Overview

- Review of Maxwell’s equations
- Skin effect
- Proximity effect
- Windings
  - Single layer and multiple-layer windings
- Dowell’s method for estimating AC losses
- Litz wire
- Core loss
- Steel
- Ferrites
Ampere’s Law

• Flowing current creates a magnetic field

\[ \oint_C H \cdot dl = \int_S J \cdot dA + \frac{d}{dt} \int_S \varepsilon_0 E \cdot dA \]

• In magnetic systems, generally there is high current and low voltage (and hence low electric field) and we can approximate for low \( d/dt \):

\[ \oint_C H \cdot dl \approx \int_S J \cdot dA \]

• The magnetic flux density integrated around a closed contour equals the net current flowing through the surface bounded by the contour
Field From Current Loop, NI = 500 A-turns

- Axisymmetric problem; coil radius R = 1"
Faraday’s Law

• A changing magnetic flux impinging on a conductor creates an electric field and hence a current (eddy current)

\[ \oint_{C} \vec{E} \cdot d\vec{l} = -\frac{d}{dt} \int_{S} \vec{B} \cdot d\vec{A} \]

• The electric field integrated around a closed contour equals the net time-varying magnetic flux density flowing through the surface bounded by the contour

• In a conductor, this electric field creates a current by:

\[ \vec{J} = \sigma \vec{E} \]

• Induction motors, brakes, etc. and eddy currents
Gauss’ Magnetic Law

- Gauss' magnetic law says that the integral of the magnetic flux density over any closed surface is zero, or:

\[ \oint_{\partial S} \vec{B} \cdot d\vec{A} = 0 \]

- This law implies that magnetic fields are due to electric currents and that magnetic charges ("monopoles") do not exist.

- Note: similar form to KCL in circuits! (We’ll use this analogy later…)

\[ B_3 A_3 = B_1 A_1 + B_2 A_2 \]
Skin Depth $\delta$

- At high frequency, magnetic fields penetrate only a finite depth into a conductor
  \[
  \delta = \sqrt{\frac{2}{\omega \mu \sigma}} = \sqrt{\frac{1}{\pi f \mu \sigma}} \quad \text{[meters]}
  \]

- A corollary to this is that current flow in a wire is in a thin layer near the surface of the wire, if $r_w >> \delta$ where $r_w =$ wire radius

- For copper at 75 °C, $\sigma \sim 5 \times 10^7 \ \Omega^{-1} \text{m}^{-1}$
  - $\delta \sim 9 \ \text{mm at 60 Hz, 0.23 mm at 100 kHz}$
Isolated Wire at Low Frequency

- Low frequency is if radius of the wire is much smaller than a skin depth
- The entire cross-sectional area of the wire is used

Example 1: 14 Gage Wire Go-And-Return

- Copper wire diameter = 1.6 mm; wire center-center spacing 10 mm. Current is +15A (RMS) in left wire, -15A in right wire
- Assume 1 meter deep into paper
Example 1: Current Density vs. Radius, 1 Hz

- 1 Hz, left wire. At 1 Hz, $\delta = 66 \text{ mm} >> r_w (0.8 \text{ mm})$ so no high frequency effects.
- Power dissipation in wire = 1.93 Watts; $R_{AC}/R_{DC} = 1$.
Example 1: Current Density vs. Radius, 60 Hz

- 60 Hz, left wire. At 60 Hz, $\delta = 8.5$ mm $>> r_w$
- Note that current density $J$ is still uniform across the wire
- Power dissipation = 1.94 Watts; $R_{AC}/R_{DC} = 1.005$
Example 1: Analysis for Small Skin Depth Limit

\[ R_{DC} = \frac{l}{\sigma(\pi r_w^2)} \]

\[ R_{AC} = \frac{l}{\sigma(\pi r_w^2 - \pi (r_w - \delta)^2)} = \frac{l}{\sigma \pi (2r_w \delta - \delta^2)} \]

\[ \frac{R_{AC}}{R_{DC}} = \frac{l}{\sigma \pi (2r_w \delta - \delta^2)} = \frac{r_w^2}{2r_w \delta - \delta^2} \]

