Power Electronics Notes 30H
Magnetic Fields from Power Cables
(Case Studies)

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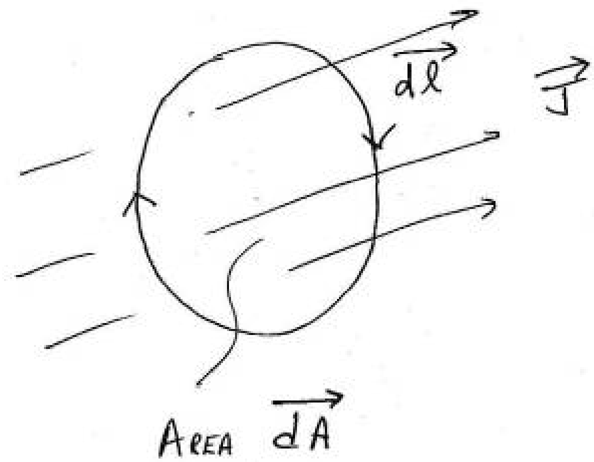
Ampere's Law

- Flowing current creates a magnetic field

$$\oint_C \vec{H} \cdot d\vec{l} = \int_S \vec{J} \cdot d\vec{A} + \frac{d}{dt} \int_S \varepsilon_o \vec{E} \cdot d\vec{A}$$



- In magnetic systems, generally there is high current and low voltage (and hence low electric field) and we can approximate for low *d/dt*:

$$\oint_C \vec{H} \cdot d\vec{l} \approx \int_S \vec{J} \cdot d\vec{A}$$

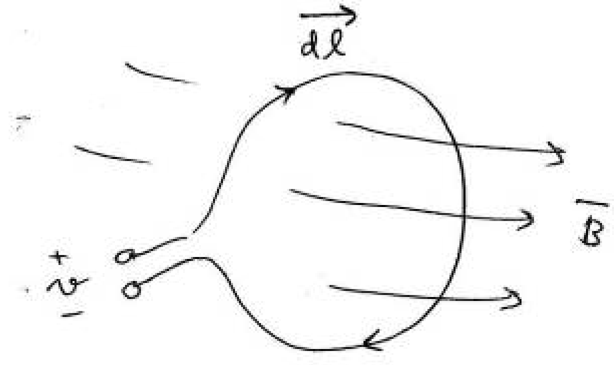
- In words: the magnetic flux density integrated around <u>any</u> closed contour equals the net current flowing through the surface bounded by the contour

André-Marie Ampère

Faraday's Law

- A changing magnetic flux impinging on a conductor creates an electric field and hence a current (eddy current)

$$\oint_C \vec{E} \cdot d\vec{l} = -\frac{d}{dt} \int_S \vec{B} \cdot d\vec{A}$$



- The electric field integrated around a closed contour equals the net time-varying magnetic flux density flowing through the surface bound by the contour
- In a conductor, this electric field creates a current by: $\vec{J} = \sigma \vec{E}$

Michael Faraday

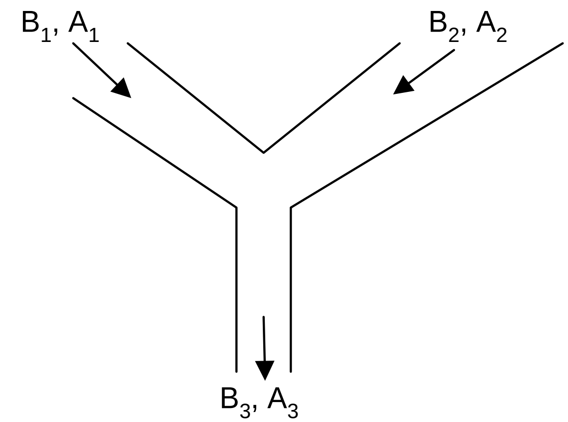
- Induction motors, brakes, etc.

Gauss' Magnetic Law

- Gauss' magnetic law says that the integral of the magnetic flux density over any closed surface is zero, or:

$$\oint_S B \cdot dA = 0$$

- This law implies that magnetic fields are due to electric currents and that magnetic charges ("monopoles") do not exist.
- Note: similar form to KCL in circuits ! (We'll use this analogy later…)



$$B_3 A_3 = B_1 A_1 + B_2 A_2$$

Carl Friedrich Gauss

Gauss' Law --- Continuity of Flux Lines

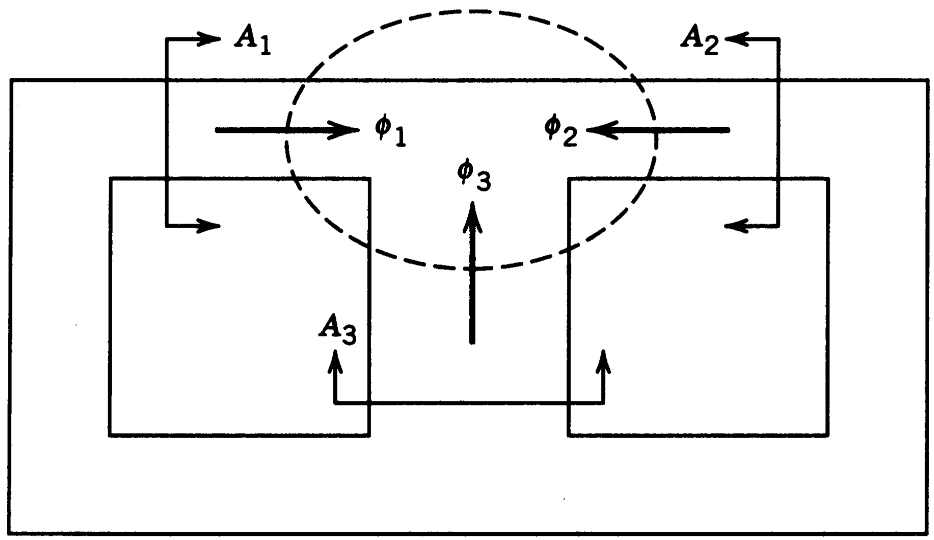


Figure 3-13 Continuity of flux.

