Power Electronics Notes 30E
High Frequency Losses in Magnetics

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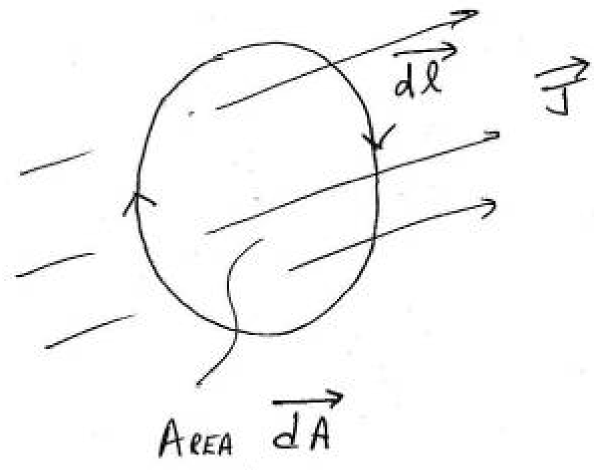
Overview

- Review of Maxwell's equations
- Skin effect
- Proximity effect
- Windings
  - Single layer and multiple-layer windings
- Dowell's method for estimating AC losses
- Litz wire
- Core loss
- Steel
- Ferrites

Ampere's Law

- Flowing current creates a magnetic field

$$\oint_C H \cdot dl = \int_S J \cdot dA + \frac{d}{dt}\int_S \varepsilon_o E \cdot dA$$



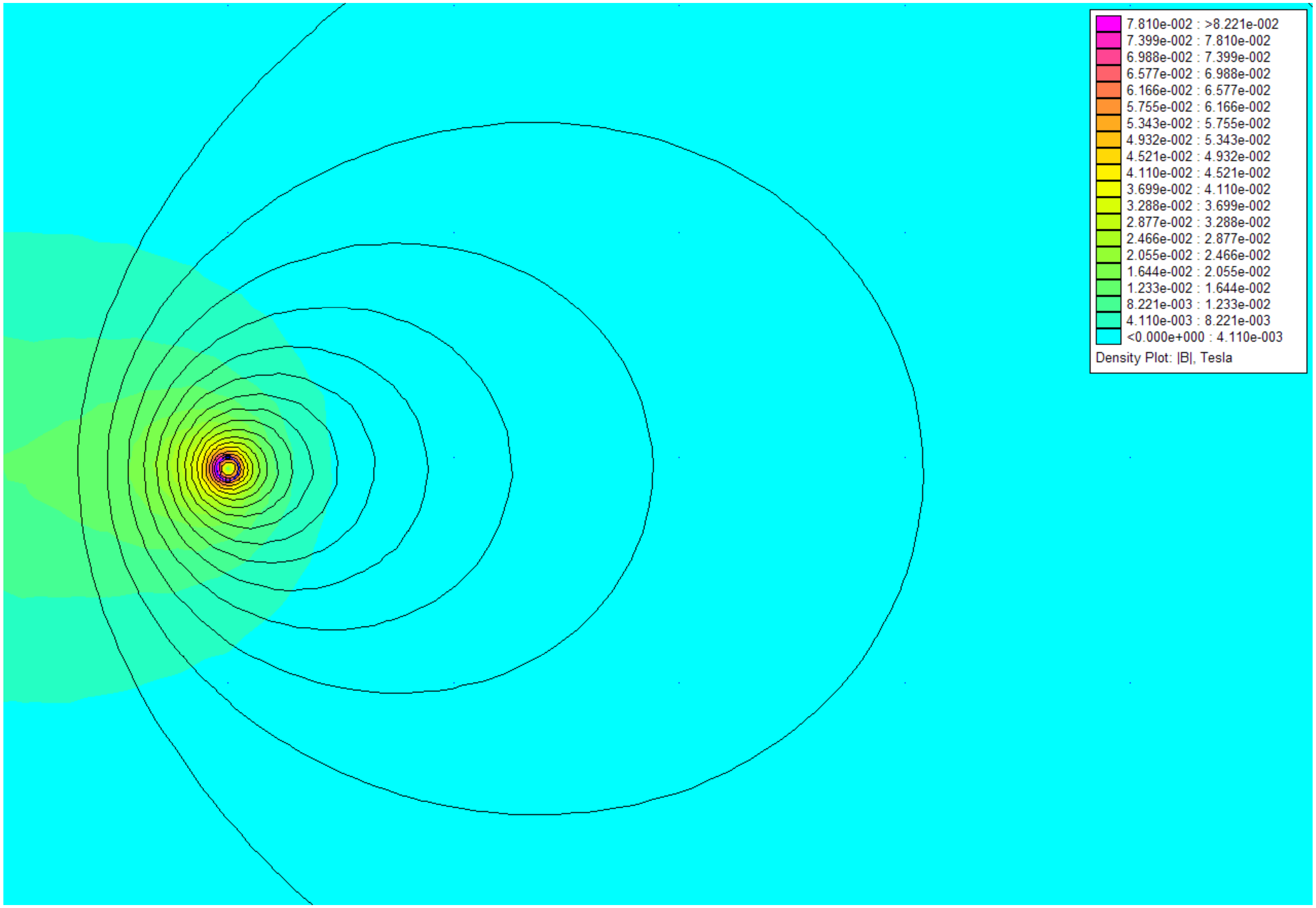
- In magnetic systems, generally there is high current and low voltage (and hence low electric field) and we can approximate for low *d/dt*:

$$\oint_C H \cdot dl \approx \int_S J \cdot dA$$

- The magnetic flux density integrated around a closed contour equals the net current flowing through the surface bounded by the contour

Field From Current Loop, NI = 500 A-turns

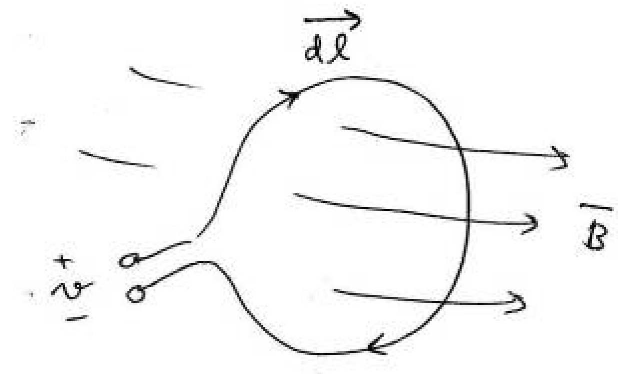
- Axisymmetric problem; coil radius R = 1”



Faraday's Law

- A changing magnetic flux impinging on a conductor creates an electric field and hence a current (eddy current)

$$\oint_C \vec{E} \cdot d\vec{l} = -\frac{d}{dt}\int_S \vec{B} \cdot d\vec{A}$$



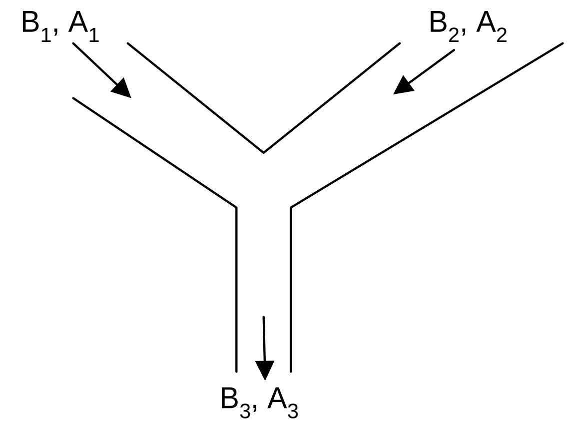
- The electric field integrated around a closed contour equals the net time-varying magnetic flux density flowing through the surface bounded by the contour
- In a conductor, this electric field creates a current by: $\vec{J} = \sigma \vec{E}$
- Induction motors, brakes, etc. and eddy currents

Gauss' Magnetic Law

- Gauss' magnetic law says that the integral of the magnetic flux density over any closed surface is zero, or:

$$\oint_S \vec{B} \cdot d\vec{A} = 0$$

- This law implies that magnetic fields are due to electric currents and that magnetic charges ("monopoles") do not exist.



$$B_3 A_3 = B_1 A_1 + B_2 A_2$$

- Note: similar form to KCL in circuits!

(We'll use this analogy later…)

Skin Depth δ

- At high frequency, magnetic fields penetrate only a finite depth into a conductor

$$\delta = \sqrt{\frac{2}{\omega\mu\sigma}} = \sqrt{\frac{1}{\pi f \mu\sigma}} \qquad \text{[meters]}$$

- A corollary to this is that current flow in a wire is in a thin layer near the surface of the wire, if $r_w >> \delta$ where r_w = wire radius
- For copper at 75 °C, $\sigma \sim 5 \times 10^7\ \Omega^{-1}m^{-1}$
  - $\delta \sim$ 9 mm at 60 Hz, 0.23 mm at 100 kHz

Isolated Wire at Low Frequency

- Low frequency is if radius of the wire is much smaller than a skin depth
- The entire cross-sectional area of the wire is used

