Power Electronics Notes 30C
Notes on Permanent Magnets

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Portions of these notes excerpted from the CD ROM accompanying Fitzgerald, Kingsley and Umans, *Electric Machinery*, 6th edition, McGraw Hill, 2003

Brief History of Permanent Magnets

- c. 1000 BC: Chinese compasses using lodestone
  - Later used to cross the Gobi desert

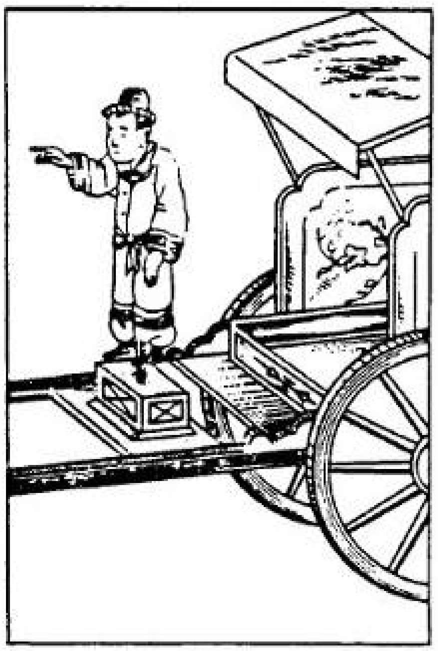


Fig. 2 *Man-shaped compass mounted on a chariot, after Kartsev [2]*

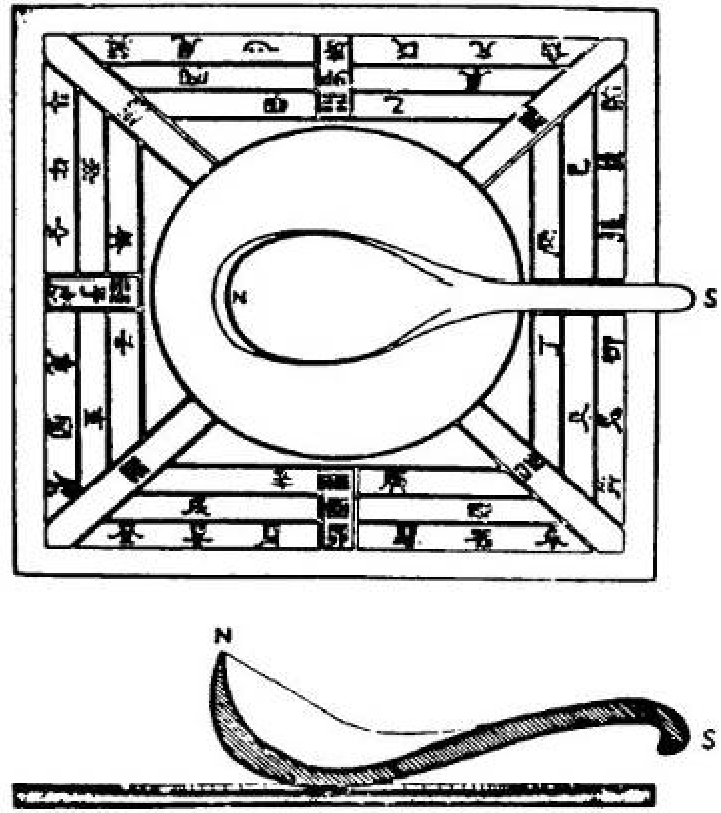


Fig. 3 *Chinese spoon compass*

Reference: K. Overshott, “Magnetism: it is permanent,” *IEE Proceedings-A*, vol. 138, no. 1, Jan. 1991, pp. 22-31

Brief History of Permanent Magnets (cont.)

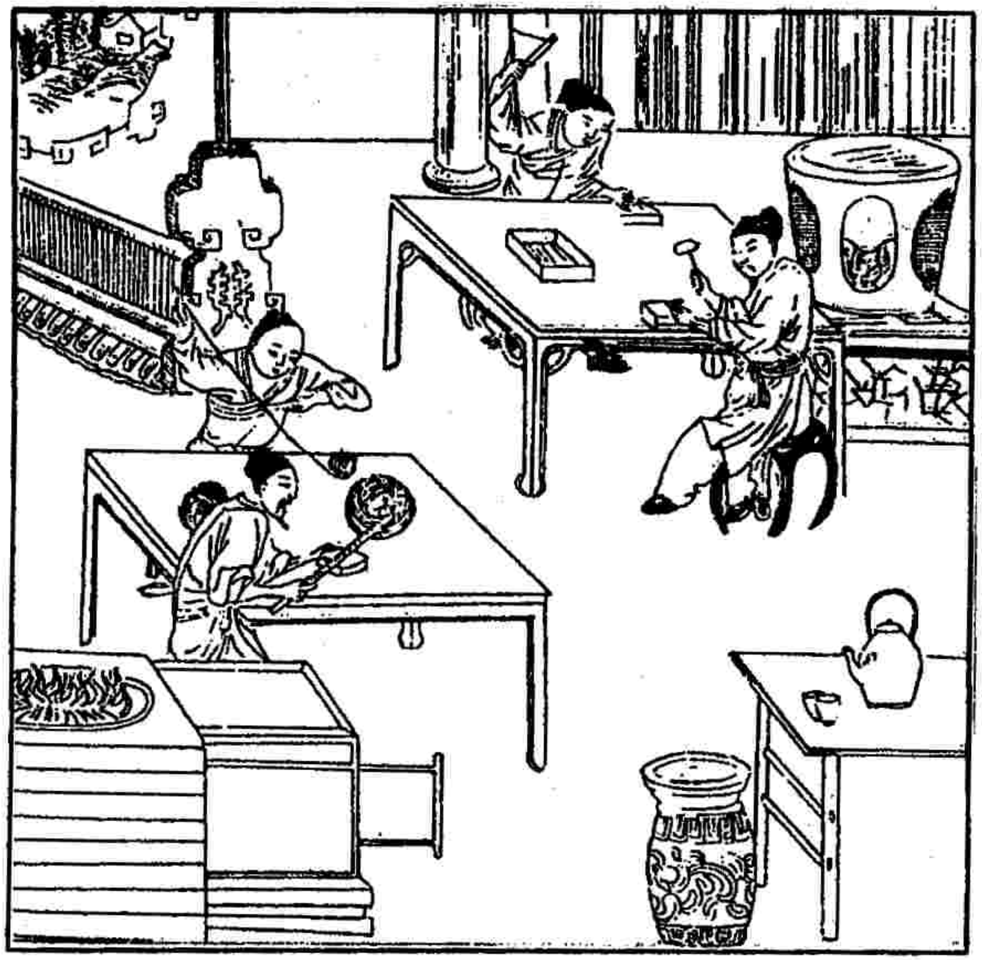


Figure 1.1 Magnetic needles for compasses are being made by craftsmen in this print of 1637. Good steel was manufactured in China from 500 A.D. onwards.