For \( r_w = 0.8 \) mm and \( \delta = 0.21 \) mm:

\[ \frac{R_{AC}}{R_{DC}} \approx 2.19 \]
Example 1: Current Density vs. Radius, 100 kHz

- At 100 kHz, skin depth $\delta = 0.21$ mm $<< r_w$
- Power dissipation = 4.27 Watts; $R_{AC}/R_{DC} = 2.2$
- Note non-uniform current distribution in the wire
Isolated Wire at High Frequency

• Induced eddy current in the wire aids current flow at the outer radius of the wire, and opposes current flow at the center of the wire.

• Result: high frequency resistance of the wire is higher than the low frequency resistance. Resistance goes up as frequency goes up.

Isolated Tubular Conductor

Reference: B. Carsten, “Calculating High Frequency Conductor Losses in Switchmode Magnetics,” HFPC ’93
Isolated Rectangular/Round Conductor

\[ P = \frac{8 \pi f}{\sqrt{Rd_c} \times 10^9} \] in cycles, \( Rd_c \) = d-c resistance in ohms per cm of length

**Fig. 3.**—Resistance ratio of rectangular conductors in terms of parameter \( p \), involving the frequency \( f \) in cycles, and \( Rd_c \) the resistance in ohms per centimeter of length. These curves are a combination of experimental and calculated results, with the dotted portions representing low-frequency results extrapolated to join on with the theoretical results for high frequencies.

Reference: B. Carsten, “Calculating High Frequency Conductor Losses in Switchmode Magnetics,” *HFPC '93*
Inductor Winding with Single Layer

• This example has $N = 4$ turns in a single layer

Fig. 8 - Inductor Winding

Transformer Winding with Single Layers

- This example has $N = 4$ turns in a primary, $N = 1$ turn in a secondary

Multiple Winding Layers and the “Proximity Effect”

- Things get more complicated when you have multiple winding layers
- This is due to the “proximity effect”
- The field from one wire affects the current distribution in adjacent wires
Transformer Winding with Multiple Layers

- Low frequency MMF diagram
- Current is uniformly distributed through all conductors since the conductors are much thinner than a skin depth

Proximity Effect --- 1 Winding Layer

- Multiple layers cause losses to increase faster than might be expected via skin depth alone.
- Let’s assume 1 foil winding layer first, with winding thickness $t$. The layer carries 1 Amp (dot into the page).
Proximity Effect --- 2 Winding Layers

- 2 layer coil (left side of coil shown only) with small skin depth limit $t >> \delta$
- Field from winding 1 induces eddy current in winding 2
- Increase is 3x

\[
\begin{align*}
\text{CURRENT} & \quad 1 \quad -1 \quad 2 \\
\text{CURRENT}^2 & \quad 1 \quad 1 \quad 4 \\
\text{LOSS} & \quad 1 \quad 5 \\
\end{align*}
\]

Low Freq Loss: $I^2R_{dc} + I^2R_{dc}$
High Freq Loss: $I^2R_{dc} + 5I^2R_{dc}$

(3x INCREASE)
Proximity Effect --- 3 Winding Layers

- 3 layer coil (left side of coil shown only)
- Increase is \((19/3)\times\)
Proximity Effect

- By the Dowell approximation, we can show that at high frequencies \( t \gg \delta \) the total winding resistance as compared to the low frequency resistance are:

\[
F_p = \frac{R_{AC}}{R_{DC}} \approx \left( \frac{t}{\delta} \right) \left( \frac{2M^2 + 1}{3} \right)
\]

