilde{f}_z = \frac{I^2}{2\gamma} L_r e^{-\frac{Z_o}{\gamma}} e^{-\frac{\tilde{z}}{\gamma}} \approx Mg\left[1 - \frac{\tilde{z}}{\gamma}\right]$$

where we make use of the fact that $e^{-x} \approx 1-x$ for $x << 1$. Therefore, there is a restoring force. For small perturbations, this suspension acts like a magnetic spring with spring constant k_z. The natural frequencies of this system are on the jω axis (Fig. 4) as in the case of a simple mass and spring. The frequency of oscillation for this system is given by:

$$f_{osc} \approx \frac{1}{2\pi}\sqrt{\frac{k_z}{M}} \approx \frac{1}{2\pi}\sqrt{\frac{g}{\gamma}} \quad .$$

This interesting result shows that it is easy to find the oscillation frequency of the suspension if we can measure (or calculate) the characteristic decay length for the terminal inductance.

Experimental results

A 1-centimeter thick aluminum plate was used for inductance measurements and levitation experiments, as a skin depth calculation shows that this is the minimum approximate thickness to achieve good magnetic shielding. Parameters of the copper test coil are given in Table 1.

Coil inductance measurements

By the energy method, magnetic forces can be calculated if the change in terminal inductance is known. The coil inductance under various test conditions was measured using a Hewlett-Packard HP4192A impedance analyzer. In a first experiment, the coil inductance in the 30 Hz to 500 Hz range was measured with the coil 10 millimeters above the aluminum plate (Fig. 5). The estimate of the skin depth shows that there should be significant shielding at 60 Hz. Hence, the terminal inductance should be greatly affected. At 60 Hz, the terminal inductance is 824 microHenries (down from the free-space value of 980 microHenries). This confirms our suspicion that there is significant magnetic shielding at 60 Hz.

In a second experiment, the 60 Hz terminal inductance was measured for coil heights of 0, 5, 10, 15 and 20 millimeters above the plate (Fig. 6). The data was curve fit to our terminal inductance expression (Eq. 4). The parameters for the curve fitting are shown in Table 2. This data was used to estimate the minimum current needed for coil lift-off and the resonant frequency of the suspension.

Coil "lift-off" and height

Since we now know the terminal inductance, the current needed to achieve levitation can be easily estimated. By the energy method, the necessary current to achieve lift-off is calculated to be:

$$I = \sqrt{\frac{2Mg}{dL(z)/dz}} = \sqrt{\frac{2(0.35)(9.81)}{280\mu H/(0.02)}} \approx 22.1A(RMS) \quad .$$

For this calculation, the term $dL(z)/dz = L_r/\gamma$. The actual current to achieve lift-

Table 1 Coil parameters

Coil mean radius	a = 4.1 cm
Coil outer radius	a_2 = 5.2 cm
Coil inner radius	a_1 = 3 cm
Coil axial thickness	b = 1.6 cm
Coil radial thickness	c = 2 cm
Coil turns	N = 107, #16 AWG copper wire
Coil inductance in free space	980 µH
Coil resistance (25C)	0.38 Ω
Coil mass	M = 0.35 kg

Table 2 Curve fit parameters for terminal inductance

L_o	980 µH
L_r	280 µH
γ	20 millimeters

Table 3 Predicted and actual coil current vs. levitation height

h (mm)	$I_{measured}$ (A-RMS)	I_{calc} (A-RMS)
0	21	22.1
10	26	28.4
20	39	36.5

off was approximately 21A (RMS), with 26A resulting in 10 millimeter levitation height (Fig. 7), and 39A resulting in 20 millimeter levitation height. The lift-off power needed is approximately 168 Watts, with 257 Watts needed at h = 10 millimeters and 577 Watts needed at h = 20 millimeters. A summary of measured and calculated current needed at a given levitation height is given in Table 3.