Reference: R. Parker, *Advances in Permanent Magnetism*, John Wiley, 1990, pp. 3

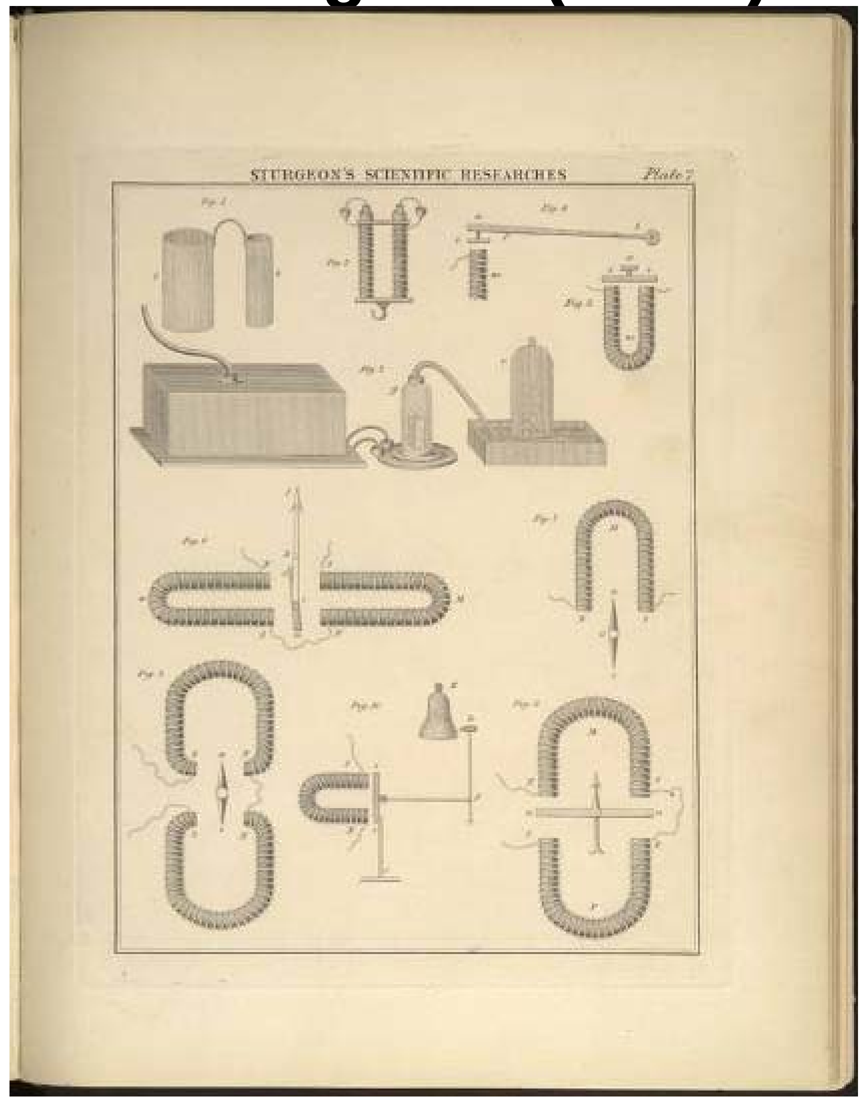
Brief History of Permanent Magnets (cont.)

- c. 200 BC: Lodestone (magnetite) known to the Greeks
  - Touching iron needles to magnetite magnetized them
- 1200 AD: French troubadour de Provins describes use of a primitive compass to magnetize needles
- 1600: William Gilbert publishes first journal article on permanent magnets
- 1819: Oersted reports that an electric current moves compass needle

References:
1. K. Overshott, "Magnetism: it is permanent," *IEE Proceedings-A*, vol. 138, no. 1, Jan. 1991, pp. 22-31
2. R. Petrie, "Permanent Magnet Material from Loadstone to Rare Earth Cobalt," *Proc. 1995 Electronics Insulation and Electrical Manufacturing and Coil Winding Conf*., pp. 63-64
3. Rollin Parker, *Advances in Permanent Magnetism*, John Wiley, 1990
4. E. Hoppe, "Geshichte des Physik," Vieweg, Braunshweig, 1926, pp. 339
5. W. Gilbert, "De Magnete 1600," translation by S. P. Thompson, 1900, republished by Basic Books, Inc., New York, 1958

Brief History of Permanent Magnets (cont.)

- c. 1825: Sturgeon invents the electromagnet, resulting in a way to artificially magnetize materials
- 7-ounce magnet was able to lift 9 pounds

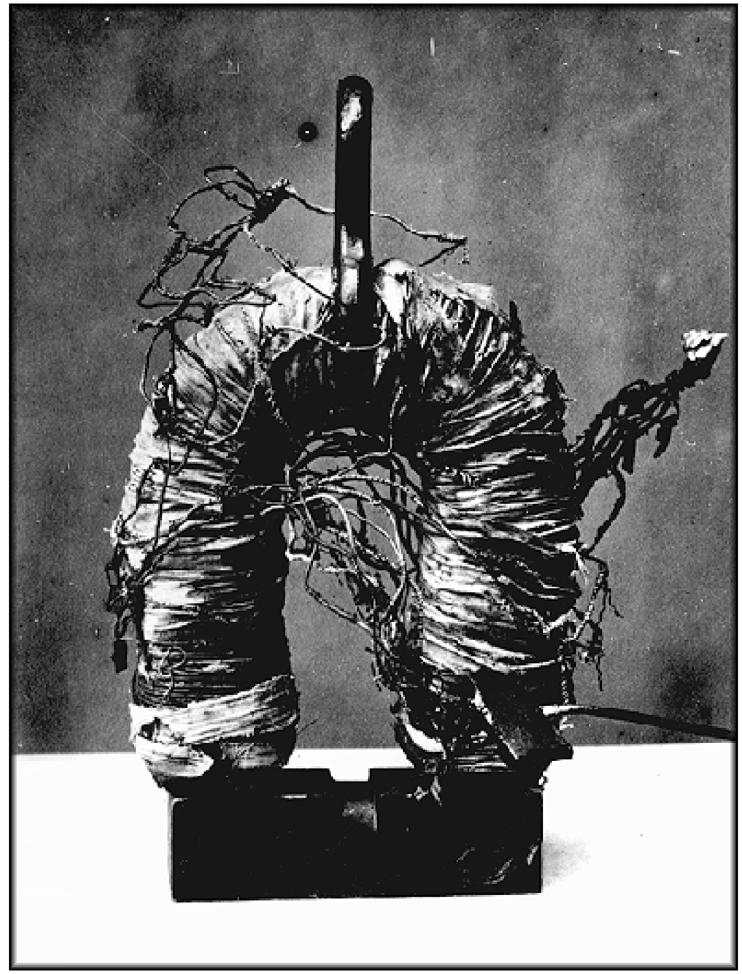


References:
1. W. Sturgeon, *Mem. Manchester Lit. Phil. Soc.*, 1846, vol. 7, pp. 625
2. Britannica Online

Brief History of Permanent Magnets (cont.)

- c. 1830: Joseph Henry (U.S.) constructs electromagnets

Joseph Henry



Reference: Smithsonian Institute archives

Brief History of Permanent Magnets (cont.)