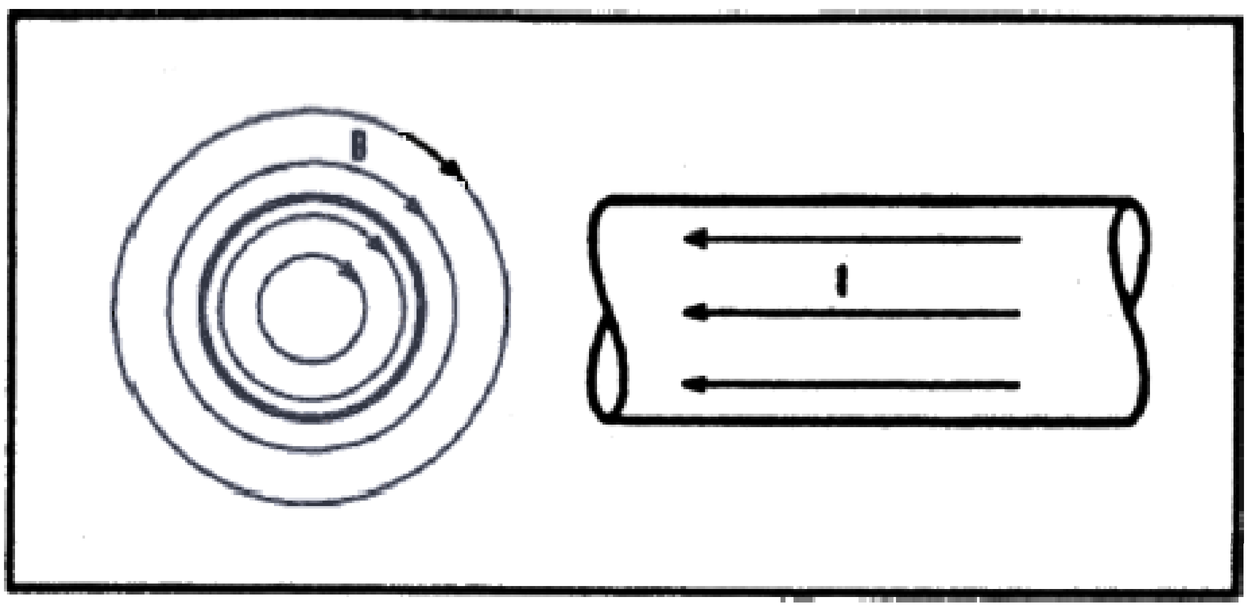
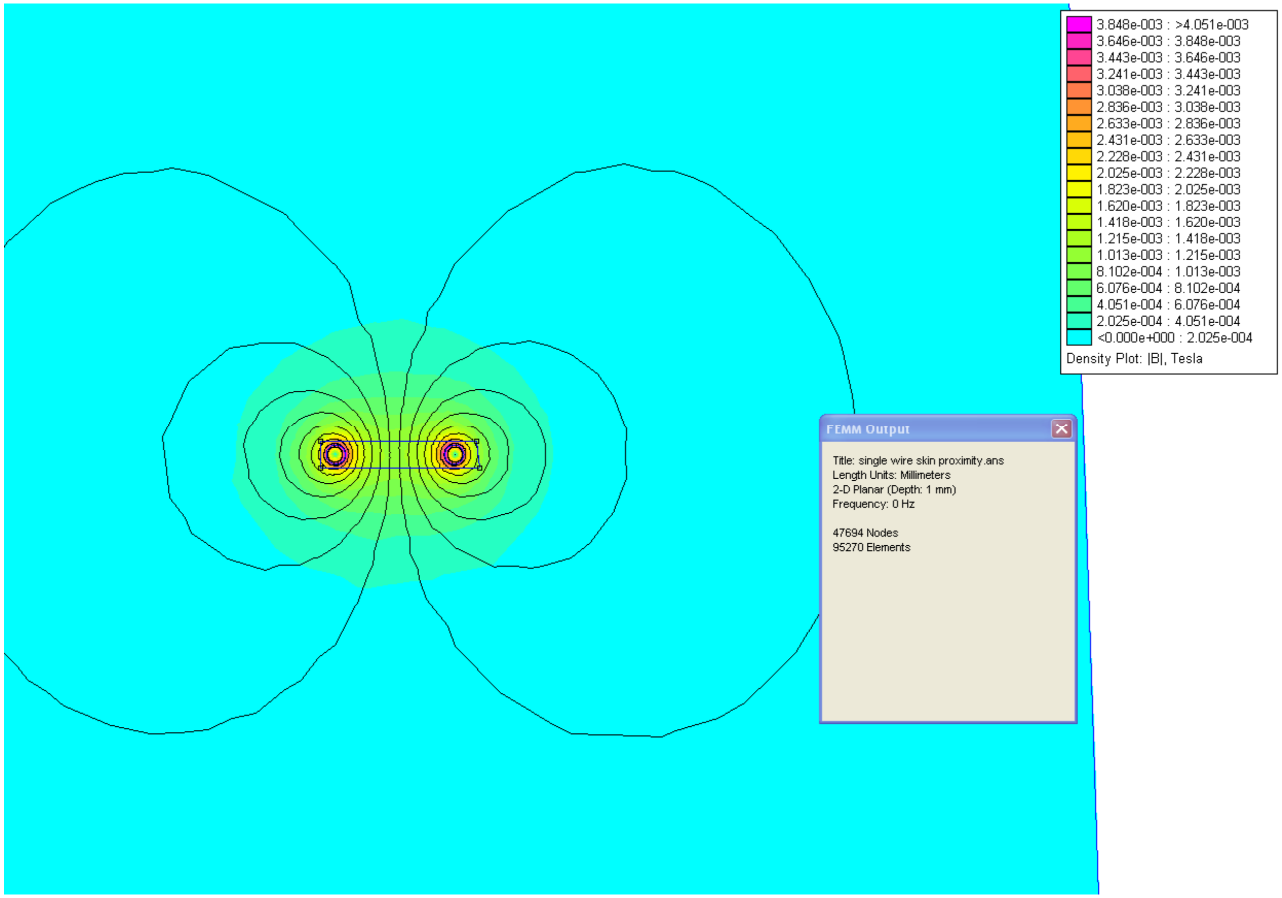


Fig. 1 - Isolated Conductor at Low Frequency

Reference: L. H. Dixon, "Eddy Current Losses in Transformer Windings and Circuit Wiring"

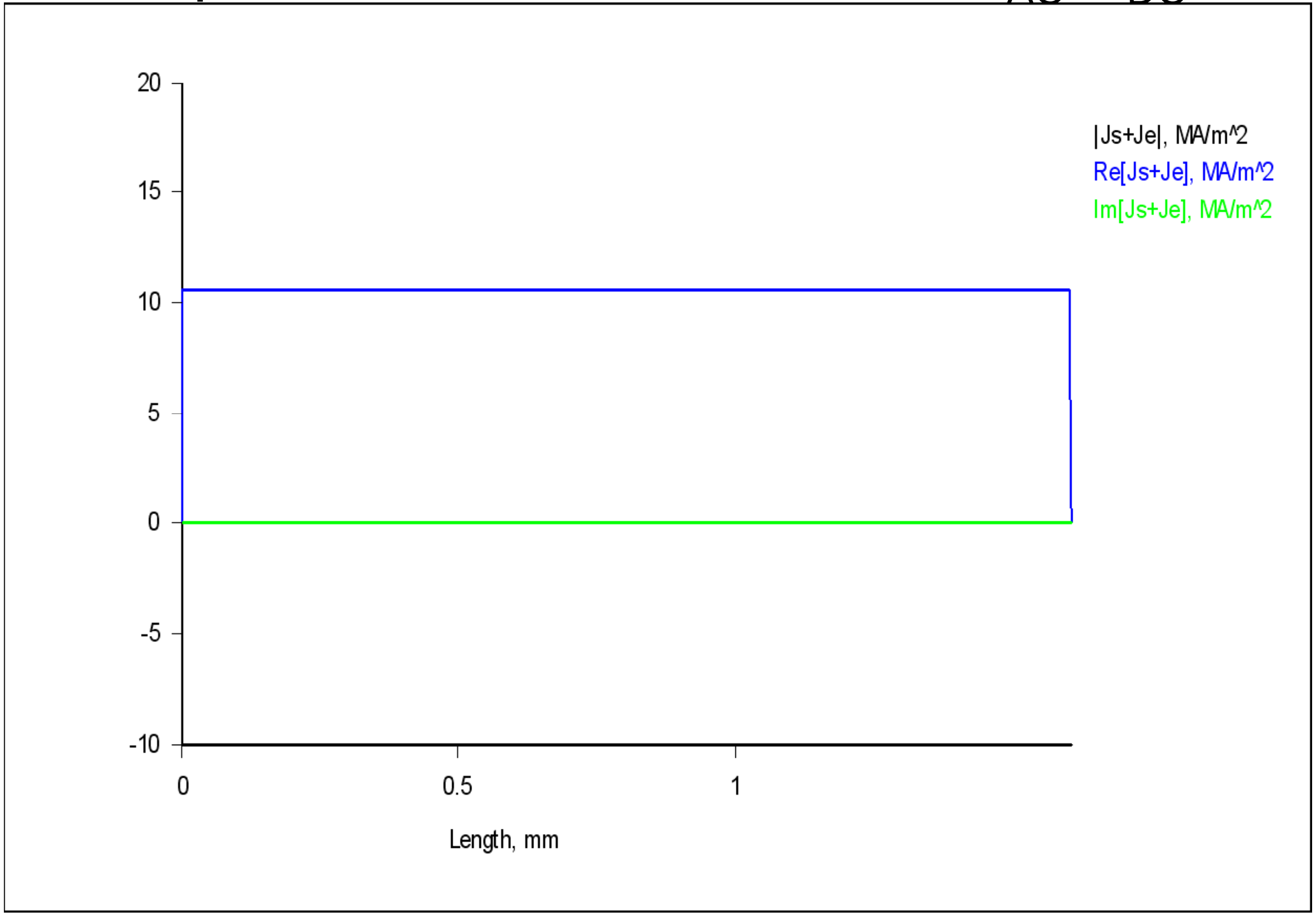
Example 1: 14 Gage Wire Go-And-Return

- Copper wire diameter = 1.6 mm; wire center-center spacing 10 mm. Current is +15A (RMS) in left wire, -15A in right wire
- Assume 1 meter deep into paper