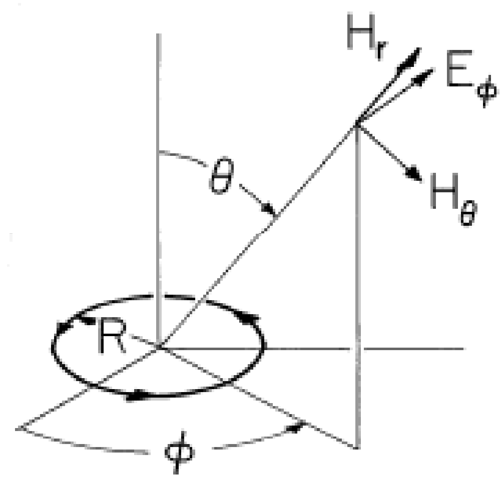
$$\phi_1 + \phi_2 + \phi_3 = 0$$

Finite-Element Analysis (FEA)

- Very useful tool for visualizing magnetic fields
- FEA is often used to simulate and predict the performance of motors, etc.
- Following we'll see some 2-dimensional (2D) FEA results to help explain Maxwell's equations

Example 1: Circular Current Loop (Dipole)

- Circular loop is a dipole with dipole moment IA (I = current, A = loop area)

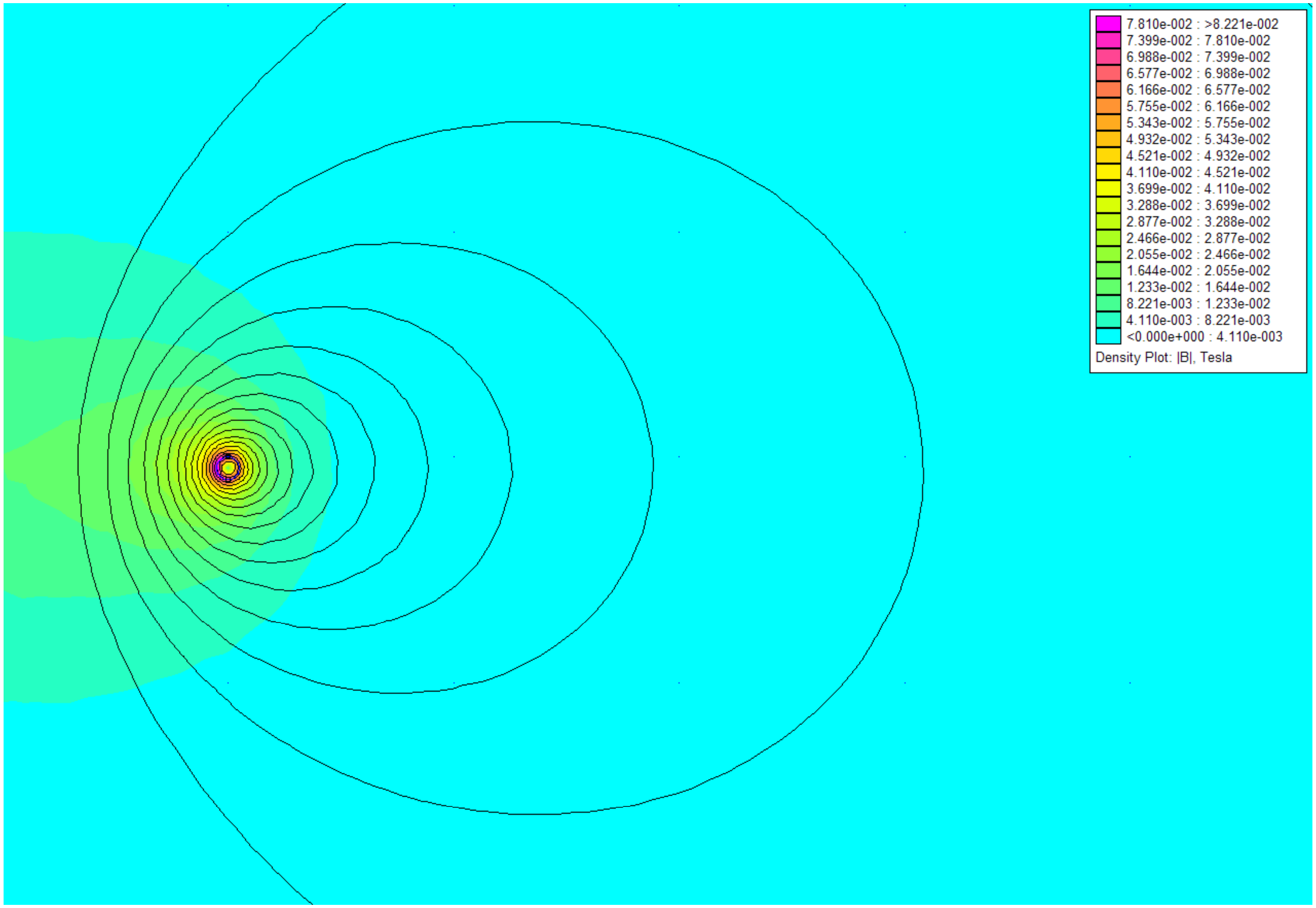


$$B_r = \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\sin\theta A_\phi = \frac{2AI}{c}\frac{\cos\theta}{r^3}$$