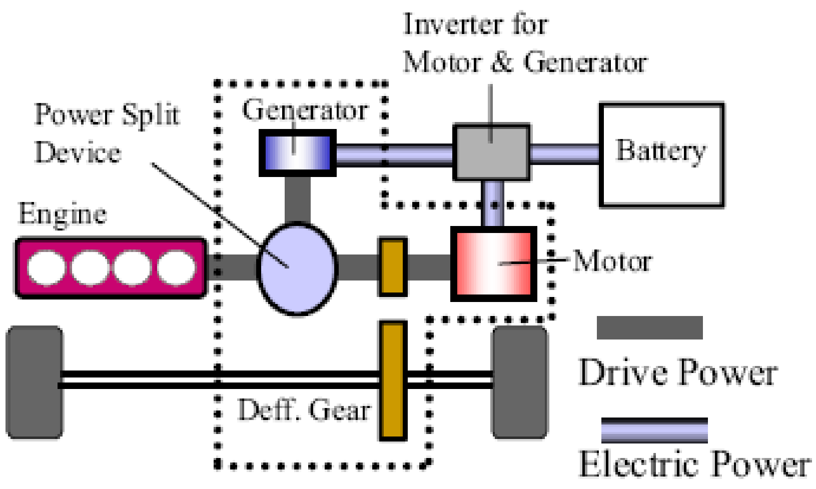
- 1917: Cobalt magnet steels developed by Honda and Takagi in Japan
- 1940: Alnico --- first "modern" material still in use
  - Good for high temperatures
- 1960: SmCo (samarium cobalt) rare earth magnets
  - Good thermal stability
- 1983: GE (later Magnequench) and Sumitomo develop neodymium iron boron (NdFeB) rare earth magnet
  - Highest energy product, but limited temperature range

References:
1. K. Overshott, "Magnetism: it is permanent," *IEE Proceedings-A*, vol. 138, no. 1, Jan. 1991, pp. 22-31
2. R. Petrie, "Permanent Magnet Material from Loadstone to Rare Earth Cobalt," *Proc. 1995 Electronics Insulation and Electrical Manufacturing and Coil Winding Conf.*, pp. 63-64

Brief History of Permanent Magnets (cont.)

- Late 1990s --- Hybrid car utilizing high strength permanent magnets

Prius and Next-Generation Hybrid



THS Transmission

Fig.2. Toyota Hybrid System.

Table 1. THS motor specifications.

System	THS		THS II	
Vehicle	Prius			SUV
E/G	1.5L			3.3L
Launch	1997	2000	2003	2005
DC Bus Voltage	About 274V		500V	650V
Max. Power	30 kW	33 kW	50 kW	123 kW
Max. Torque	305 Nm	350 Nm	400 Nm	333 Nm
Max. Speed	6000 rpm		6700 rpm	12400 rpm

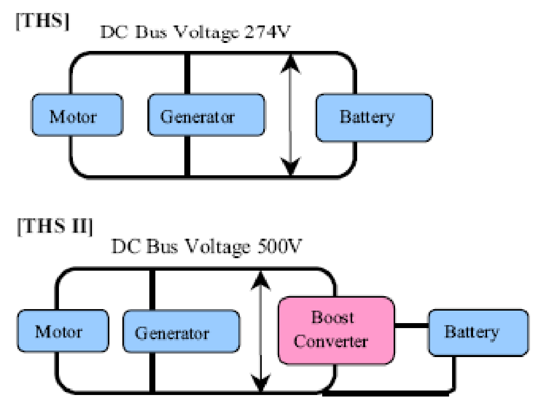


Fig.3. High voltage motors drive system employing a boost DC/DC converter.

Reference: M. Kamiya, "Development of Traction Drive Motors for the Toyota Hybrid System"

Maglev System #1

- Magnemotion (Acton, MA)

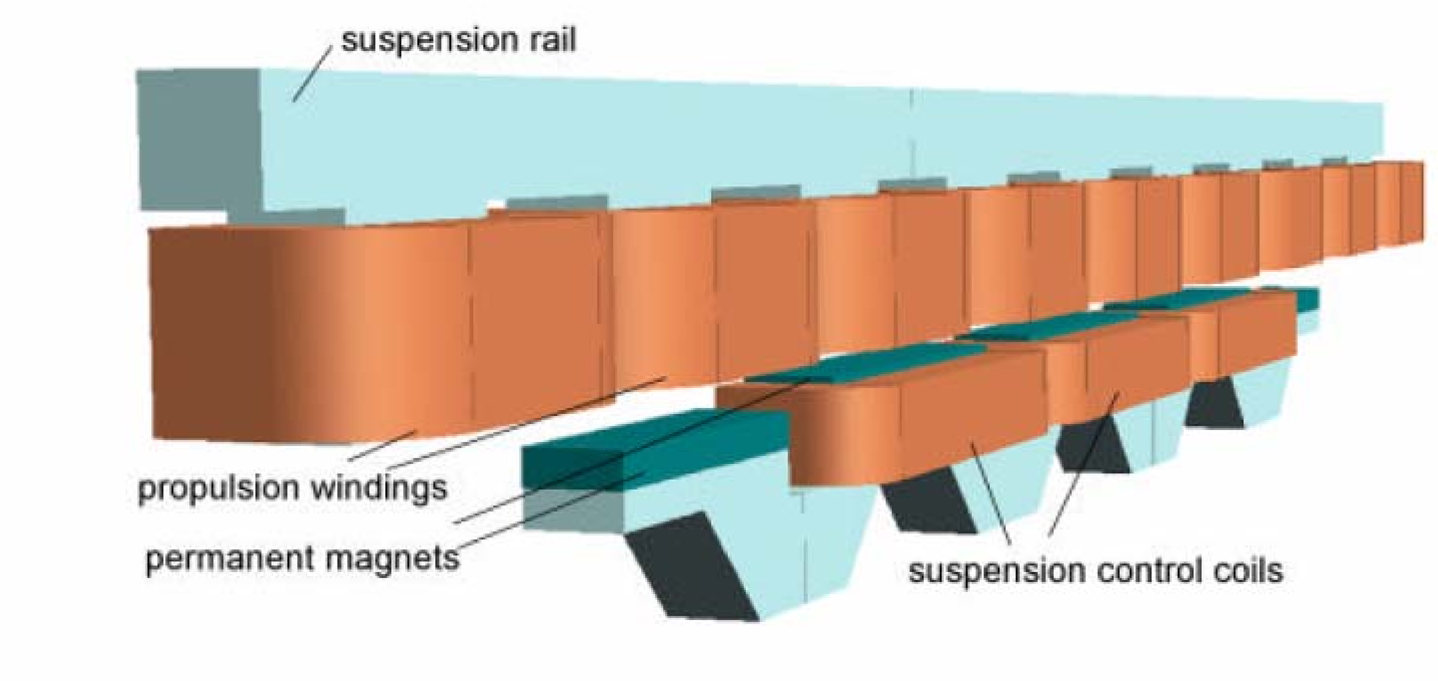


Fig. 2.1. A vehicle's magnet pod attracted upwards to a suspension rail.