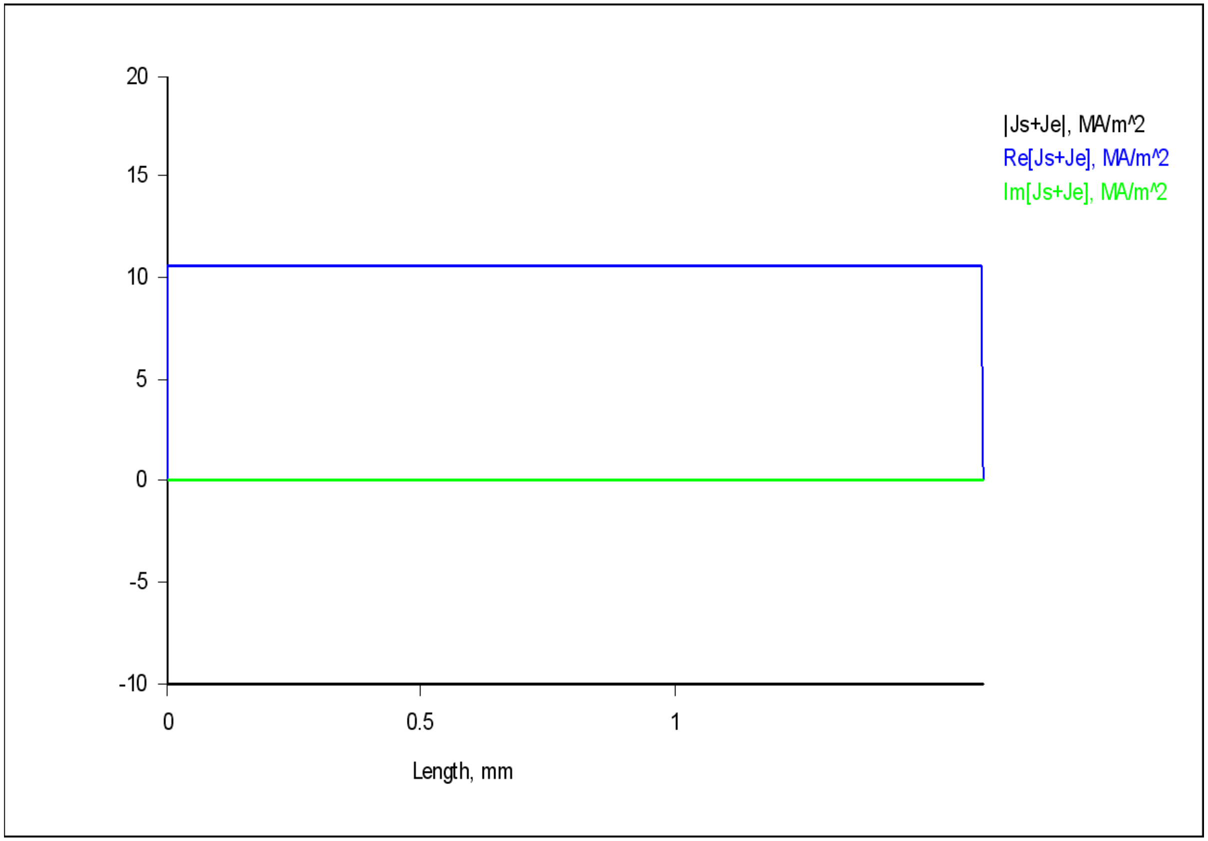
Example 1: Current Density vs. Radius, 1 Hz

- 1 Hz, left wire. At 1 Hz, δ = 66 mm >> r_w (0.8 mm) so no high frequency effects
- Power dissipation in wire = 1.93 Watts; $R_{AC}/R_{DC} = 1$



Example 1: Current Density vs. Radius, 60 Hz

- 60 Hz, left wire. At 60 Hz, δ = 8.5 mm >> r_w
- Note that current density J is still uniform across the wire
- Power dissipation = 1.94 Watts; R_{AC}/R_{DC} = 1.005



Example 1: Analysis for Small Skin Depth Limit

$R_{DC} = \frac{l}{\sigma\left(\pi r_w^2\right)}$

$R_{AC} = \frac{l}{\sigma\left(\pi r_w^2 - \pi\left(r_w - \delta\right)^2\right)} = \frac{l}{\sigma\pi\left(2r_w\delta - \delta^2\right)}$

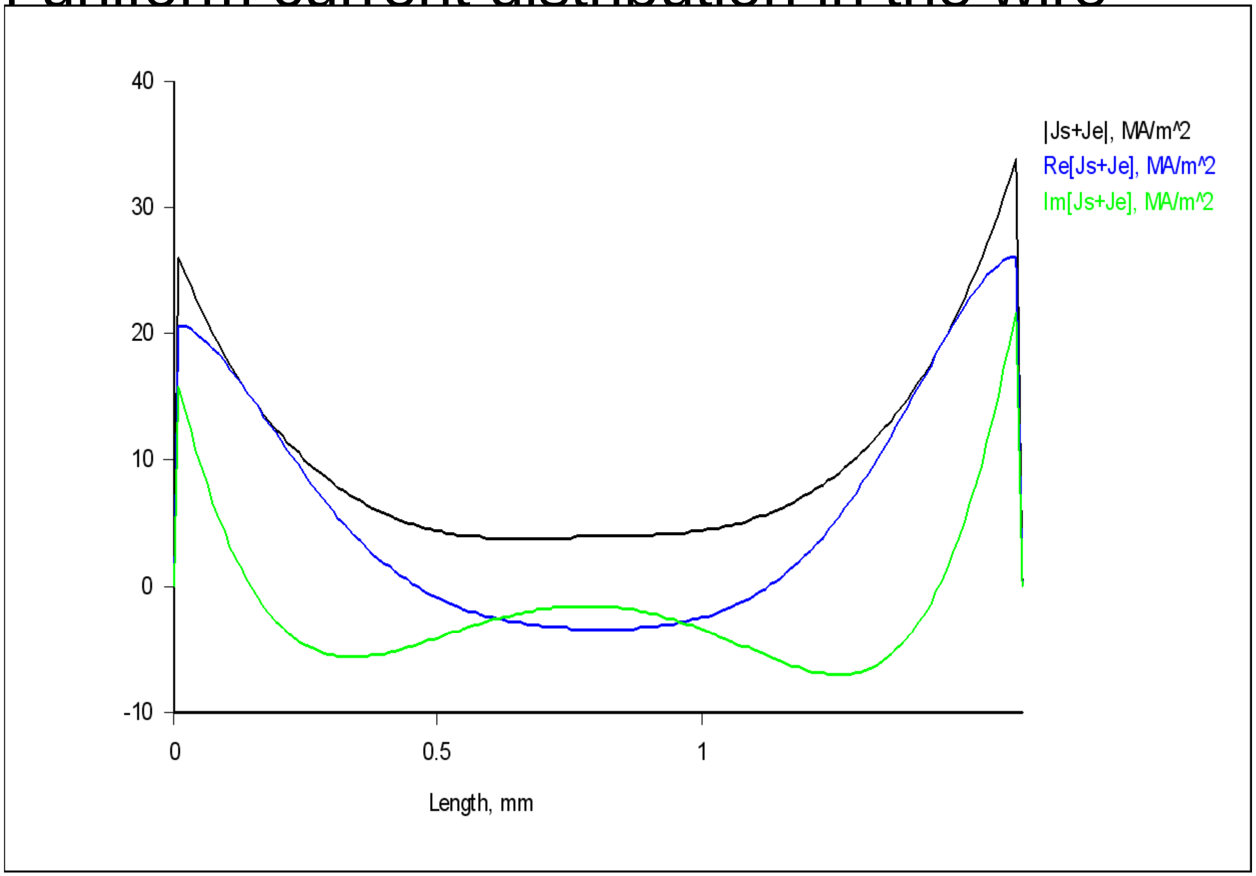
$$\frac{R_{AC}}{R_{DC}} = \frac{\frac{l}{\sigma\pi(2r_w\delta - \delta^2)}}{\frac{l}{\sigma\left(\pi r_w^2\right)}} = \frac{r_w^2}{2r_w\delta - \delta^2}$$

For r_w = 0.8 mm and δ = 0.21 mm:

$$\frac{R_{AC}}{R_{DC}} \approx 2.19$$

Example 1: Current Density vs. Radius, 100 kHz

- At 100 kHz, skin depth $\delta = 0.21$ mm << r_w
- Power dissipation = 4.27 Watts; $R_{AC}/R_{DC} = 2.2$
- Note non-uniform current distribution in the wire



Isolated Wire at High Frequency

- Induced eddy current in the wire aids current flow at the outer radius of the wire, and opposes current flow at the center of the wire
- Result: high frequency resistance of the wire is higher than the low frequency resistance. Resistance goes up as frequency goes up

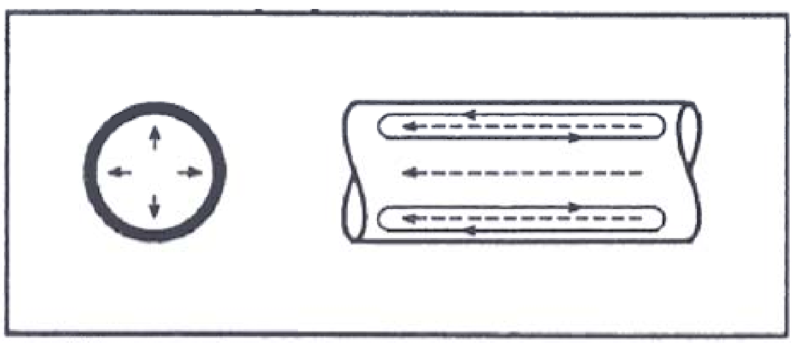


Fig. 2 - Eddy Current at High Frequency

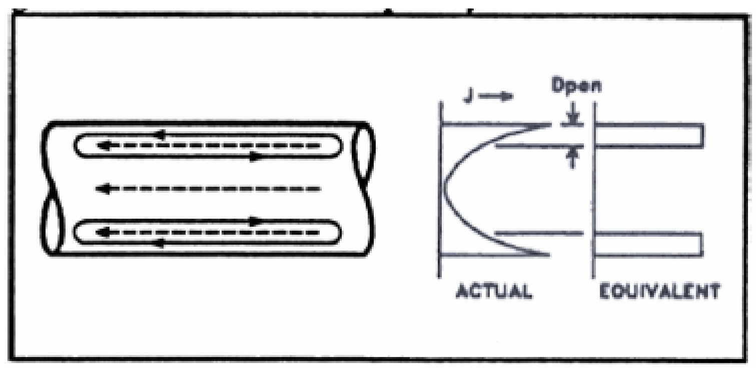


Fig. 3 - High Frequency Current Distribution

Reference: L. H. Dixon, “Eddy Current Losses in Transformer Windings and Circuit Wiring”