$$B_\theta = -\frac{1}{r}\frac{\partial}{\partial r}rA_\phi = \frac{AI}{c}\frac{\sin\theta}{r^3}.$$

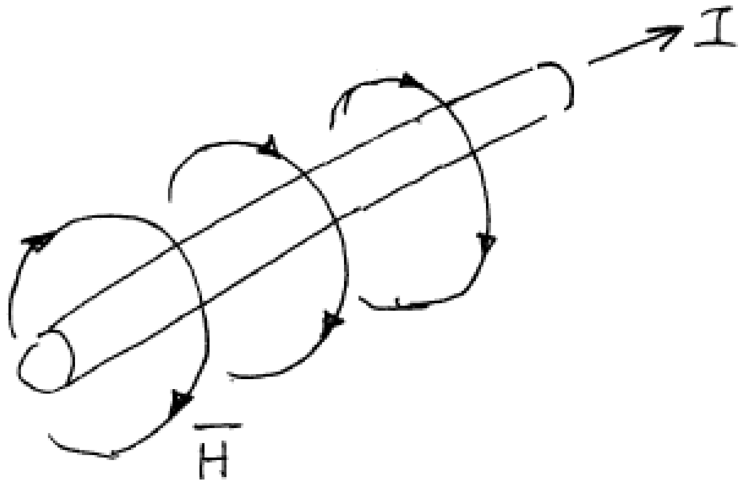
Example 1: Field From Current Loop, NI = 500 A-Turns

- Coil radius R = 1”; plot from 2D finite-element analysis



Example 2: Isolated Wire, Return Far Away

- This one is easily solved



By Ampere's Law: $\oint_C \bar{H} \cdot d\bar{l} = \int_S \bar{J} \cdot d\bar{A}$

or $2\pi r H_\phi = I$

$$H_\phi = \frac{I}{2\pi r}$$

$$B_\phi = \mu_0 H_\phi = \frac{\mu_0 I}{2\pi r}$$

$\mu_0 = 4\pi \times 10^{-7}$ H/m

(magnetic permeability of free space)

Example: $I = 15A$
$r = 1m$

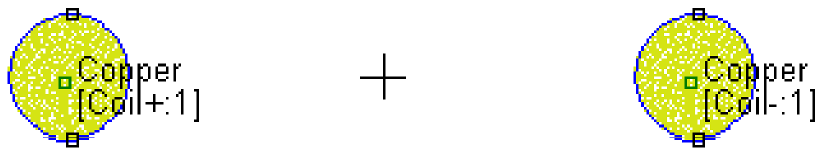
$$B_\phi = \frac{(4\pi \times 10^{-7})(15)}{2\pi(1)} = 3 \times 10^{-6} \text{ Tesla}$$

$= 0.03$ Gauss

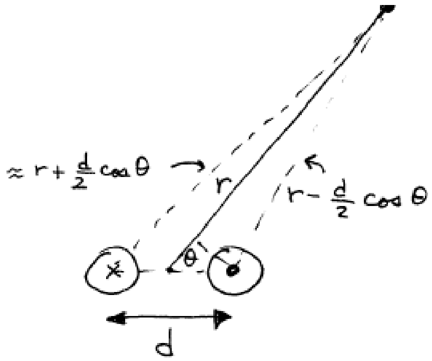
(Earth's magnetic flux density ≈ 0.5 Gauss)

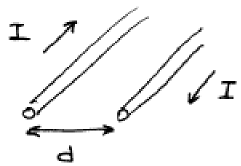
Example 3: Magnetic Dipole, 2D

- Wires are long into page
- What's the magnetic flux density ***B*** far away?
- Assume I = 15A; wire-wire spacing d = 10 mm; r_{wire}=2 mm



Example 3: Magnetic Dipole, 2D





By superposition

$$B \cong \frac{\mu_o I}{2\pi}\left[\frac{1}{r+\frac{d}{2}\cos\theta} - \frac{1}{r-\frac{d}{2}\cos\theta}\right]$$

Assume $r >> d$ (far away from wires)

$$B \cong \frac{\mu_o I}{2\pi r}\left[\frac{1}{1+\frac{d}{2r}\cos\theta} - \frac{1}{1-\frac{d}{2r}\cos\theta}\right]$$