Reference: R. Thornton, T. Clark and B. Perreault, "Linear Synchronous Motor Propulsion of Small Transit Vehicles," *Proceedings of the 2004 ASME/IEEE Joint Rail Conference*, April 6-8, 2004, Baltimore MD, pp. 101-107

Maglev System #2

- General Atomics (San Diego, CA)

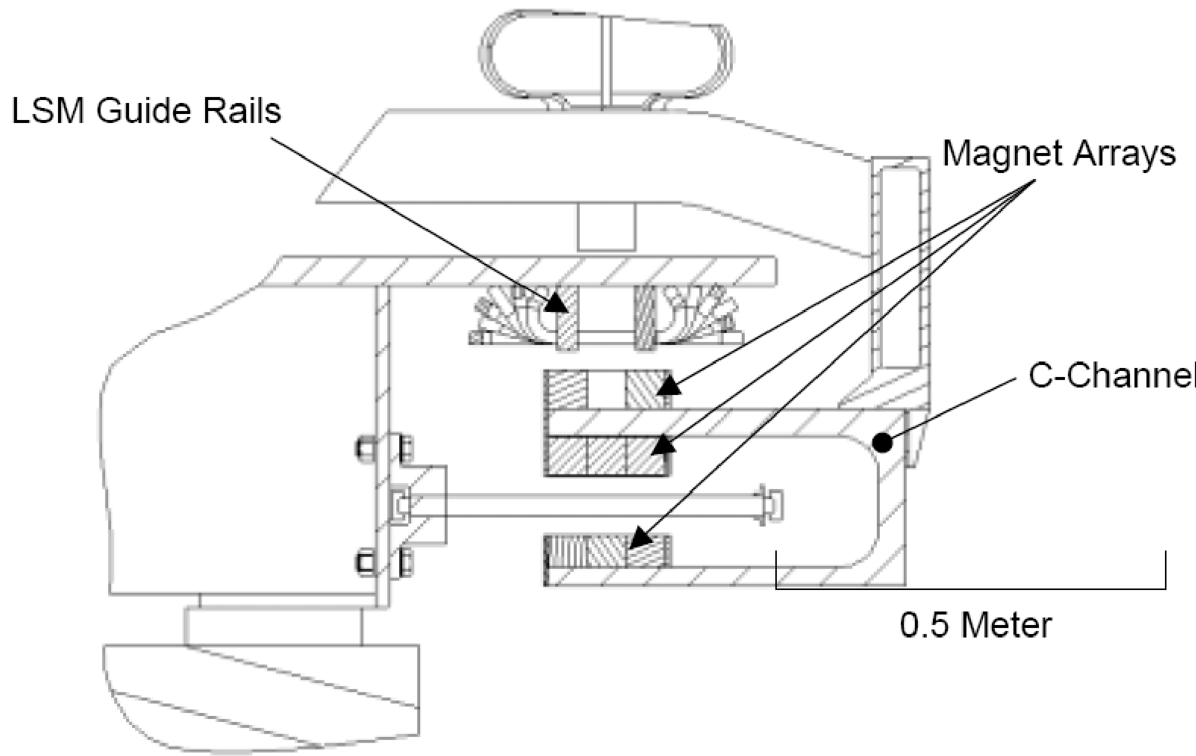
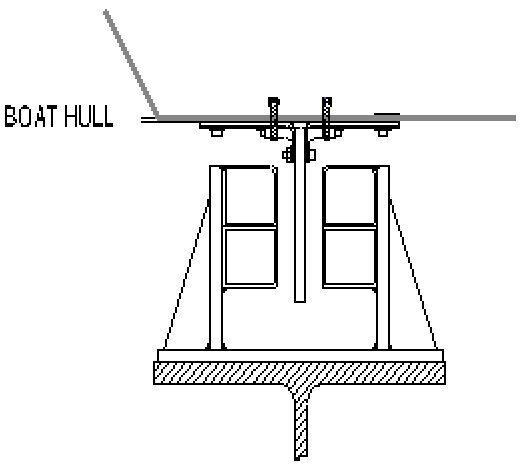


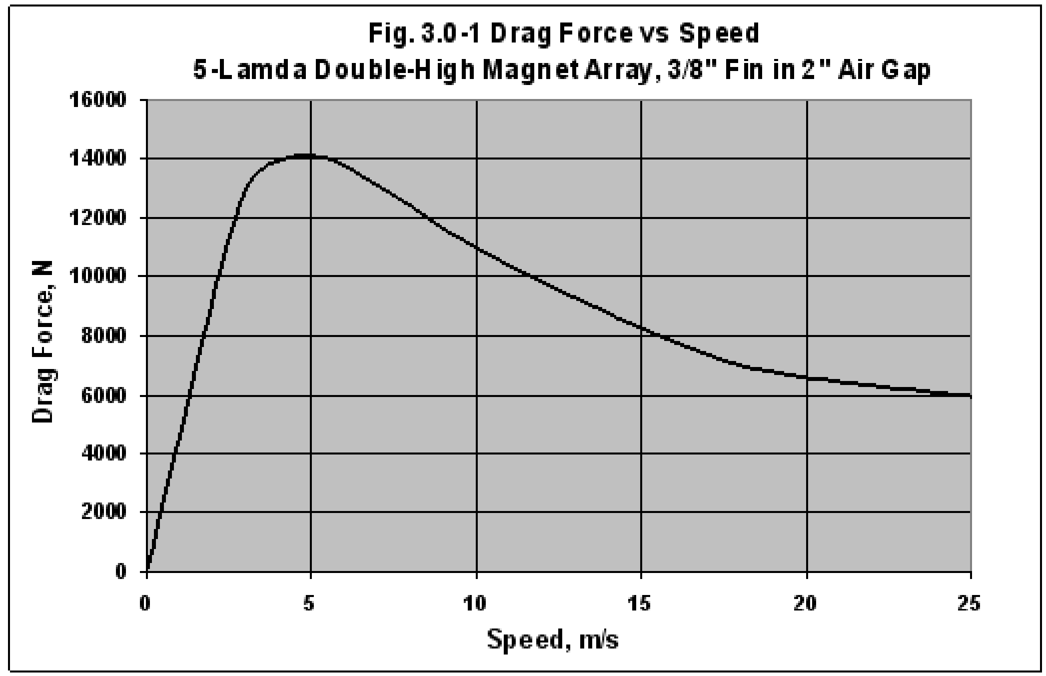
Figure 2-9
Cross-section of Magnet System Assembly

Reference: U.S. Department of Transportation (Federal Transit Administration), "Low Speed Maglev Technology Development Program – Final Report," FTA-CA-26-7025-02.1, March 2002.

Eddy Current Brake

- Magnetar Corp. (Seal Beach, CA)





Magnetizing Permanent Magnets

- Material is placed inside magnetizing fixture
- Magnetizing coil is energized with a current producing sufficient field to magnetize the PM material

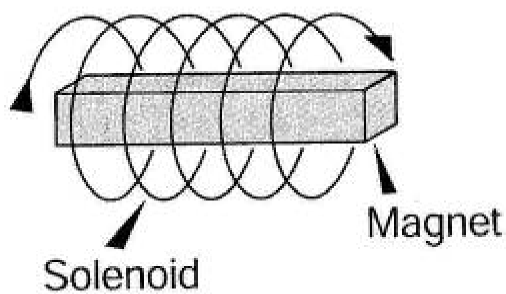


FIGURE 1.33 Magnetization of a bar magnet using a solenoid

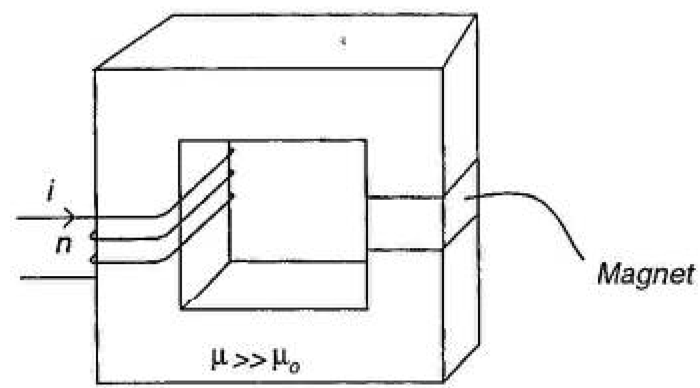


FIGURE 1.34 Conventional magnetizing fixture.