<table>
<thead>
<tr>
<th>( M )</th>
<th>( \frac{R_{AC}}{R_{DC}} \approx \left( \frac{t}{\delta} \right) \left( \frac{2M^2 + 1}{3} \right) ) for ( t \gg \delta )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( \left( \frac{t}{\delta} \right) )</td>
</tr>
<tr>
<td>2</td>
<td>( 3 \left( \frac{t}{\delta} \right) )</td>
</tr>
<tr>
<td>3</td>
<td>( \frac{19}{3} \left( \frac{t}{\delta} \right) )</td>
</tr>
<tr>
<td>4</td>
<td>( 11 \left( \frac{t}{\delta} \right) )</td>
</tr>
<tr>
<td>5</td>
<td>( 17 \left( \frac{t}{\delta} \right) )</td>
</tr>
<tr>
<td>6</td>
<td>( \frac{73}{3} \left( \frac{t}{\delta} \right) )</td>
</tr>
<tr>
<td>7</td>
<td>( 33 \left( \frac{t}{\delta} \right) )</td>
</tr>
<tr>
<td>8</td>
<td>( 43 \left( \frac{t}{\delta} \right) )</td>
</tr>
<tr>
<td>9</td>
<td>( \frac{163}{3} \left( \frac{t}{\delta} \right) )</td>
</tr>
<tr>
<td>10</td>
<td>( 67 \left( \frac{t}{\delta} \right) )</td>
</tr>
<tr>
<td>11</td>
<td>( 81 \left( \frac{t}{\delta} \right) )</td>
</tr>
</tbody>
</table>
Proximity + Skin Effect --- Dowell Method

- In the Dowell method, round wires are approximated by an equivalent foil, and a 1D solution is done.

Reference: B. Carsten, "Calculating High Frequency Conductor Losses in Switchmode Magnetics," HFPC '93
Proximity + Skin Effect --- Dowell Method


Reference: B. Carsten, “Calculating High Frequency Conductor Losses in Switchmode Magnetics,” *HFPC ’93*
Proximity + Skin Effect --- Dowell Method

- This is a general result (but 1D), over all range of wire diameters and d/δ
- The total resistance increase is given by:

\[
F_p = \frac{R_{AC}}{R_{DC}} = \varphi \left[ G_1(\varphi) + \left( \frac{2}{3} \right) \left( M^2 - 1 \right) (G_1(\varphi) - 2G_2(\varphi)) \right]
\]

\[
\varphi = \sqrt{\eta} \left( \frac{\sqrt{\pi}}{2} \right) \left( \frac{d}{\delta} \right)
\]

\[
G_1(\varphi) = \frac{\sinh(2\varphi) + \sin(2\varphi)}{\cosh(2\varphi) - \cos(2\varphi)}
\]

\[
G_2(\varphi) = \frac{\sinh(\varphi)\cos(\varphi) + \cosh(\varphi)\sin(\varphi)}{\cosh(2\varphi) - \cos(2\varphi)}
\]
Proximity + Skin Effect --- Dowell Method

- Note that for tiny wires $F_p \to 1$
Proximity + Skin Effect --- Dowell Method

Calculates $R_{ac}/R_{dc}$ vs. $t$/skindepth and number of winding layers $M$

$F_p = R_{ac}/R_{dc}$

$\phi = \text{eff layer thickness}/\text{skindep}$
function proxy

% Calculates proximity and skin effect losses in windings using Dowell method
% Define a = d/skindepth where d is wire diameter
% M = number of winding layers
% Reference, Brickson, 2nd edition, pp. 500-522

clear all
for M=1:10
    a=0.1:0.01:2;
    phi=sqrt(pi)*a/2;
    G1=(sinh(2*phi)+sin(2*phi))./(cosh(2*phi)-cos(2*phi));
    G2=(sinh(phi).*cos(phi)+cosh(phi).*sin(phi))./(cosh(2*phi)-cos(2*phi));
    Fp=phi.*(G1+(2/3)*(M^2-1)*(G1-2*G2));
    semilogy(a,Fp,'k')
    text(1.5,Fp(round(length(Fp)*0.75)),['M=',num2str(M)])
hold on
end
|
title('Calculates Rac/Rdc vs. d/skindepth and number of winding layers M')
xlabel('Wire diam/skin depth')
ylabel('Rac/Rdc')
grid; hold off;
Example 2: Using Dowell’s Method