Remember

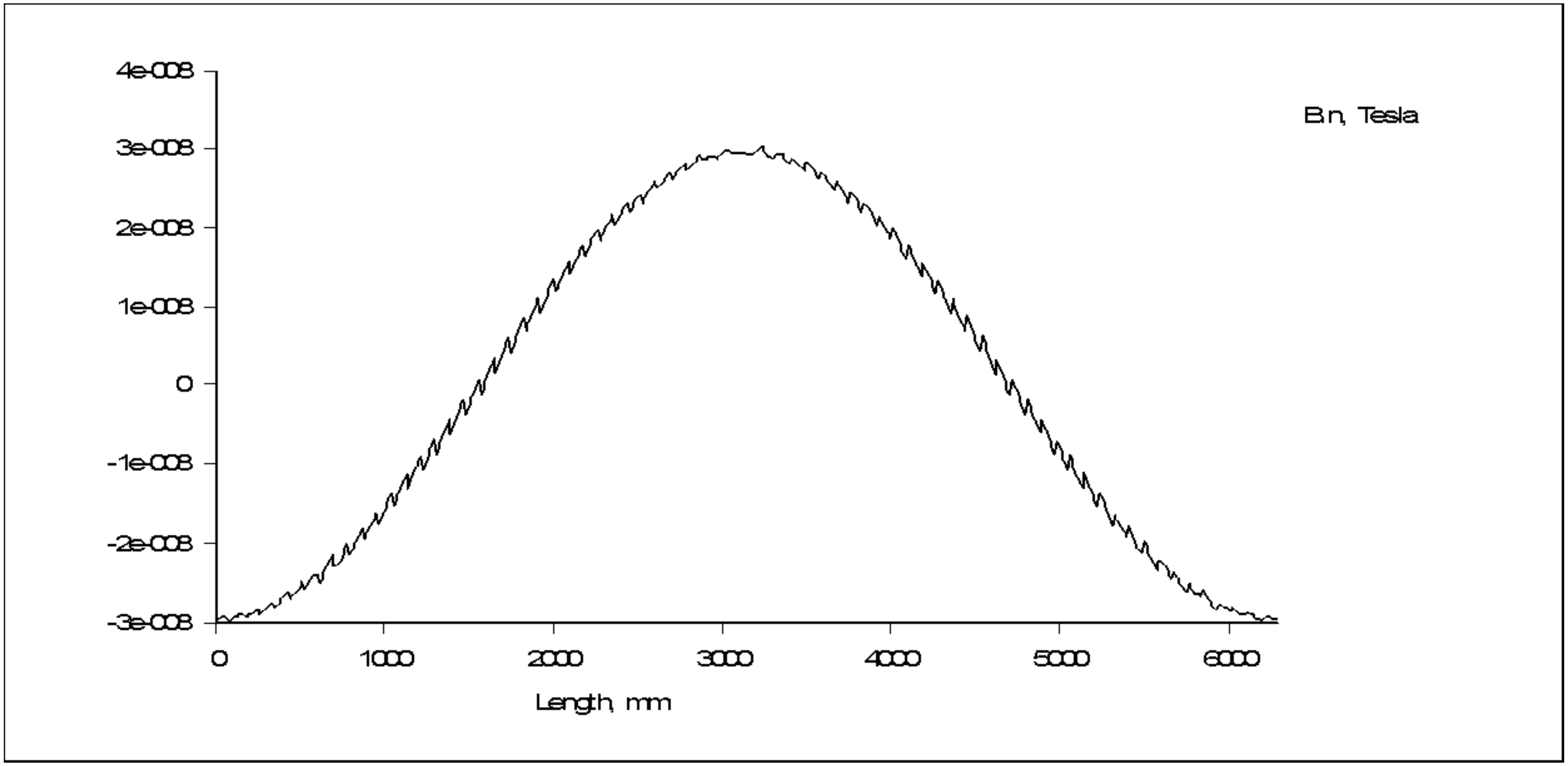
$$\frac{1}{1+x} \approx 1-x \quad \text{if} \quad x<<1$$

$$B \approx \frac{\mu_o I}{2\pi r}\left[\cancel{1} - \frac{d}{2r}\cos\theta - \left[\cancel{1} + \frac{d}{2r}\cos\theta\right]\right]$$

$$\therefore \bar{B} \approx \frac{-\mu_o I}{2\pi r}\frac{d}{r}\cos\theta \cong \frac{-\mu_o I d}{2\pi r^2}\cos\theta$$