Resonant frequency of suspension

The curve fit shows that the characteristic decay length for inductance is approximately 20 millimeters. Therefore, our estimate for resonant frequency (given by Eq. 8) is $f_{osc} \approx$ 3.5 Hz. The coil was levitated at a height 10 millimeters above the plate and bounced, and a resonant frequency of approximately 4 Hz was measured.

Magnetic scaling laws

Magnetic scaling laws show that large magnetic, elements are more efficient in energy conversion than smaller ones. Conversely, small-scale levitation experiments are likely to be very power hungry (or unable to levitate at all before they burn up). For the test coil, this effect can be quantified by considering the ratio of the lift force to the power dissipation. For the thin disk coil, the inductance L is approximately proportional to a, the coil radius.

The lift force is proportional to $dL(z)/dz$, which is also proportional to the coil radius a. The power dissipation is proportional to the resistance of the coil. This, in turn, is proportional to a/bc, the ratio of current path length to coil cross-sectional area. Therefore, the ratio of lift force to power loss is proportional to bc, or the cross-sectional area of the coil. If all coil lengths are scaled up by the same factor l, the ratio of lift force to power dissipation increases by the factor l^2, or the length squared.

To achieve lift-off of the test coil in this experiment, approximately 168 Watts is dissipated in the coil. Furthermore, as the coil heats up, more power must be dissipated for the same levitation height. The reason is the winding resistance increases with temperature. (The temperature coefficient of the resistance of copper is approximately = 0.4% per degree C.) So, the test coil gets hot, and can only be run for a few seconds at a time. As shown by the magnetic scaling laws a larger coil could be levitated for longer periods of time.

Conclusions

We presented a simple demonstration of eddy-current magnetic levitation using a small copper coil, energized with 60 Hz AC and levitated over an aluminum plate. The processes that generate magnetic forces are identified using Maxwell's equations. Through this experiment, a method for determining lift-off power, levitation height and suspension resonant frequency was shown.

The principles outlined in this article have numerous applications to magnetic levitation, induction heating and other electrodynamic processes involving induced eddy currents. Scaling laws show how to size a suspension conductor and power supply for a given levitated load.