Reference: E. Furlani, _Permanent Magnet and Electromechanical Devices_, Academic Press, 2001, pp. 57

Pictorial View of Magnetization Process

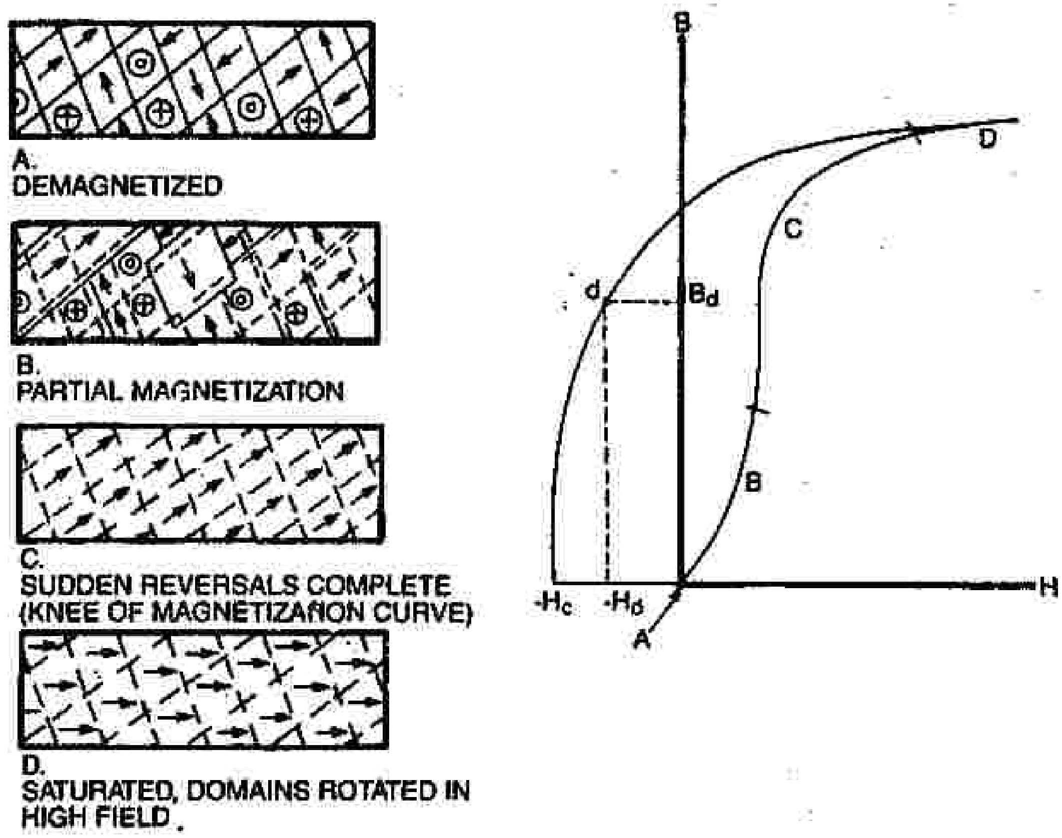
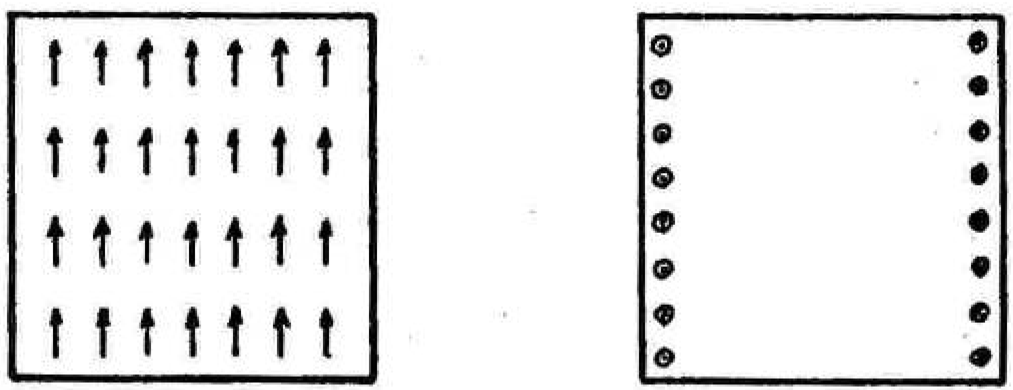


Figure 3.6 Pictorial explanation of magnetization curve in a ferromagnetic bar.

Reference: R. Parker, _Advances in Permanent Magnetism_, John Wiley, 1990, pp. 49

Permanent Magnets

- External effects of PMs can be modeled as surface current



(a) (b)

Figure 1.4. Uniformly magnetized magnet (*a*), which may be modeled by a current density over its boundary (*b*).

Reference: P. Campbell, *Permanent Magnet Materials and their Applications*, Cambridge University Press, 1994, pp. 7

Permanent Magnets

- After magnetization, magnetization vector **M** has value of either $+M_{sat}$ or $–M_{sat}$. H_{ci} is the temperature-dependent field strength (A/m) which causes the **M** vector to flip direction

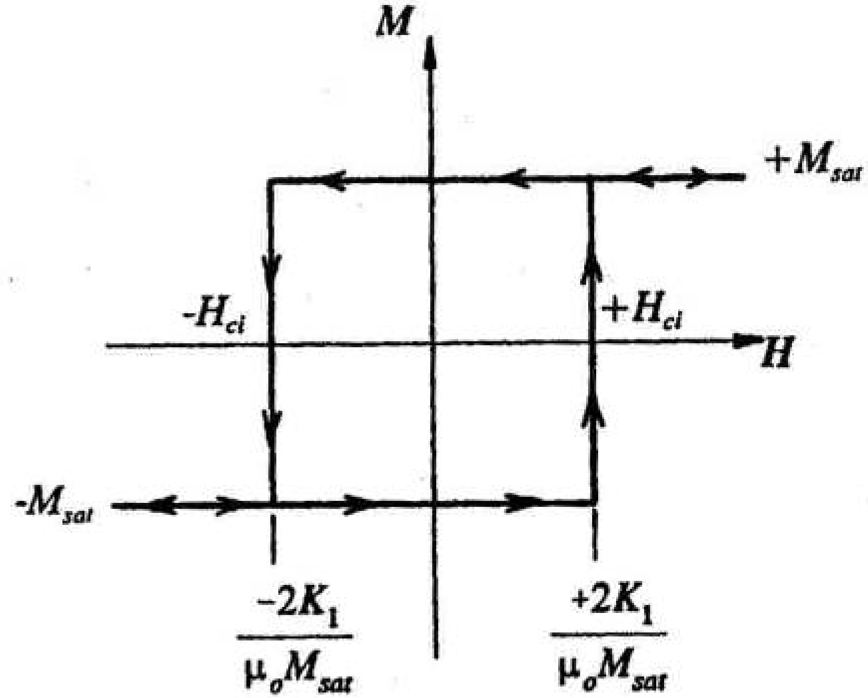


Figure 1.10. Intrinsic magnetization characteristic for an elemental volume of a magnet.