Isolated Tubular Conductor

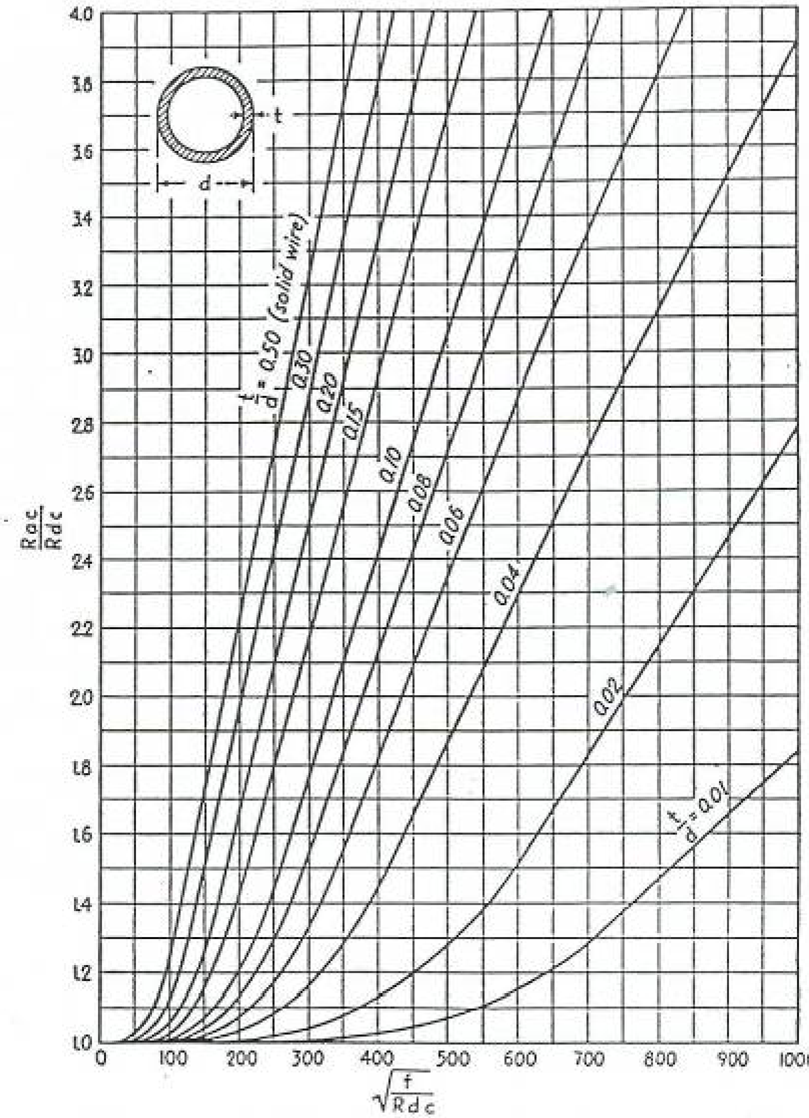


FIG. 2.—Resistance ratio of isolated nonmagnetic tubular conductors in terms parameter $\sqrt{f/R_{dc}}$, where f is the frequency in cycles, and R_{dc} the direct-current resistar per thousand feet.

Reference: B. Carsten, "Calculating High Frequency Conductor Losses in Switchmode Magnetics," *HFPC '93*

Isolated Rectangular/Round Conductor

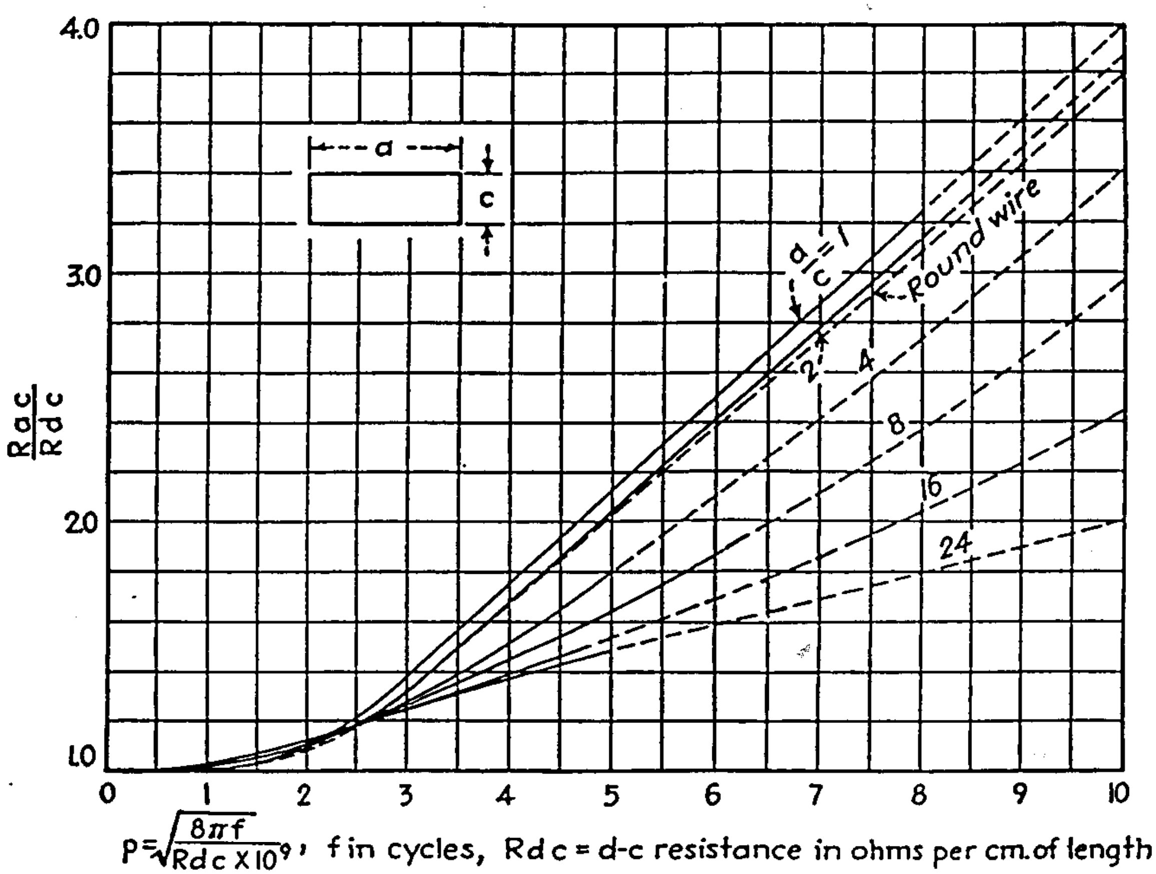


FIG. 3.—Resistance ratio of rectangular conductors in terms of parameter p, involving the frequency f in cycles, and R_{dc} the resistance in ohms per centimeter of length. These curves are a combination of experimental and calculated results, with the dotted portions representing low-frequency results extrapolated to join on with the theoretical results for high frequencies.

Reference: B. Carsten, "Calculating High Frequency Conductor Losses in Switchmode Magnetics," *HFPC '93*

Inductor Winding with Single Layer

- This example has N = 4 turns in a single layer

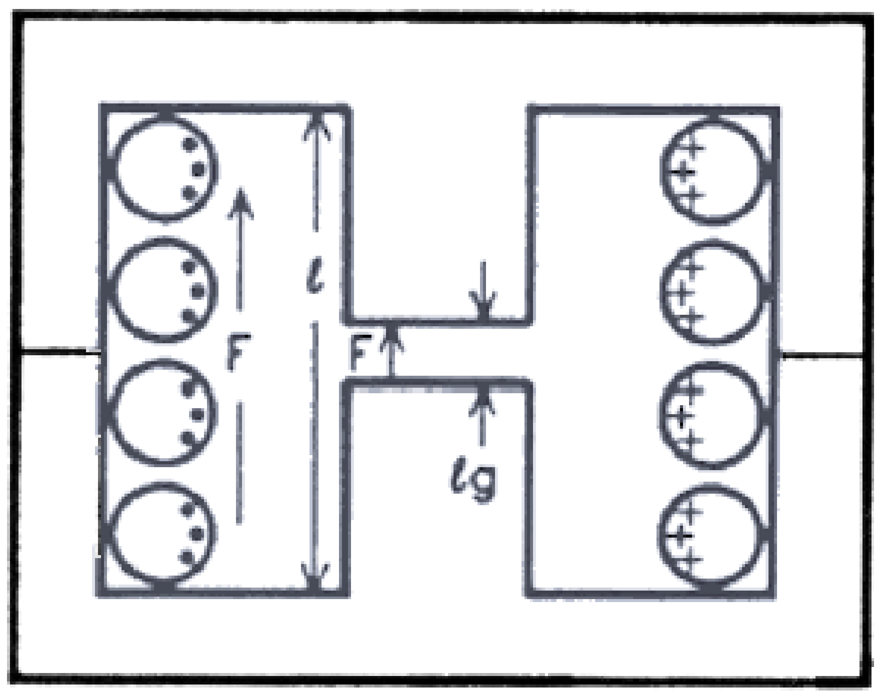


Fig. 8 - Inductor Winding

Reference: L. H. Dixon, “Eddy Current Losses in Transformer Windings and Circuit Wiring”

Transformer Winding with Single Layers

- This example has N = 4 turns in a primary, N = 1 turn in a secondary

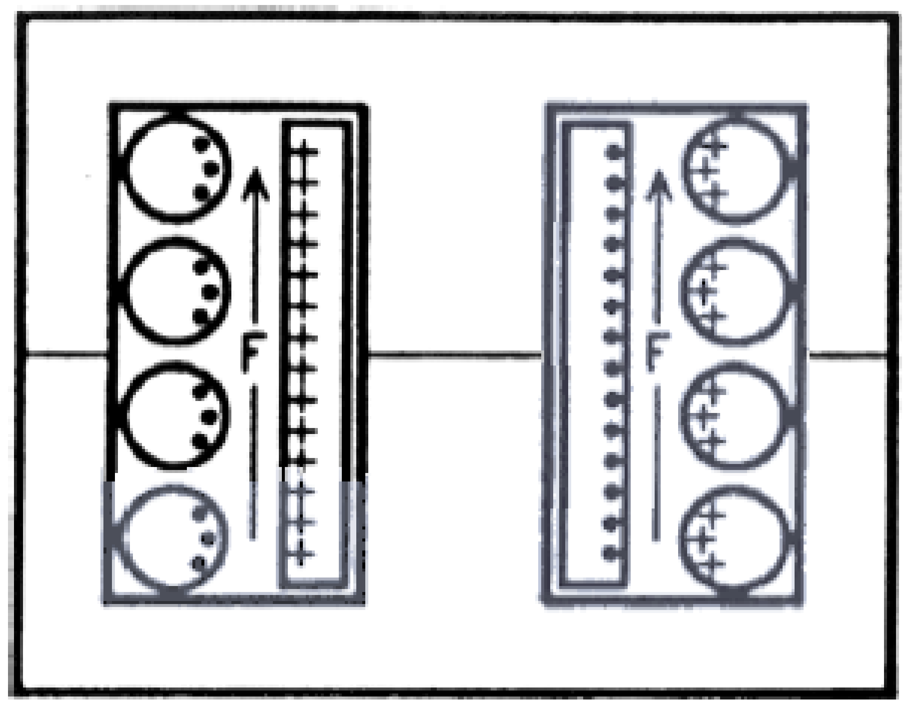


Fig. 9 - Transformer Windings

Reference: L. H. Dixon, “Eddy Current Losses in Transformer Windings and Circuit Wiring”