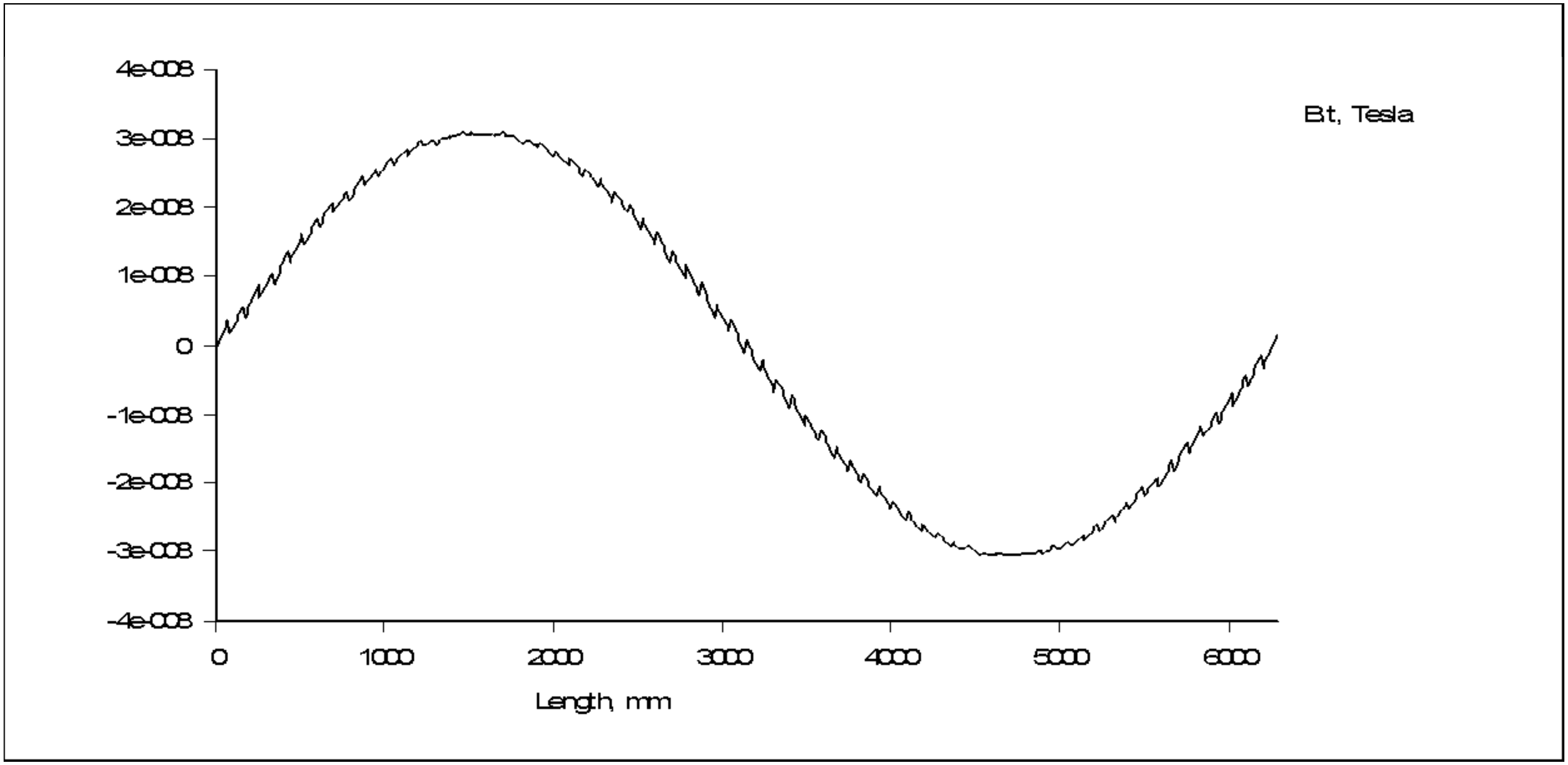
Example 3: Magnetic Dipole, 2D

- Field r = 1 meter away (r >> d), radial component



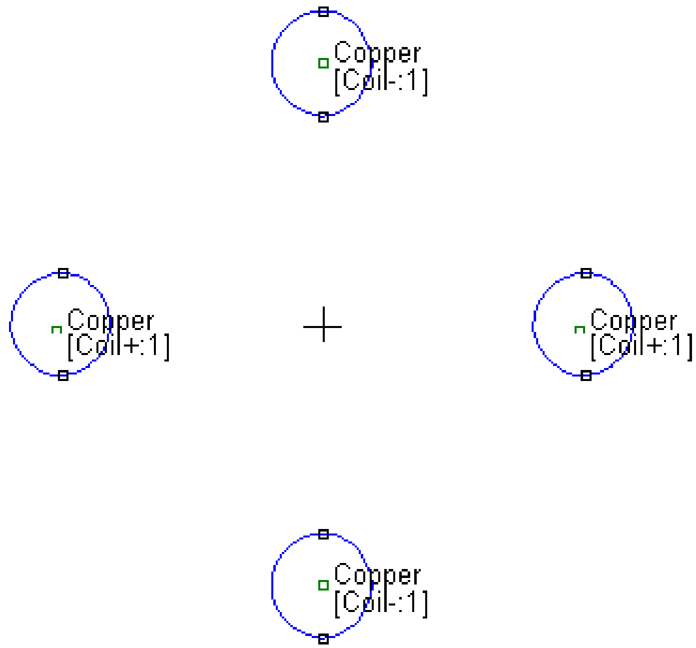
Example 3: Magnetic Dipole, 2D