Reference: P. Campbell, *Permanent Magnet Materials and their Applications*, Cambridge University Press, 1994, pp. 14-15

Permanent Magnets with High H_{ci}

- Constitutive relationship: $\mathbf{B} = \mu_o(\mathbf{H}+\mathbf{M})$
- Since **M** has values of either $+M_{sat}$ or $-M_{sat}$, it follows that the slope of the BH curve for the permanent magnet is $\sim\mu_o$; this holds for NdFeB at moderate temperature

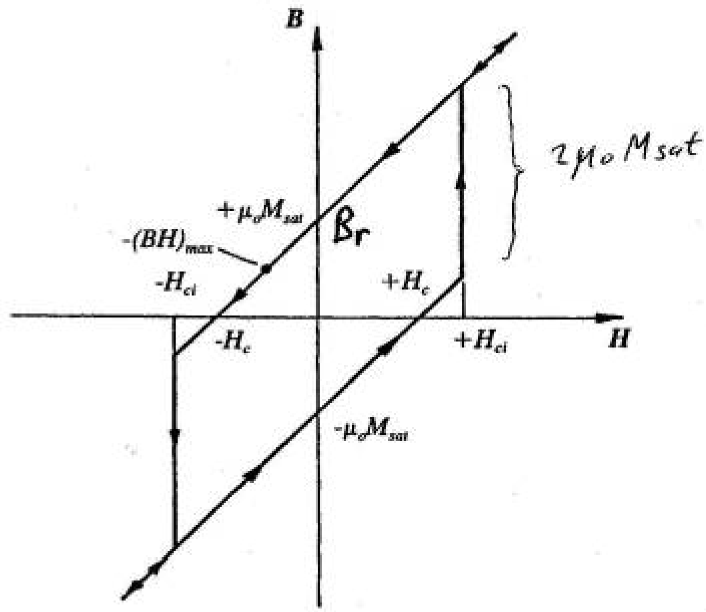


Figure 1.11. *B versus H* characteristic for a magnet. (with $H_{ci} > M_{sat}$)

Reference: P. Campbell, *Permanent Magnet Materials and their Applications*, Cambridge University Press, 1994, pp. 15, 23

Permanent Magnets with Lower H_{ci}

- Note that H_{ci} is temperature-dependent
- At higher temperature, the "knee" in the 2nd quadrant shows up. Operation with H inside the magnet less than H_{ci} results in irreversible demagnetization

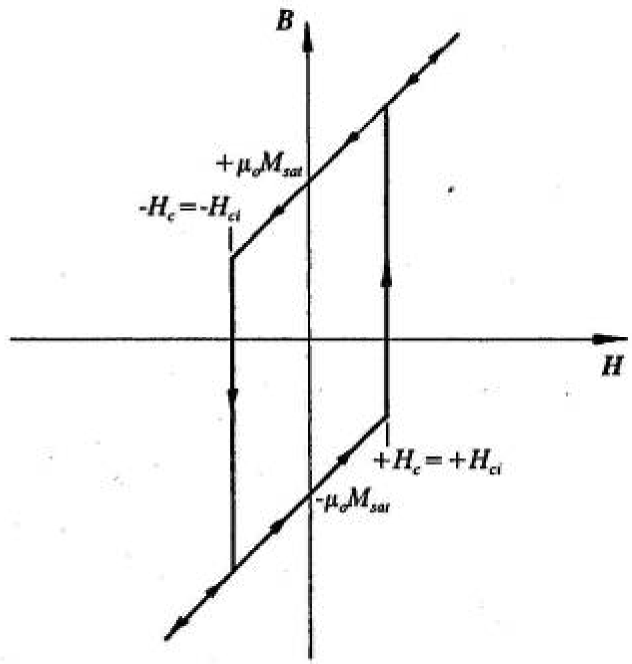


Figure 1.17. ***B*** *versus* ***H*** characteristic for a magnet with $|H_{ci}| < M_{sat}$.

Reference: P. Campbell, *Permanent Magnet Materials and their Applications*, Cambridge University Press, 1994, pp. 15, 23

Demagnetization Curves of Ceramic 8

- Typical sintered ceramic magnet

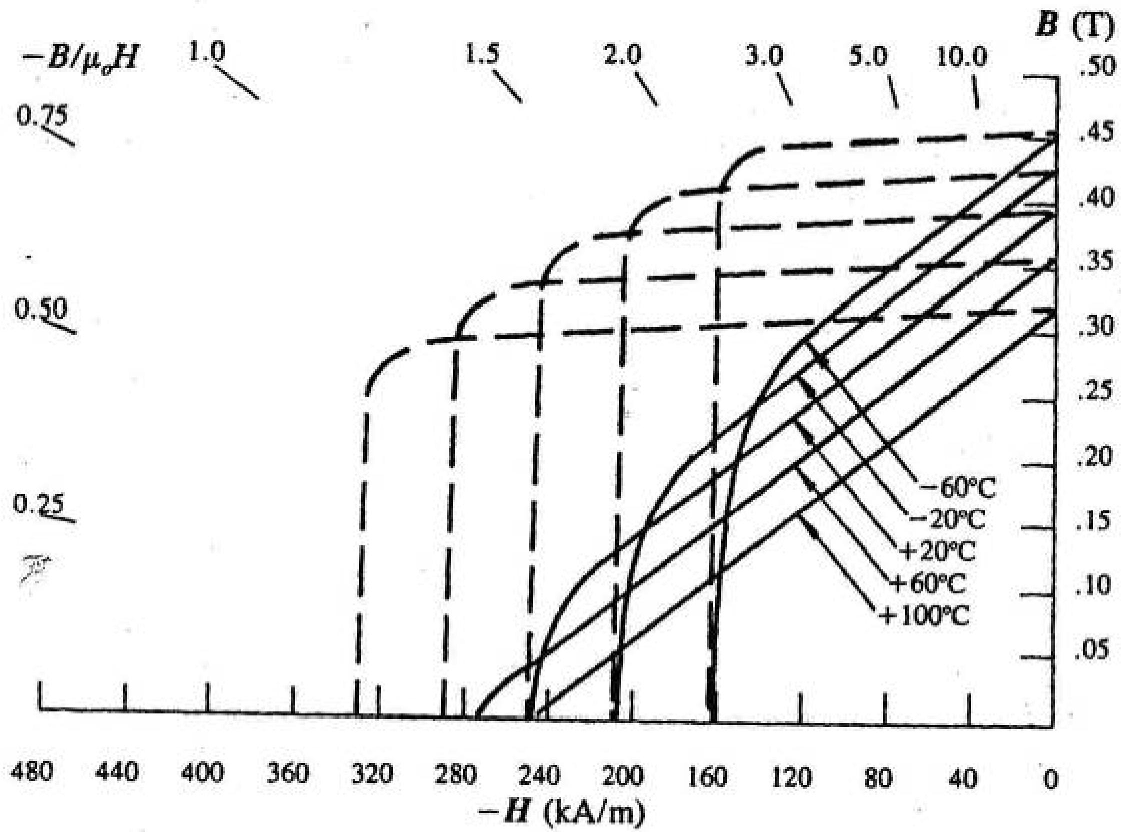


Figure 3.3. Demagnetization curves of Ceramic 8 at various temperatures.