Multiple Winding Layers and the “Proximity Effect”

- Things get more complicated when you have multiple winding layers
- This is due to the “proximity effect”
- The field from one wire affects the current distribution in adjacent wires

Transformer Winding with Multiple Layers

- Low frequency MMF diagram
- Current is uniformly distributed through all conductors since the conductors are much thinner than a skin depth

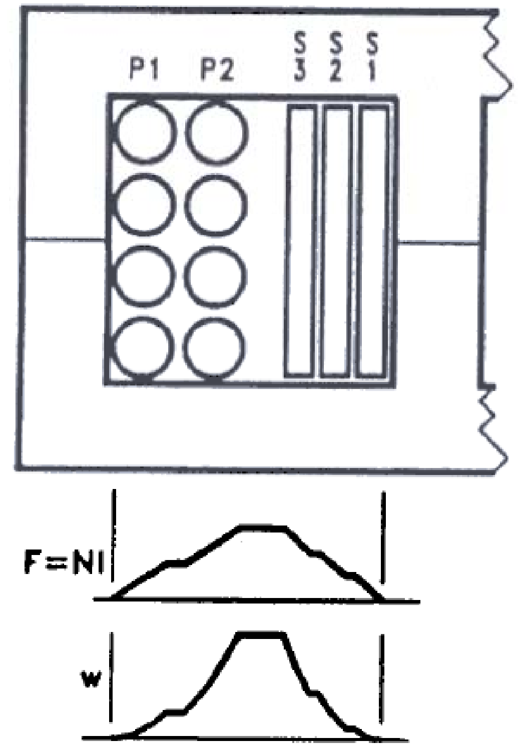


Fig. 10 - Multiple Layer Winding

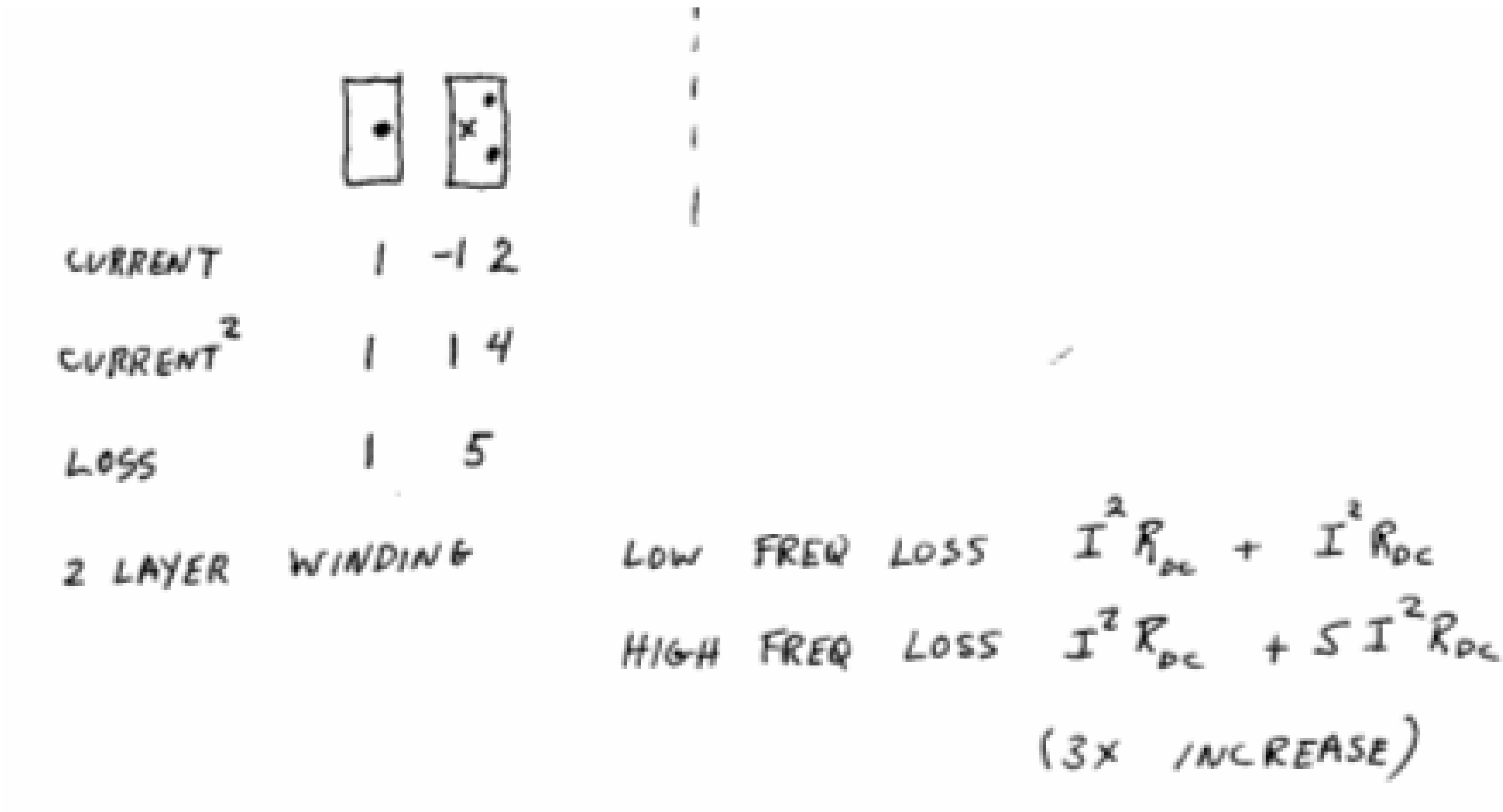
Reference: L. H. Dixon, "Eddy Current Losses in Transformer Windings and Circuit Wiring"

Proximity Effect --- 1 Winding Layer

- Multiple layers cause losses to increase faster than might be expected via skin depth alone
- Let's assume 1 foil winding layer first, with winding thickness *t*. The layer carries 1 Amp (dot into the page)

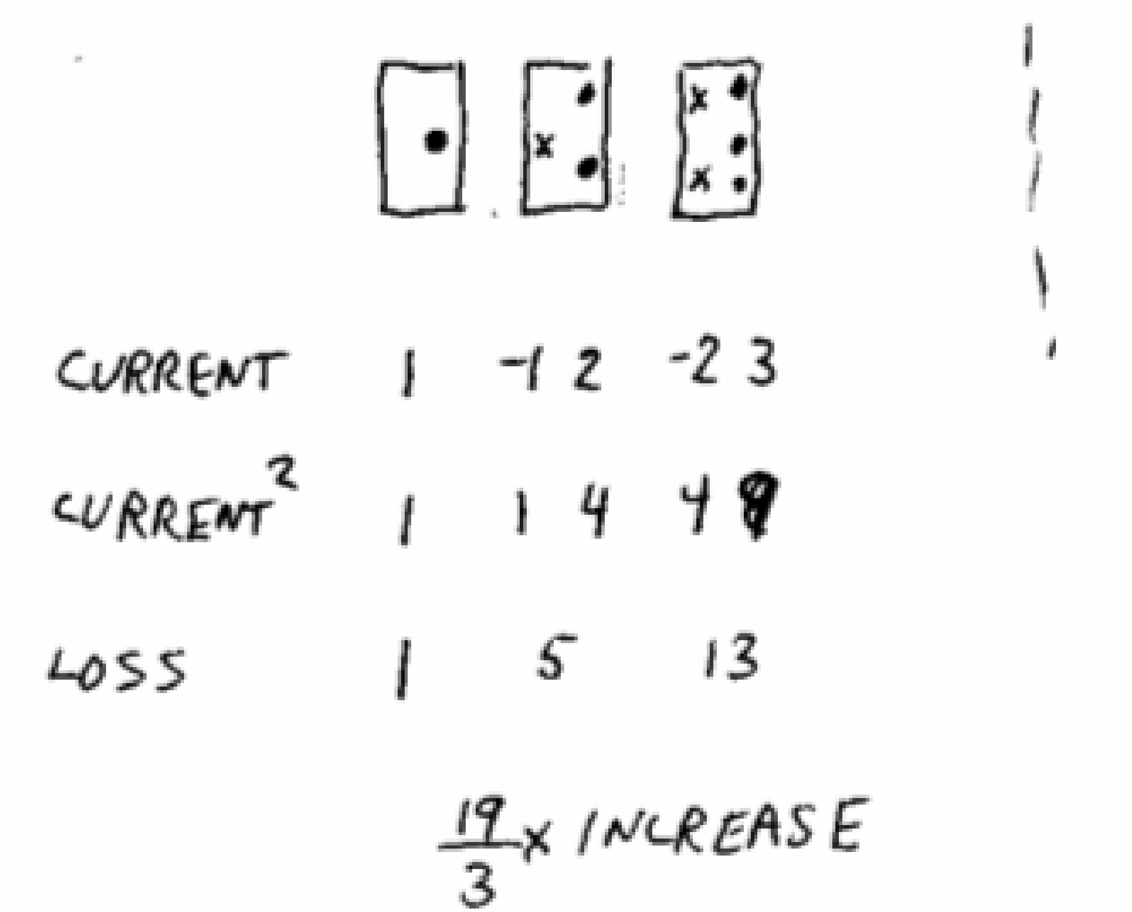
Proximity Effect --- 2 Winding Layers

- 2 layer coil (left side of coil shown only) with small skin depth limit t >> δ
- Field from winding 1 induces eddy current in winding 2
- Increase is 3x



Proximity Effect --- 3 Winding Layers

- 3 layer coil (left side of coil shown only)
- Increase is (19/3)×



Proximity Effect

- By the Dowell approximation, we can show that at high frequencies (t >> δ) the total winding resistance as compared to the low frequency resistance are:

$$F_p = \frac{R_{AC}}{R_{DC}} \approx \left(\frac{t}{\delta}\right)\left(\frac{2M^2+1}{3}\right)$$