- Field r = 1 meter away, theta component

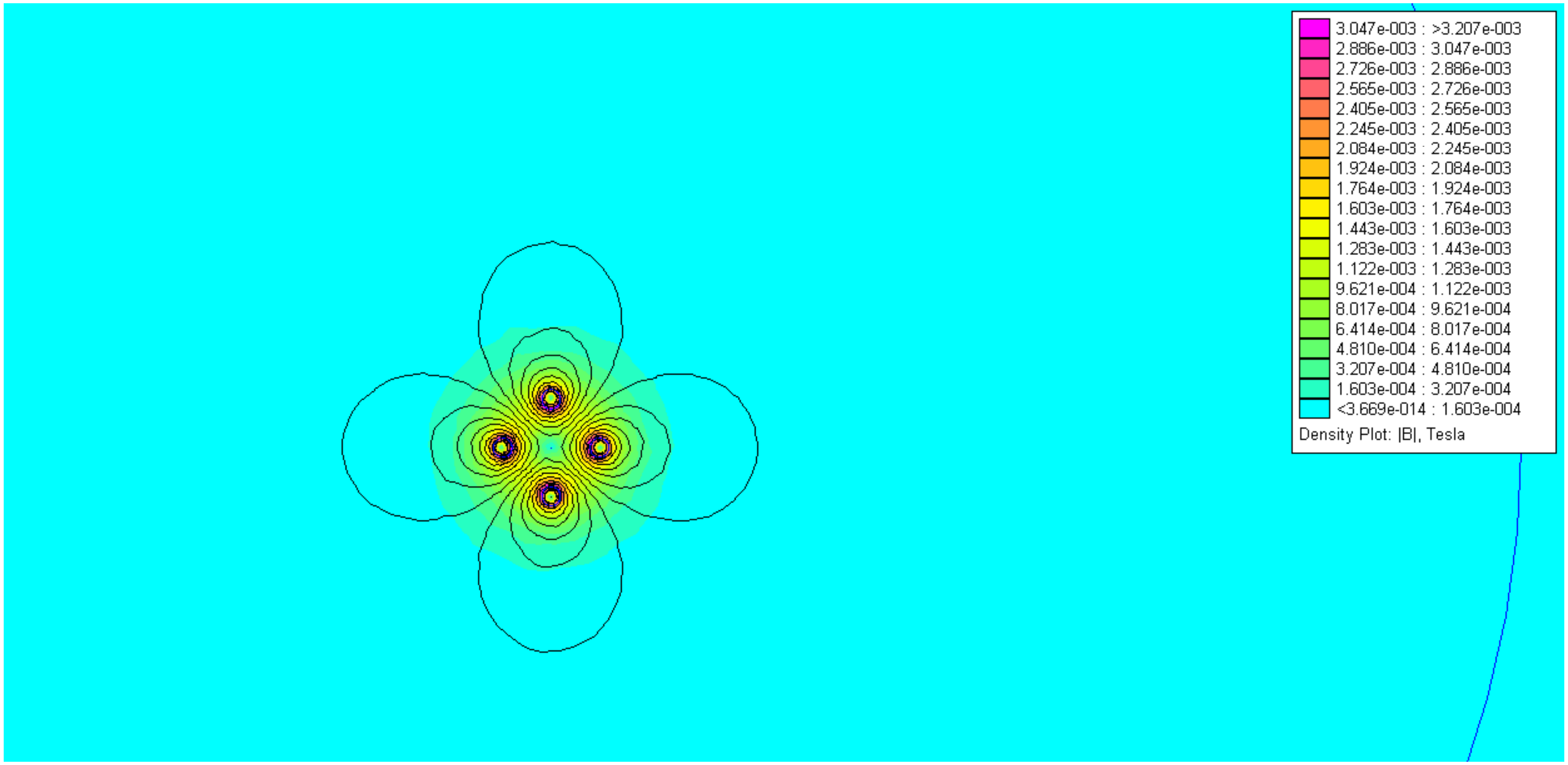


Example 4: Magnetic Quadrupole, 2D

- Assume I = 15A; d = 10 mm; r_{wire}=2 mm
- Note orientation of + and - currents