- N=490, 24 gage wire, 75 kHz
- 7 layers, 70 turns each
- Skin depth $\delta = 0.26$ mm
- Wire diam. $d = 0.51$ mm
- $\eta = 0.7910$
- $\varphi = 1.5461$
- $N = 7$, $\phi = 1.5461$, Dowell predicts $F_p = 26$
Example 2: FEA Result

- $R_{dc} = 4.8 \, \Omega$
- $R_{ac} = 73.4 \, \Omega$
- $F_p = R_{ac}/R_{dc} = 15.3$ from FEA
Optimum Winding Thickness vs. Number of Layers

- Dowell’s method shows that optimal winding thickness is phi (φ) near or a little less than a skin depth. Note: \( \phi = \text{eff layer thickness/skin dep} \)
Comment on Dowell Method

- Note that it’s a 1D approximation to a 2D or 3D problem
- Seems to do better for tall coils
How to Keep AC Losses Low

• Conductor thickness must be small to keep AC losses to a minimum
• Keep number of winding layers low
• Sometimes foil is used for low voltage, high current windings
• In transformers, you can interleave the windings
• If foil is not appropriate, multiple conductors can be used. Litz wire is the name for multiple, twisted strands
Litz Wire

• Multiple small conductors
• Disadvantages
  – Cost
  – Reduced copper fill factor
  – Manufacturing
The term Litz wire is derived from the German word *Litzenband*, meaning woven wire. Generally defined, it is a wire constructed of individual film insulated wires bunched or braided together in a uniform pattern of twists and length of lay.

The multistrand configuration minimizes the power losses otherwise encountered in a solid conductor due to the "skin effect", or the tendency of radio frequency current to be concentrated at the surface of the conductor.
Litz Wire Designation

• For instance, type 100/40 Litz has 100 paralleled strands of #40 AWG wire
## Core Loss

<table>
<thead>
<tr>
<th>Core type</th>
<th>$B_{\text{sat}}$</th>
<th>Relative core loss</th>
<th>Applications</th>
</tr>
</thead>
<tbody>
<tr>
<td>Laminations</td>
<td>1.5 - 2.0 T</td>
<td>high</td>
<td>50-60 Hz transformers, inductors</td>
</tr>
<tr>
<td>iron, silicon steel</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Powdered cores</td>
<td>0.6 - 0.8 T</td>
<td>medium</td>
<td>1 kHz transformers, 100 kHz filter inductors</td>
</tr>
<tr>
<td>powdered iron, molypermalloy</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ferrite</td>
<td>0.25 - 0.5 T</td>
<td>low</td>
<td>20 kHz - 1 MHz transformers, ac inductors</td>
</tr>
<tr>
<td>Manganese-zinc, Nickel-zinc</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Core Loss

• From purely first-order analysis, we can see that eddy current losses scale as $B^2f^2$ and hysteresis power loss increases proportionally with frequency. In practice, measurements are done on magnetic cores to determine the functional dependence. Manufacturers sometimes provide curve fit of the form:

$$P_L = Af^\alpha (\Delta B)^\beta$$

with curve fit parameters $\alpha$ is usually in the range of $1.0 \sim 2.0$ and $\beta$ is in the range $2.0 \sim 3.0$, but this varies from material to material.
Core Loss for 3F3

![Core Loss Graph](image)

**Fig. 6** Specific power loss as a function of peak flux density with frequency as a parameter.

Fig. 22  Performance factor \((f \times B_{\text{max}})\) at \(P_V = 500\, \text{mW/cm}^3\) as a function of frequency for power ferrite materials.
Example 3: From IEEE Paper

\[ p_C = K1 \cdot f^{K2} \cdot B^{K3} \]  

where

- \( p_C \) [kW/m\(^3\)] — core power loss density
- \( f \) [Hz] — frequency
- \( B \) [T] — flux density
- \( K1, K2, \) and \( K3 \) — curve fitting formula constants.