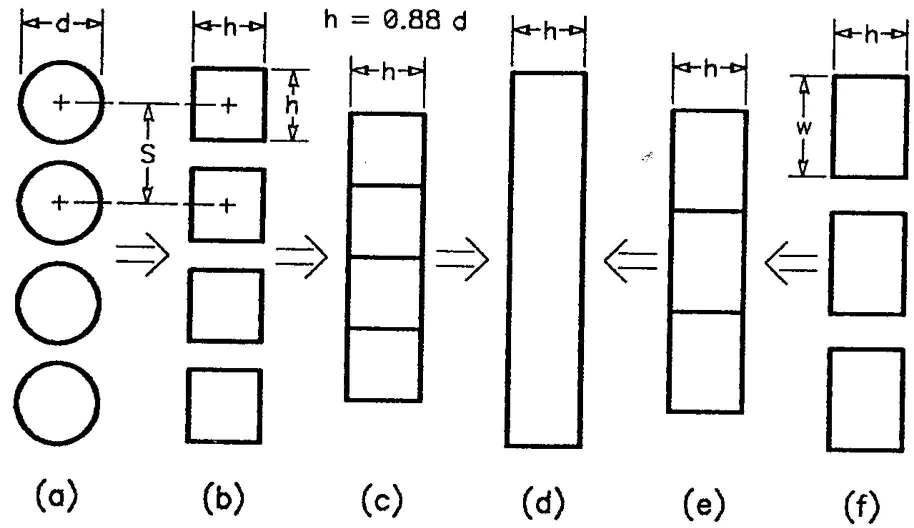
M	$\frac{R_{AC}}{R_{DC}} \approx \left(\frac{t}{\delta}\right)\left(\frac{2M^2+1}{3}\right)$ for t >> δ
1	$\left(\frac{t}{\delta}\right)$
2	$3\left(\frac{t}{\delta}\right)$
3	$\left(\frac{19}{3}\right)\left(\frac{t}{\delta}\right)$
4	$11\left(\frac{t}{\delta}\right)$
5	$17\left(\frac{t}{\delta}\right)$
6	$\left(\frac{73}{3}\right)\left(\frac{t}{\delta}\right)$
7	$33\left(\frac{t}{\delta}\right)$
8	$43\left(\frac{t}{\delta}\right)$
9	$\left(\frac{163}{3}\right)\left(\frac{t}{\delta}\right)$
10	$67\left(\frac{t}{\delta}\right)$
11	$81\left(\frac{t}{\delta}\right)$

Proximity + Skin Effect --- Dowell Method

- In the Dowell method, round wires are approximated by an equivalent foil, and a 1D solution is done

B: Round Wire or Strap to Foil Winding Transformation

Dowell (and others later) transform wire to "equivalent foil" by the process shown in Figure 2.2 below. Round wires (a) are first replaced with square wires of equal area (b), squeezed together (c) and "stretched" axially to cover the original winding breadth (d). The new foil thickness "h" is equal to the thickness of the square wire, and the resistivity is raised by a factor of S/h to maintain the same resistance. A similar process can be applied to strap windings; (f) → (e) → (d).



Wire or Strap to Equivalent Foil Transformation

Figure 2.2

Reference: B. Carsten, "Calculating High Frequency Conductor Losses in Switchmode Magnetics," *HFPC '93*

Proximity + Skin Effect --- Dowell Method

- Classic reference: P. L. Dowell, "Effects of Eddy Currents in Transformer Windings," *Proceedings of the IEE*, vol. 13, no. 8, August 1966, pp. 1387-1394

Reference: B. Carsten, "Calculating High Frequency Conductor Losses in Switchmode Magnetics," *HFPC '93*

Proximity + Skin Effect --- Dowell Method

- This is a general result (but 1D), over all range of wire diameters and d/δ
- The total resistance increase is given by:

$$F_p = \frac{R_{AC}}{R_{DC}} = \varphi\left[G_1(\varphi) + \left(\frac{2}{3}\right)\left(M^2 - 1\right)\left(G_1(\varphi) - 2G_2(\varphi)\right)\right]$$

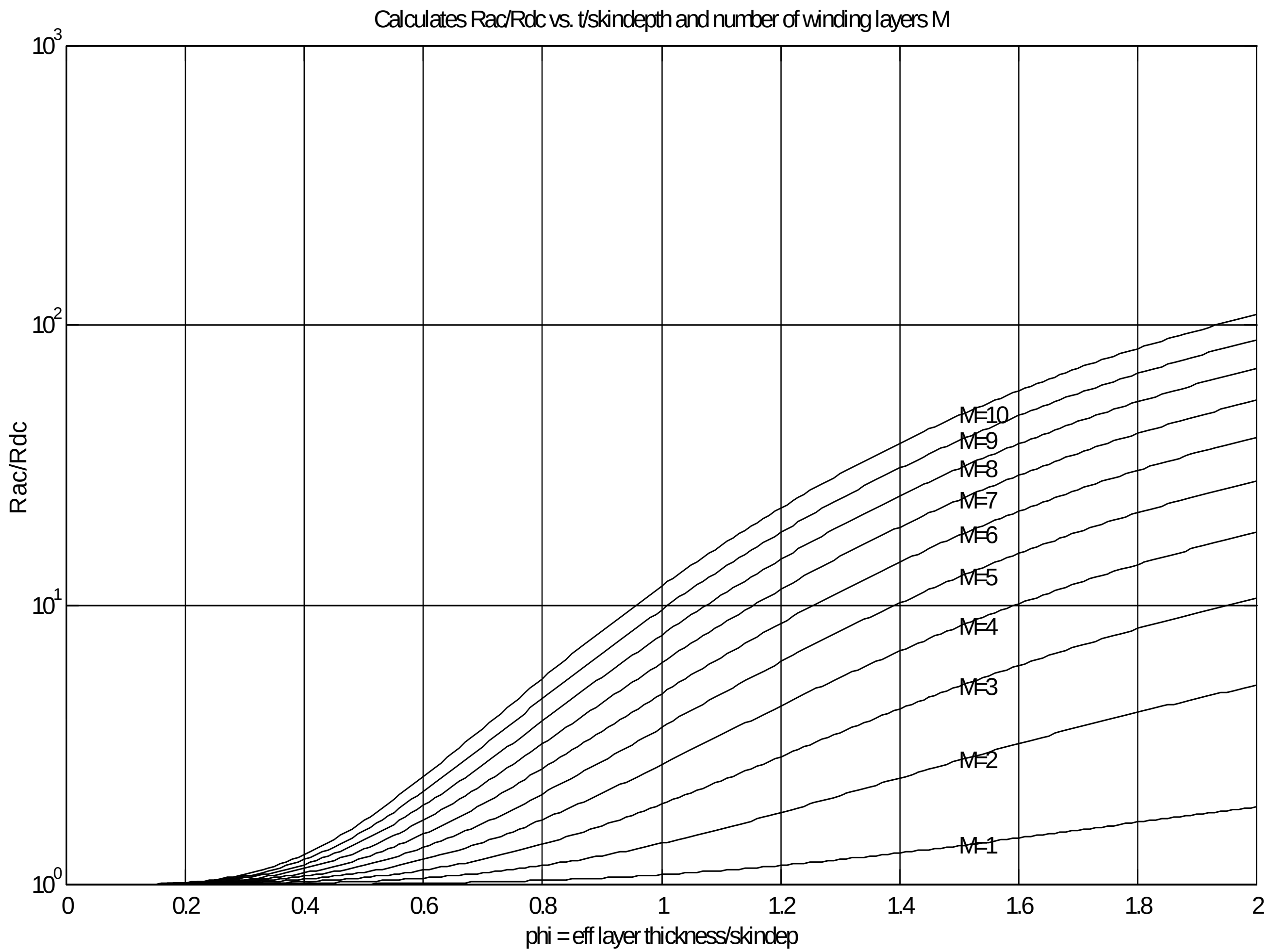
$$\varphi = \sqrt{\eta}\left(\frac{\sqrt{\pi}}{2}\right)\left(\frac{d}{\delta}\right)$$

$$G_1(\varphi) = \frac{\sinh(2\varphi) + \sin(2\varphi)}{\cosh(2\varphi) - \cos(2\varphi)}$$

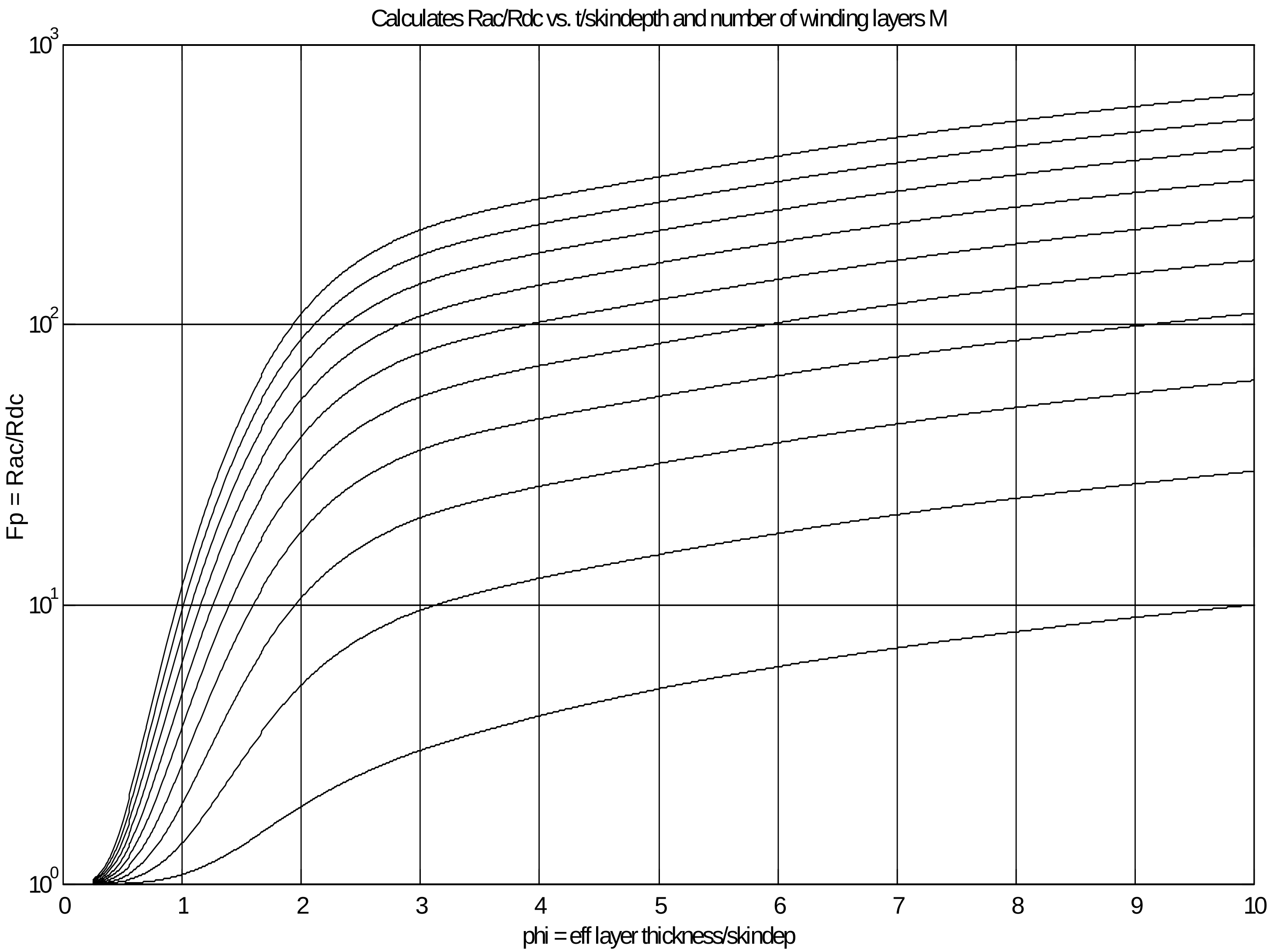
$$G_2(\varphi) = \frac{\sinh(\varphi)\cos(\varphi) + \cosh(\varphi)\sin(\varphi)}{\cosh(2\varphi) - \cos(2\varphi)}$$