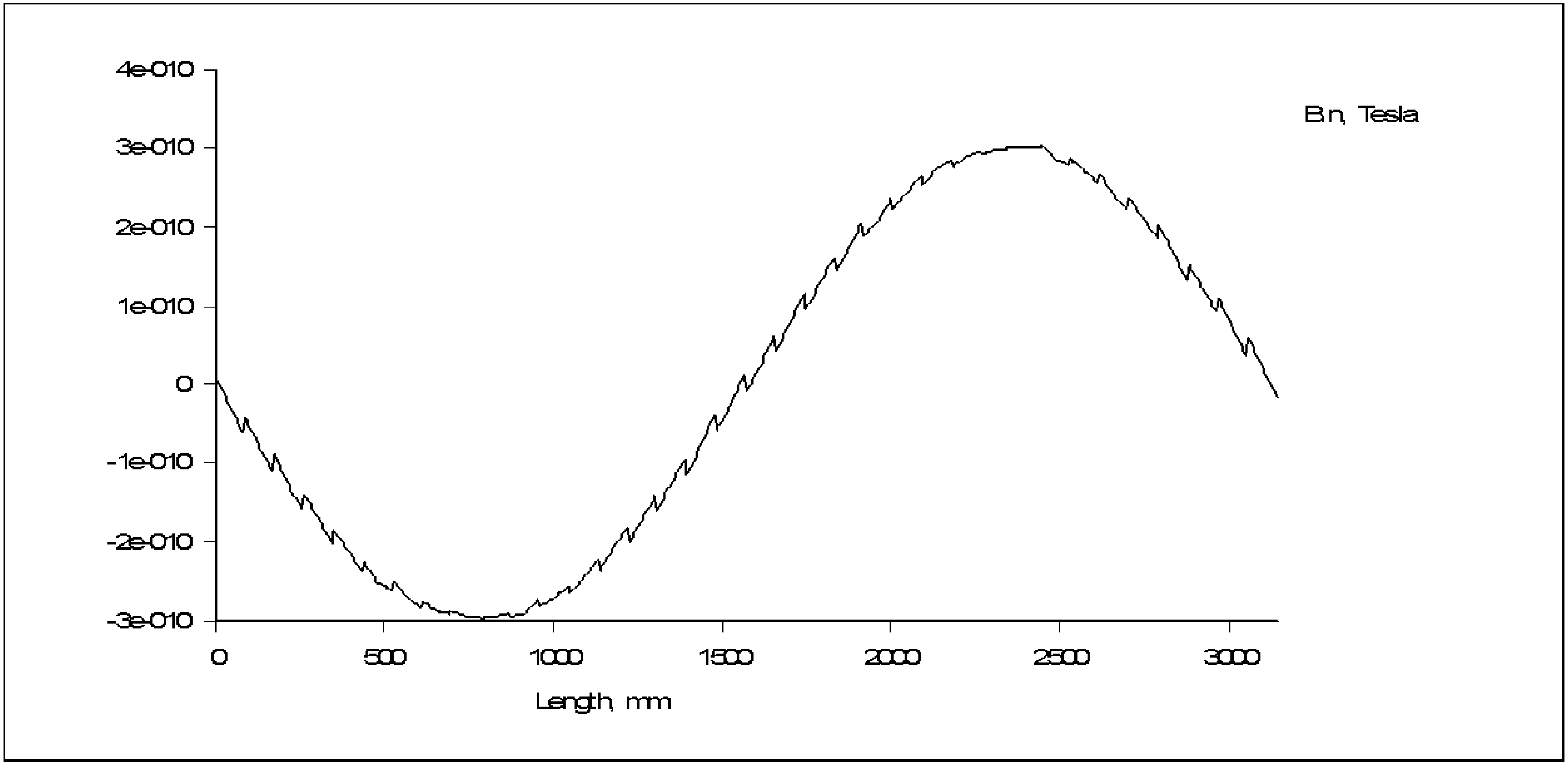


Example 4: Magnetic Quadrupole, 2D



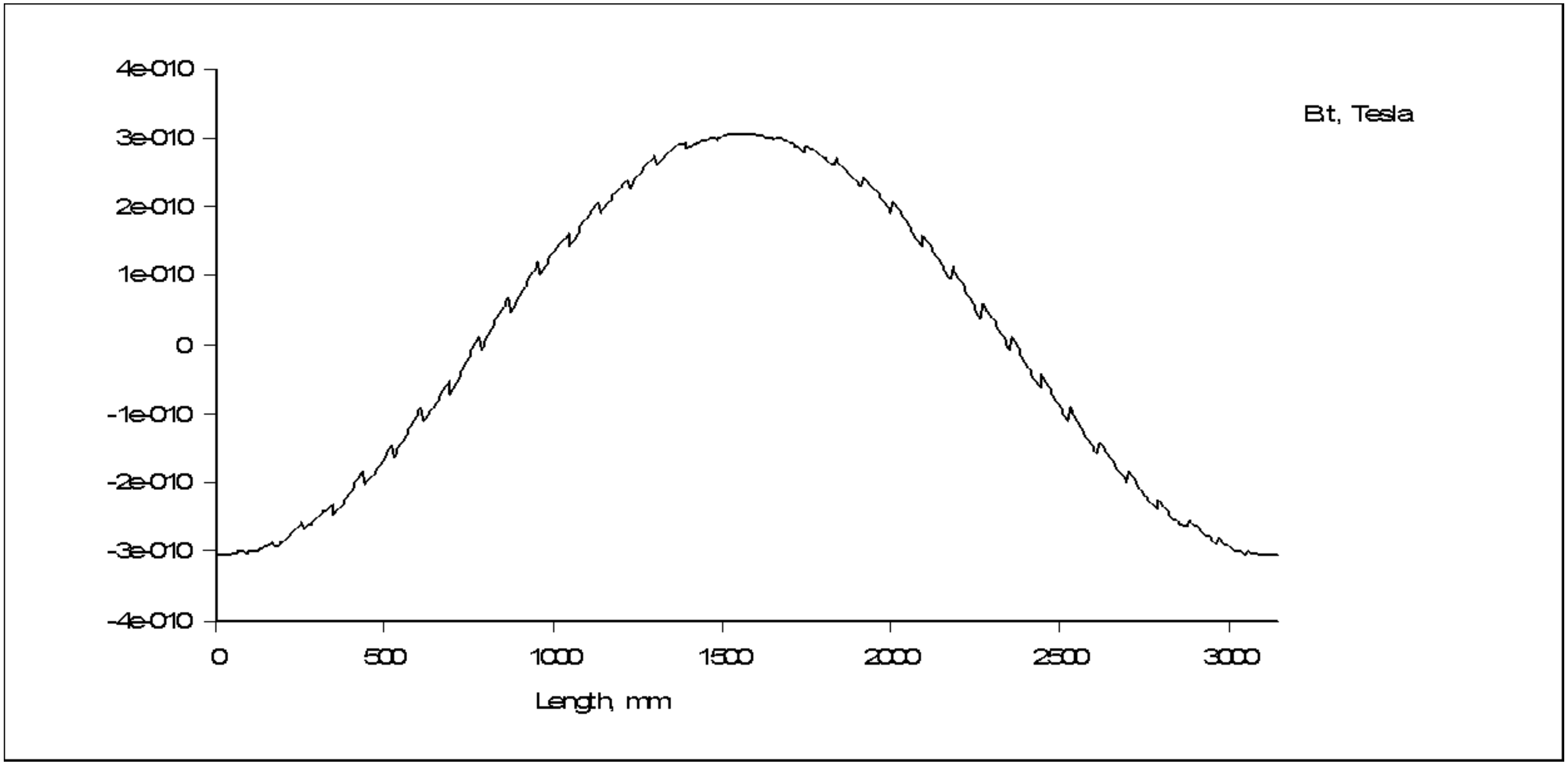
Example 4: Magnetic Quadrupole, 2D

- Field r = 1 meter away, radial component. Fields decay as $1/r^3$



Example 4: Magnetic Quadrupole, 2D

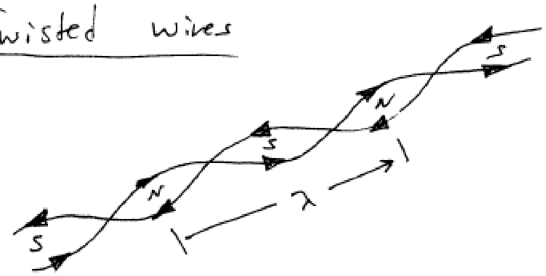
- Field r = 1 meter away, theta component. Fields decays as $1/r^3$



Example 5: Twisted Pair

- Field r = 1 meter away, theta component. Fields decays as e^{-kr}

Twisted wires



- λ is twisting wavelength