Reference: P. Campbell, *Permanent Magnet Materials and their Applications*, Cambridge University Press, 1994, pp. 62

Demagnetization Curves of NdFeB

- Strong neodymium-iron-boron

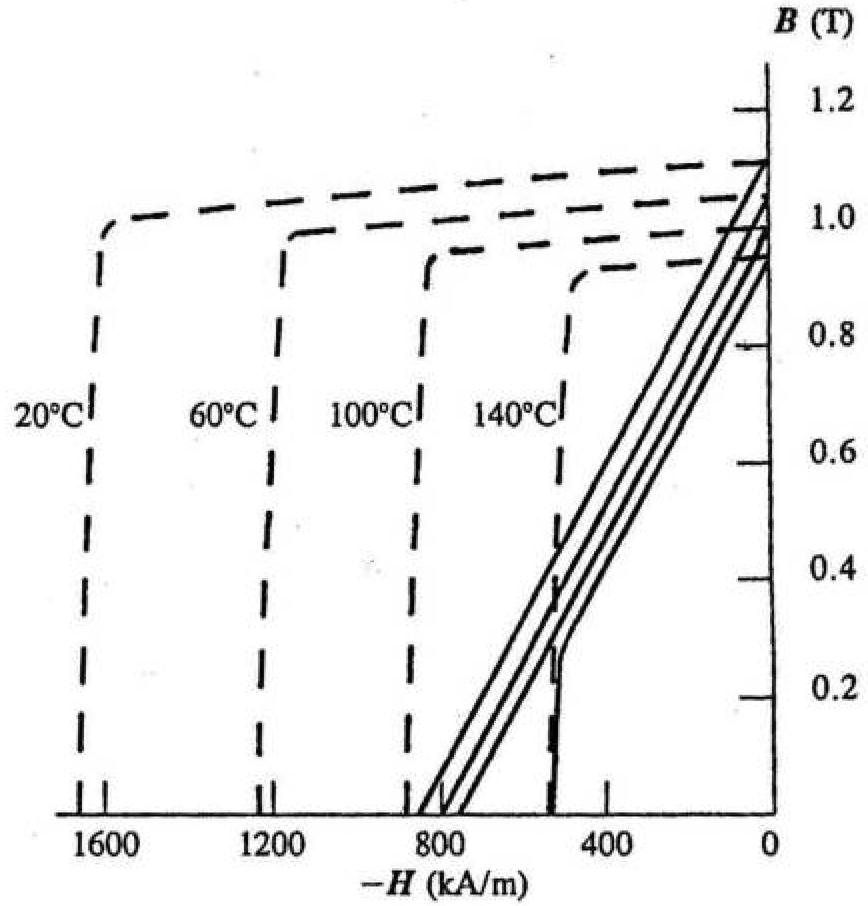


Figure 3.14. Demagnetization curves of (Nd, Dy)–Fe–B at various temperatures.

Reference: P. Campbell, *Permanent Magnet Materials and their Applications*, Cambridge University Press, 1994, pp. 74

Curie Temperature

- The “Curie temperature” is the temperature at which the magnetization is totally destroyed.
- The practical maximum operating temperature for a permanent magnet is well below the Curie temperature

Reference: P. Campbell, *Permanent Magnet Materials and their Applications*, Cambridge University Press, 1994, pp. 74

Maximum Working Temperature

- The practical maximum operating temperature for a permanent magnet is well below the Curie temperature

Magnetic materials have a wide range of working temperatures. The following chart list the various materials and their maximum working temperature. NdFeB material comes in many different heat tolerances but as the heat tolerance increases the maximum available flux density decreases:

Material	Maximum Working Temperature	
	ºC	ºF
Ceramic	400	752
Alnico	540	1004
SmCo 1,5	260	500
SmCo 2, 17	350	662
NdFeB N	80	176
NdFeB M	100	212
NdFeB H	120	248
NdFeB SH	150	302
NdFeB UH	180	356
NdFeB EH	200	392

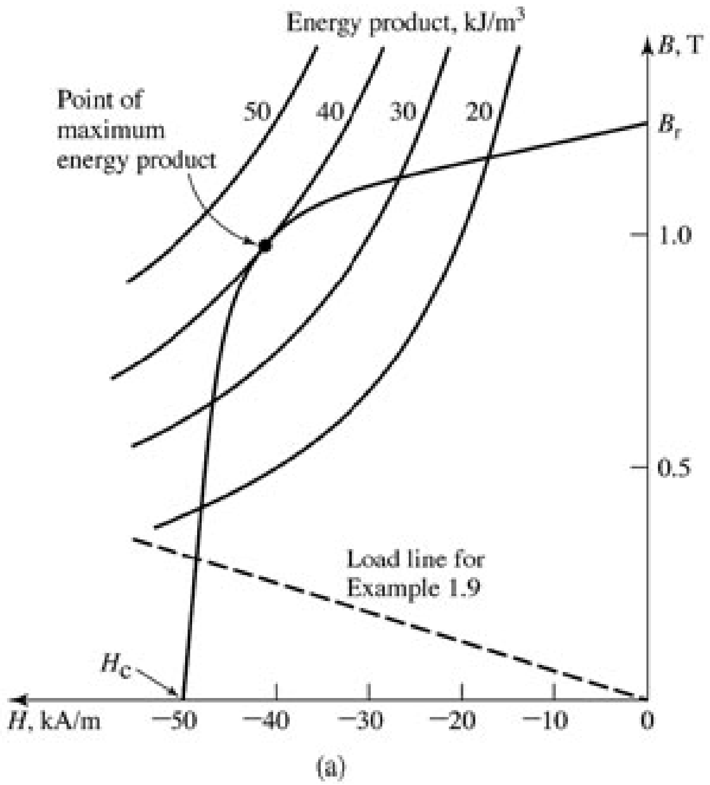
Table 3 - Working Temperatures

Reference: Magcraft, “Permanent Magnet Selection and Design Handbook”