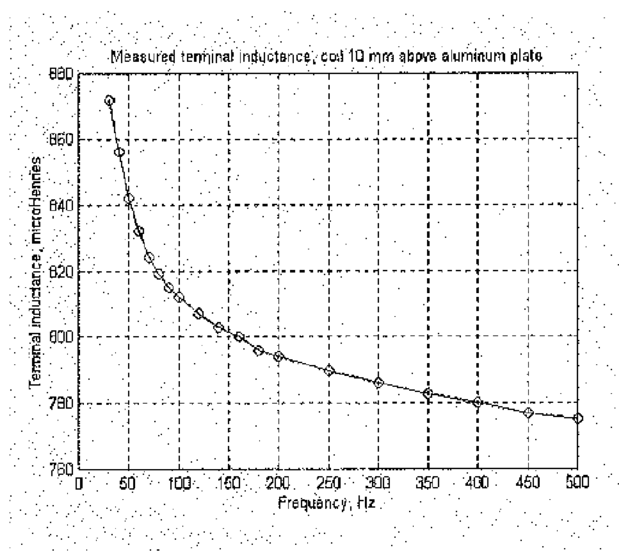

Fig. 5 Measured terminal inductance, coil 10 millimeters above aluminum plate

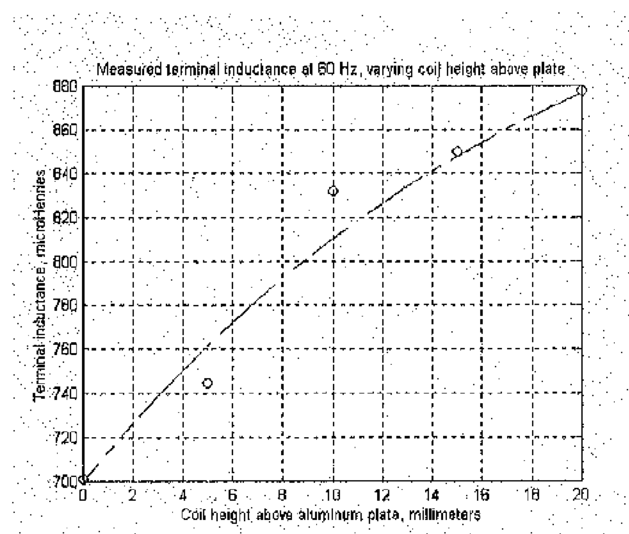

Fig. 6 60 Hz coil inductance, for various coil heights above aluminum plate. Dotted line is curve fit to data

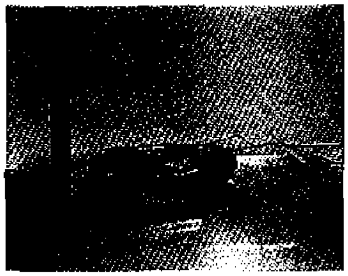

Fig. 7 Copper coil levitated approximately 10 millimeters above aluminum plate.

Read more about it

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About the author

Marc T. Thompson (M '92) was born in Vinalhaven Island, Maine. He received the B.S. E.E. degree from the Massachusetts Institute of Technology (M.I.T.) in 1985, the M.S.E.E. in 1992, the Electrical Engineer's degree in 1994, and the Ph.D. in 1997.

Presently he is an engineering consultant and Adjunct Associate Professor of Electrical Engineering at Worcester Polytechnic Institute, Worcester Massachusetts. At W.P.I., he teaches intuitive methods for analog circuit, magnetic, thermal and power electronics design. His main research at MIT concerned the design and test of high-temperature superconducting suspensions for MAGLEV and the implementation of magnetically-based ride quality control. Other areas of his research and consulting interest include planar magnetics, power electronics, high speed analog design, induction heating, IC packaging for improved thermal and electrical performance, use of scaling laws for electrical and magnetic design, and high speed laser diode modulation techniques. He has worked as a consultant in analog, electromechanics, mechanical and magnetics design, and holds 2 patents. Currently he works on a variety of consulting projects including high power and high speed laser diode modulation, eddy-current brake design for amusement applications, flywheel energy storage for satellites, and magnetic tracking for inter-body catheter positioning and is a consultant for Magnemotion, Inc., United States Department of Transportation, Edward M. Pribonic, PE Inc., and Polaroid Corporation.

Index of symbols

a	Coil mean radius (4.1 cm)
a_2	Coil outer radius (5.2 cm)
a_1	Coil inner radius (3 cm)
B	Magnetic flux density (Tesla)
b	Coil axial thickness (1.6 cm)
c	Coil radial thickness (2.2 cm)
E_m	Magnetic stored energy (Joules)
F	Force density (N/m^3)
f_z	Levitation force (N)
F_Z	Equilibrium levitation force = Mg = 3.4 N
$\tilde{f}_z$	Perturbed levitation force
g	Acceleration due to gravity (9.81 m/s^2)
h	Levitation height, from bottom of coil to top of plate
I	Coil current (A)
J	Current density (A/m^2)
k_z	Magnetic spring constant (N/m)
l	Coil length scale
L_o	Inductance of coil with plate far away (980 μH)
L_r	Inductance reduction when coil location $z = 0$
M	Mass of levitation coil (0.35 kg)
N	Coil turns (= 107)
t	Conducting plate thickness (1 cm)
z	Axial position of coil above plate
Z_0	Equilibrium position of coil above plate
$\tilde{z}$	Perturbed coil location above plate
μ_o	Magnetic permeability of free space $4\pi\times10^{-7}$ H/m
σ	Electrical conductivity (Ω-m)
δ	Skin depth (m)
γ	Decay length for inductance above plate (20 mm)
f_{osc}, ω_{osc}	Oscillation frequency of levitated body (Hz, radians/sec)