Proximity + Skin Effect --- Dowell Method

- Note that for tiny wires $F_p \rightarrow 1$



Proximity + Skin Effect --- Dowell Method

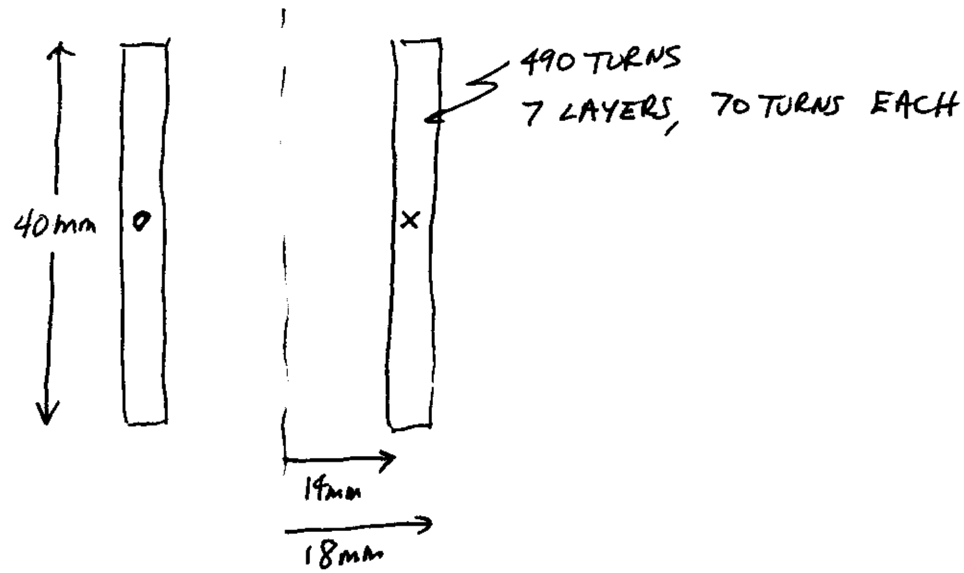


Proximity + Skin Effect --- Dowell Method

```
function proxy

% Calculates proximity and skin effect losses in windings using Dowell method
% Define a = d/skindepth where d is wire diameter
% M = number of winding layers
% Reference, Erickson, 2nd edition, pp. 508-522

clear all
for M=1:10
   a=0.1:0.01:2;
   phi=sqrt(pi)*a/2;
   G1=(sinh(2*phi)+sin(2*phi))./(cosh(2*phi)-cos(2*phi));
   G2=(sinh(phi).*cos(phi)+cosh(phi).*sin(phi))./(cosh(2*phi)-cos(2*phi));
   Fp=phi.*(G1+(2/3)*(M^2-1)*(G1-2*G2));
   semilogy(a,Fp,'k')
   text(1.5,Fp(round(length(Fp)*0.75)),['M=',num2str(M)])
   hold on
end

title('Calculates Rac/Rdc vs. d/skindepth and number of winding layers M')
xlabel('Wire diam/skin depth')
ylabel('Rac/Rdc')
grid; hold off;
```

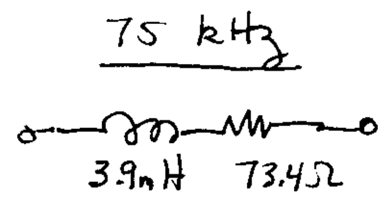
Example 2: Using Dowell's Method

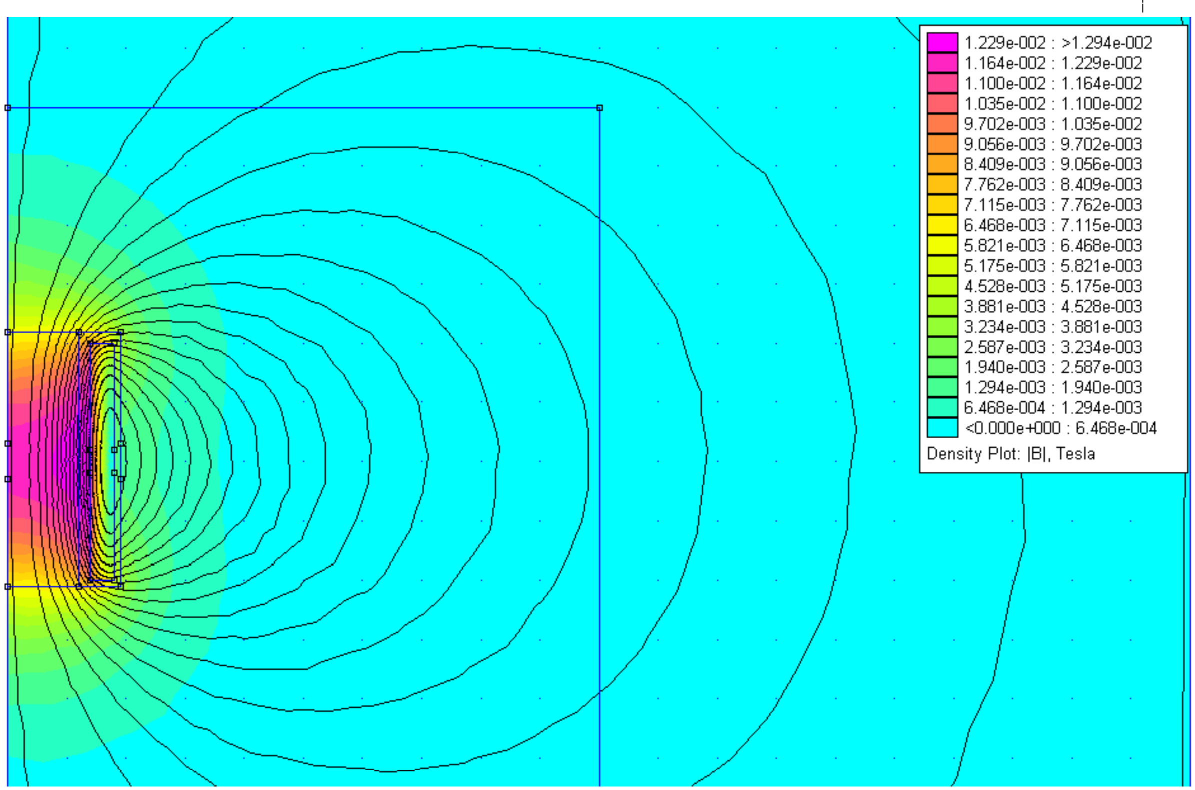
- N=490, 24 gage wire, 75 kHz
- 7 layers, 70 turns each
- Skin depth δ= 0.26 mm
- Wire diam. d = 0.51 mm
- η = 0.7910
- φ = 1.5461
- N = 7, phi = 1.5461, Dowell predicts Fp = 26



Example 2: FEA Result

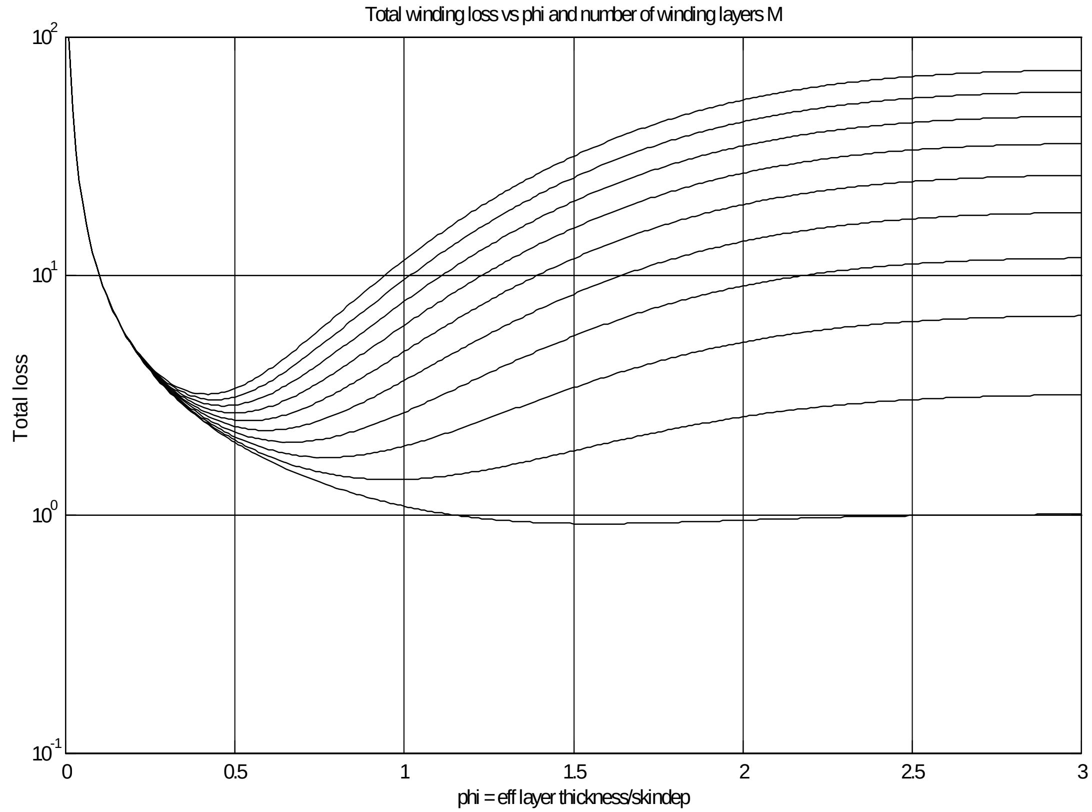
- $R_{dc} = 4.8\ \Omega$
- $R_{ac} = 73.4\ \Omega$
- $F_p = R_{ac}/R_{dc} = 15.3$ from FEA