The values of \( K1, K2, \) and \( K3 \) for the FERROXCUBE ferrite grades at core temperature 100°C, which provide an approximation accuracy about 20%, can be found in [17]. For the case of 3C80 grade, these values are \( K1 = 16.7, K2 = 1.3, K3 = 2.5. \)

**Example 4: From Magnetics, Inc.**

Table 3: Power Loss

<table>
<thead>
<tr>
<th>Material</th>
<th>Frequency</th>
<th>a</th>
<th>c</th>
<th>d</th>
</tr>
</thead>
<tbody>
<tr>
<td>K</td>
<td>f&lt;500 kHz</td>
<td>0.0530</td>
<td>1.60</td>
<td>3.15</td>
</tr>
<tr>
<td></td>
<td>500 kHz≤f&lt;1 MHz</td>
<td>0.00113</td>
<td>2.19</td>
<td>3.10</td>
</tr>
<tr>
<td></td>
<td>f≥1 MHz</td>
<td>1.77*10^-9</td>
<td>4.13</td>
<td>2.98</td>
</tr>
<tr>
<td>R</td>
<td>f&lt;100 kHz</td>
<td>0.074</td>
<td>1.43</td>
<td>2.85</td>
</tr>
<tr>
<td></td>
<td>100 kHz≤f&lt;500 kHz</td>
<td>0.036</td>
<td>1.64</td>
<td>2.68</td>
</tr>
<tr>
<td></td>
<td>f≥500 kHz</td>
<td>0.014</td>
<td>1.84</td>
<td>2.2</td>
</tr>
<tr>
<td>P</td>
<td>f&lt;100 kHz</td>
<td>0.158</td>
<td>1.36</td>
<td>2.86</td>
</tr>
<tr>
<td></td>
<td>100 kHz≤f&lt;500 kHz</td>
<td>0.0434</td>
<td>1.63</td>
<td>2.62</td>
</tr>
<tr>
<td></td>
<td>f≥500 kHz</td>
<td>7.36*10^-7</td>
<td>3.47</td>
<td>2.54</td>
</tr>
<tr>
<td>F</td>
<td>f≤10 kHz</td>
<td>0.790</td>
<td>1.06</td>
<td>2.85</td>
</tr>
<tr>
<td></td>
<td>10 kHz≤f&lt;100 kHz</td>
<td>0.0717</td>
<td>1.72</td>
<td>2.66</td>
</tr>
<tr>
<td></td>
<td>100 kHz≤f&lt;500 kHz</td>
<td>0.0573</td>
<td>1.66</td>
<td>2.68</td>
</tr>
<tr>
<td></td>
<td>f≥500 kHz</td>
<td>0.0126</td>
<td>1.88</td>
<td>2.29</td>
</tr>
<tr>
<td>J</td>
<td>f&lt;20 kHz</td>
<td>0.245</td>
<td>1.39</td>
<td>2.50</td>
</tr>
<tr>
<td></td>
<td>f&gt;20 kHz</td>
<td>0.00458</td>
<td>2.42</td>
<td>2.50</td>
</tr>
<tr>
<td>W</td>
<td>f&lt;20 kHz</td>
<td>0.300</td>
<td>1.26</td>
<td>2.00</td>
</tr>
<tr>
<td></td>
<td>f&gt;20 kHz</td>
<td>0.00382</td>
<td>2.32</td>
<td>2.62</td>
</tr>
<tr>
<td>H</td>
<td>f&lt;20 kHz</td>
<td>0.148</td>
<td>1.50</td>
<td>2.25</td>
</tr>
<tr>
<td></td>
<td>f&gt;20 kHz</td>
<td>0.135</td>
<td>1.62</td>
<td>2.15</td>
</tr>
</tbody>
</table>

**FORMAT:** $P_L = a f^c B^d$


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High Frequency Losses in Magnetics
References

• L. H. Dixon, “Eddy Current Losses in Transformer Windings and Circuit Wiring” available from Unitrode
• B. Carsten, “Calculating High Frequency Conductor Losses in Switchmode Magnetics,” HFPC ’93