- Fields decay as e^{-kr} where

$$k \text{ ("wavenumber")} = \frac{2\pi}{\lambda}$$

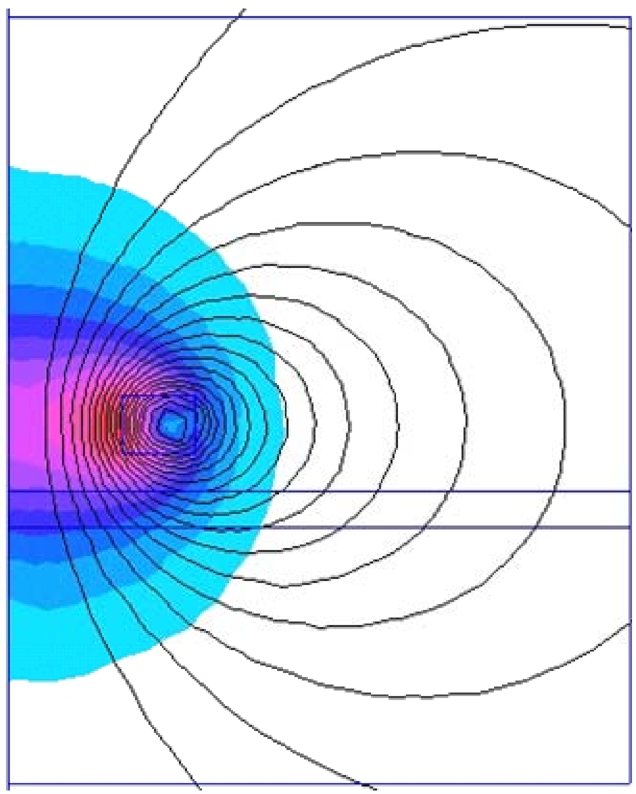
- So, the more twists you use per foot, the faster the field decays.
- Example $\lambda = 0.1$ m

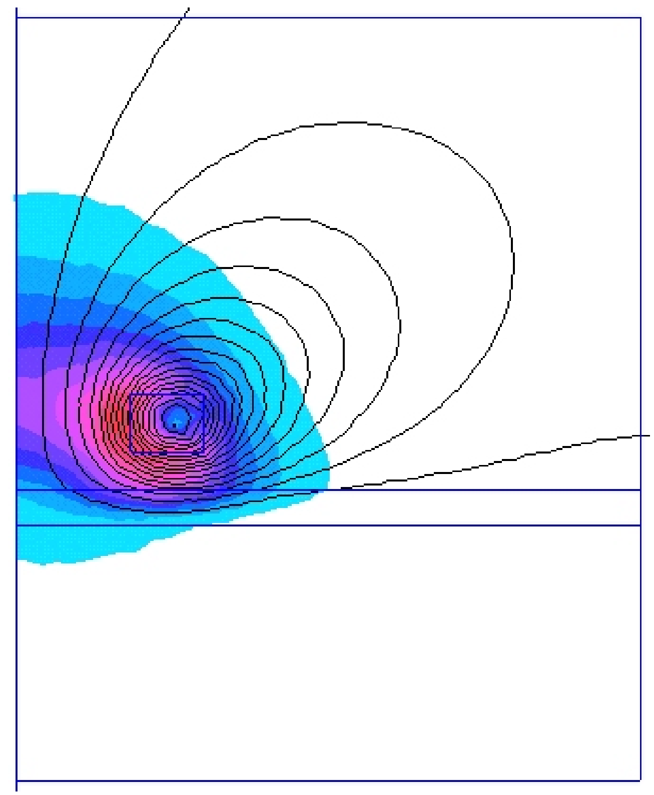
$$k = 2\pi/0.1 = 62.8$$

At r=1m, $e^{-kr} = e^{-62.8}$ ⇒ VERY SMALL NUMBER $\sim 10^{-28}$

AC Magnetic Shielding

- Flux density plots at DC and 60 Hz
- At 60 Hz, currents induced in plate via magnetic induction create lift force





Good Reference