Optimum Winding Thickness vs. Number of Layers

- Dowell's method shows that optimal winding thickness is phi (φ) near or a little less than a skin depth. Note: phi = about a skin depth



Comment on Dowell Method

- Note that it's a 1D approximation to a 2D or 3D problem
- Seems to do better for tall coils

How to Keep AC Losses Low

- Conductor thickness must be small to keep AC losses to a minimum
- Keep number of winding layers low
- Sometimes foil is used for low voltage, high current windings
- In transformers, you can interleave the windings
- If foil is not appropriate, multiple conductors can be used. Litz wire is the name for multiple, twisted strands

Litz Wire

- Multiple small conductors
- Disadvantages
  - Cost
  - Reduced copper fill factor
  - Manufacturing

Litz Wire

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LITZ Wire

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Technical Data On Our Products

Specialty Products
Multifilar
Twistite
Microsquare

Insulated Products
Round Magnet Wire
Square Magnet Wire
Bondable Magnet Wire
Litz Wire

Bare Wire Products
Copper

The term Litz wire is derived from the German word **litzendraht** meaning woven wire. Generally defined, it is a wire constructed of individual film insulated wires bunched or braided together in a uniform pattern of twists and length of lay.

The multistrandconfiguration minimizes the power losses otherwise encountered in a solid conductor due to the "skin effect", or the tendency of radio frequency current to be concentrated at the surface of the conductor.

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Litz Wire Designation

- For instance, type 100/40 Litz has 100 paralleled strands of #40 AWG wire

Core Loss

Core type	B_{sat}	Relative core loss	Applications
Laminations iron, silicon steel	1.5 - 2.0 T	high	50-60 Hz transformers, inductors
Powdered cores powdered iron, molypermalloy	0.6 - 0.8 T	medium	1 kHz transformers, 100 kHz filter inductors
Ferrite Manganese-zinc, Nickel-zinc	0.25 - 0.5 T	low	20 kHz - 1 MHz transformers, ac inductors

Reference: Erickson and Maksimovic, _Fundamentals of Power Electronics_, 2nd edition, 2001

Core Loss

• From purely first-order analysis, we can see that eddy current losses scale as B^2f^2 and hysteresis power loss increases proportionally with frequency. In practice, measurements are done on magnetic cores to determine the functional dependence. Manufacturers sometimes provide curve fit of the form:

$$P_L = Af^{\alpha}(\Delta B)^{\beta}$$

with curve fit parameters α is usually in the range of 1.0 ~ 2.0 and β is in the range 2.0 ~ 3.0, but this varies from material to material

Core Loss for 3F3

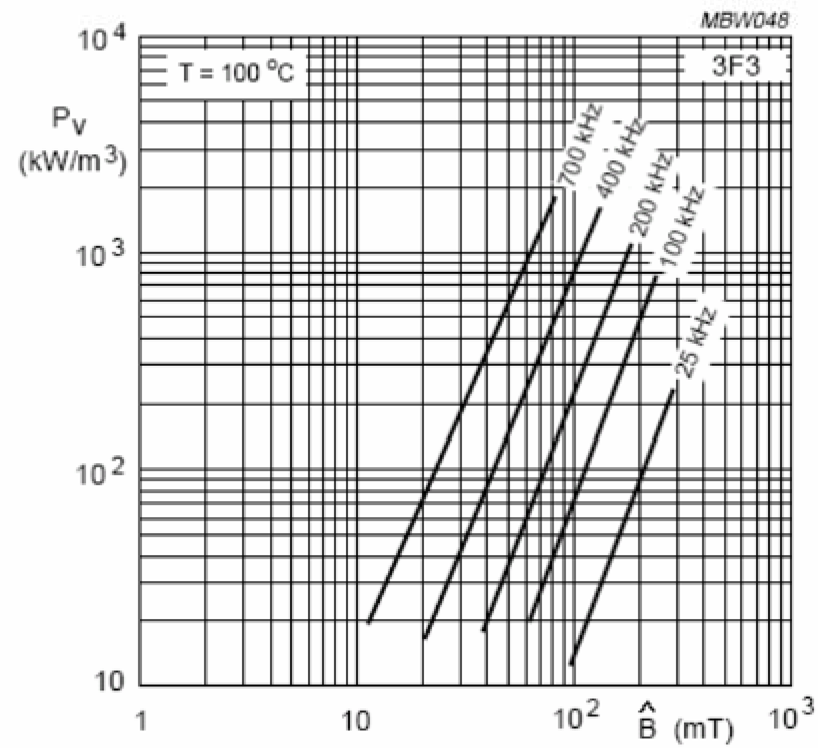

Fig.6 Specific power loss as a function of peak flux density with frequency as a parameter.

Reference: http://ferroxcube.com/prod/assets/3f3.pdf

Ferrite Core Performance Factor

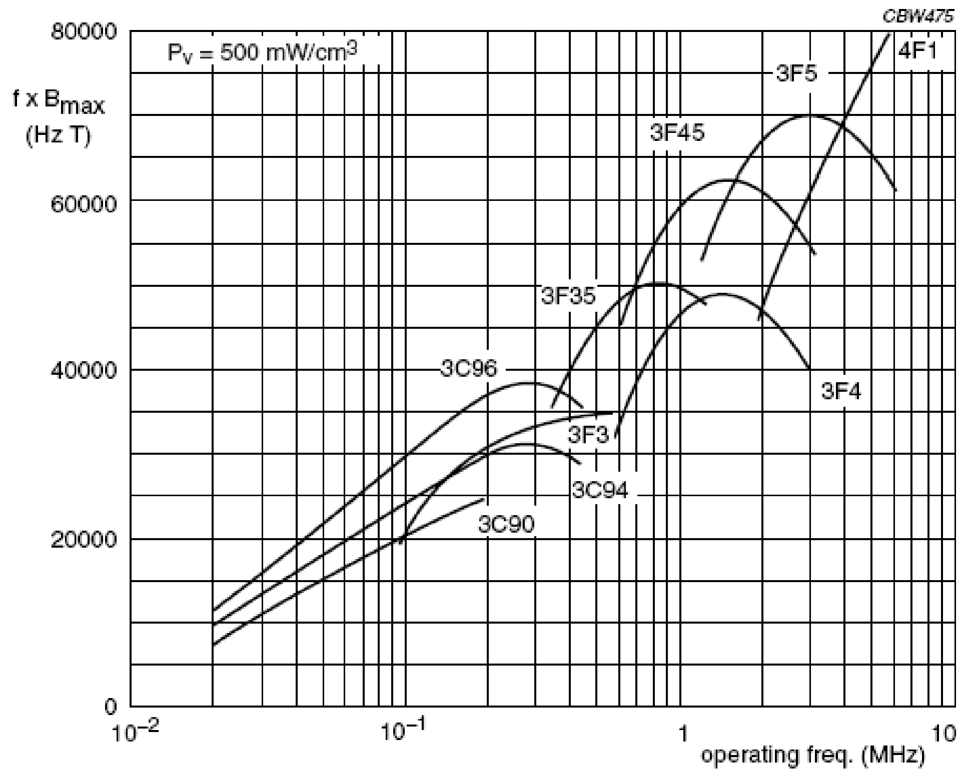

Fig.22 Performance factor ($f \times B_{max}$) at PV = 500 mW/cm^3 as a function of frequency for power ferrite materials.

Example 3: From IEEE Paper

$$p_C = K1 \cdot f^{K2} \cdot B^{K3} \tag{1}$$

where

p_C [kW/m^3]	—core power loss density
f [Hz]	—frequency
B [T]	—flux density
$K1$, $K2$, and $K3$	—curve fitting formula constants.

The values of $K1$, $K2$, and $K3$ for the FERROXCUBE ferrite grades at core temperature 100°C, which provide an approximation accuracy about 20%, can be found in [17]. For the case of 3C80 grade, these values are $K1 = 16.7$, $K2 = 1.3$, $K3 = 2.5$.

Reference: Petkov, "Optimum Design of a High-Power, High-Frequency Transformer," *IEEE Transactions on Power Electronics*, vol. 11, no. 1, January 1996, pp. 33-42

Example 4: From Magnetics, Inc.

Table 3 Power Loss

Material	Frequency	a	c	d
K at 80°C	**f<500 kHz** **500 kHz≤f<1 MHz** **f≥1 MHz**	0.0530 0.00113 $1.77*10^{-9}$	1.60 2.19 4.13	3.15 3.10 2.98
R at 100°C	**f<100 kHz** **100 kHz≤f<500 kHz** **f≥500 kHz**	0.074 0.036 0.014	1.43 1.64 1.84	2.85 2.68 2.2
P at 80°C	**f<100 kHz** **100 kHz≤f<500 kHz** **f≥500 kHz**	0.158 0.0434 $7.36*10^{-7}$	1.36 1.63 3.47	2.86 2.62 2.54
F at 25°C	**f≤10 kHz** **10 kHz<f<100 kHz** **100 kHz≤f<500 kHz** **f≥500 kHz**	0.790 0.0717 0.0573 0.0126	1.06 1.72 1.66 1.88	2.85 2.66 2.68 2.29
J at 25°C	**f≤20 kHz** **f>20 kHz**	0.245 0.00458	1.39 2.42	2.50 2.50
W at 25°C	**f≤20 kHz** **f>20 kHz**	0.300 0.00382	1.26 2.32	2.60 2.62
H at 25°C	**f≤20 kHz** **f>20 kHz**	0.148 0.135	1.50 1.62	2.25 2.15

FORMAT: $P_L = af^cB^d$ P_L in mW/cm^3, B in kG, f in kHz

Reference: http://www.mag-inc.com/pdf/fc-s7.pdf

References

- L. H. Dixon, "Eddy Current Losses in Transformer Windings and Circuit Wiring" available from Unitrode
- P. L. Dowell, "Effects of Eddy Currents in Transformer Windings," *Proceedings of IEE,* vol. 113, no. 8, August 1966
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- R. W. Erickson and D. Maksimovic, *Fundamentals of Power Electronics*, 2nd edition, Kluwer, 2001