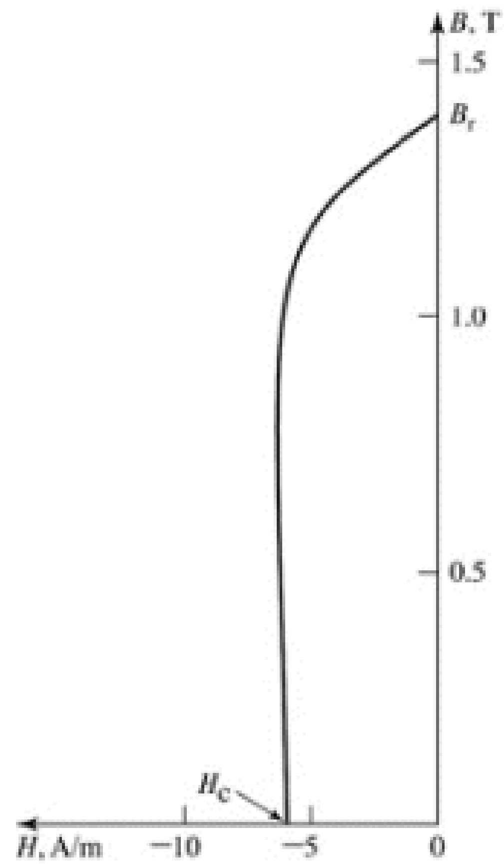
Permanent Magnets vs. Steel

- Note that PM has much higher coercive force

Permanent magnet: Alnico 5

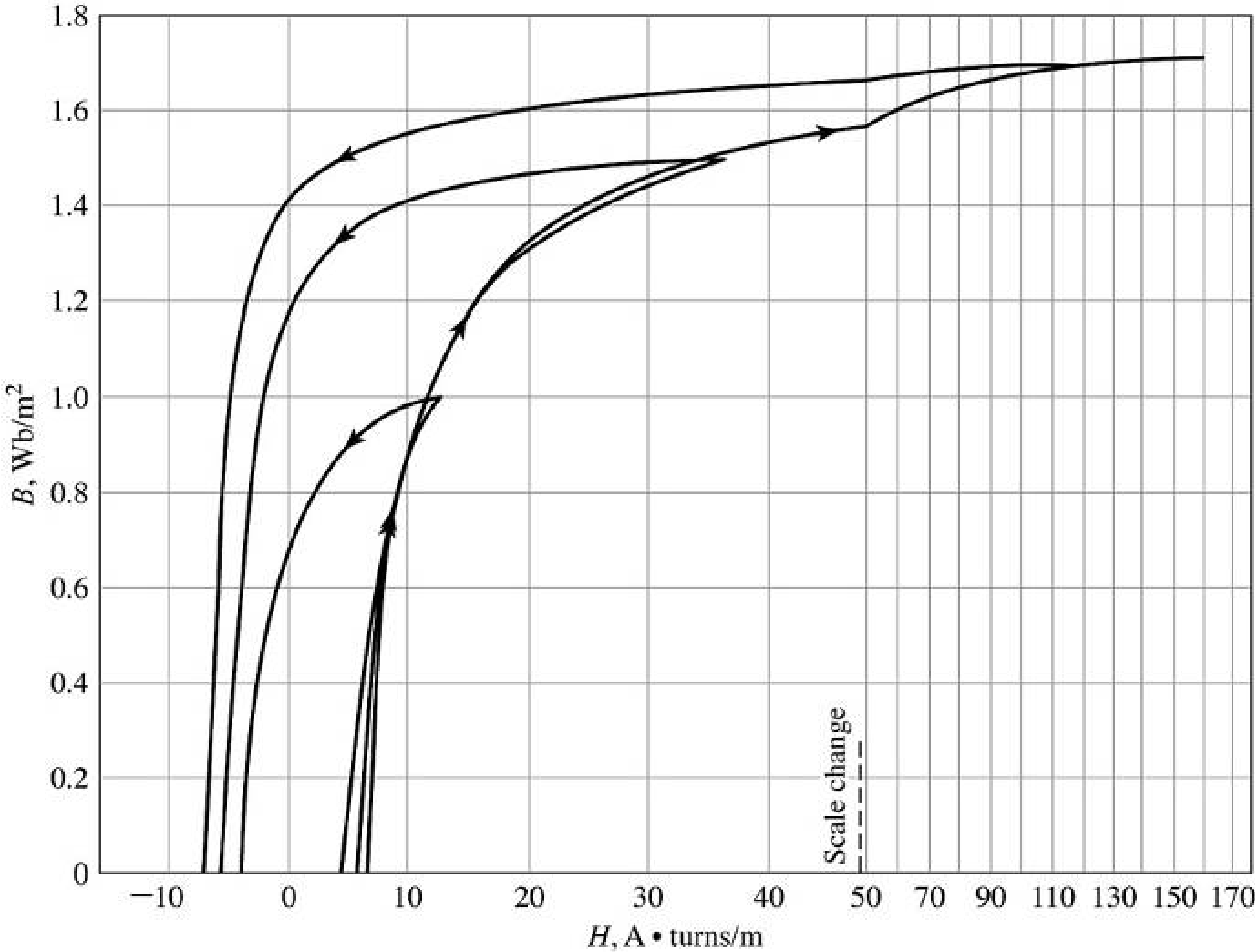


M-5 steel



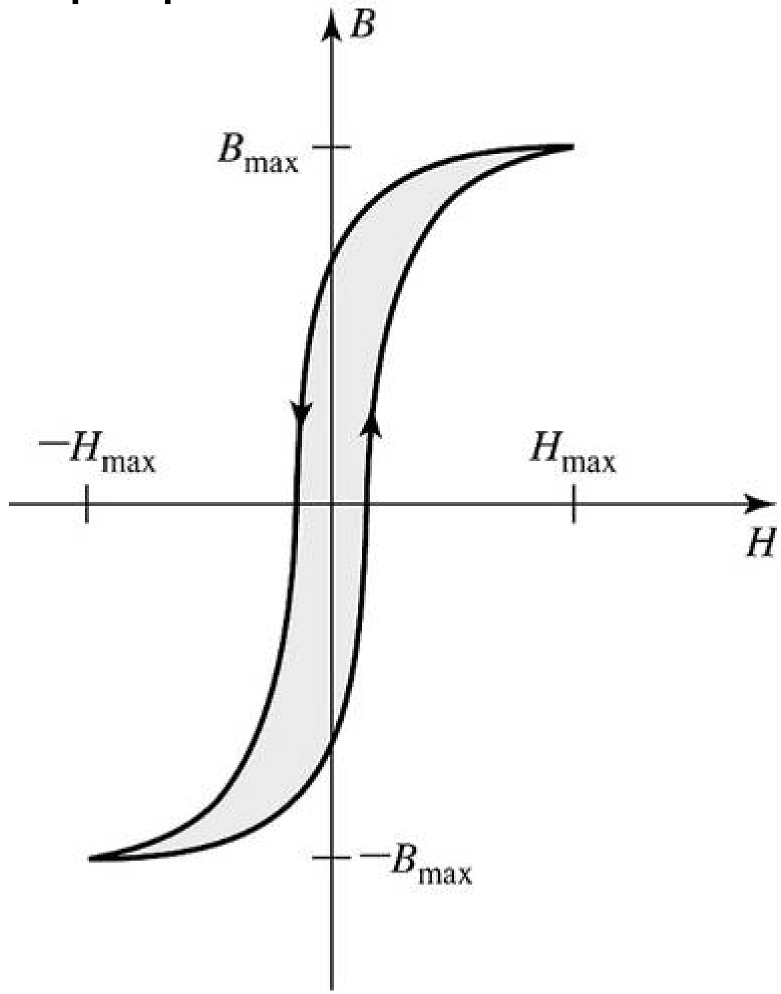
B-H Loop for M-5 Grain-Oriented Steel

- Only the top half of the loops shown for steel 0.012” thick



Hysteresis Loop

- Hysteresis loss proportional to shaded area



Permanent Magnets

- "Soft" magnetic materials such as magnetic steel can behave as very weak permanent magnets
- Permanent magnets, or "hard" magnetic materials, have a high coercive force H_c and can produce significant flux in an airgap

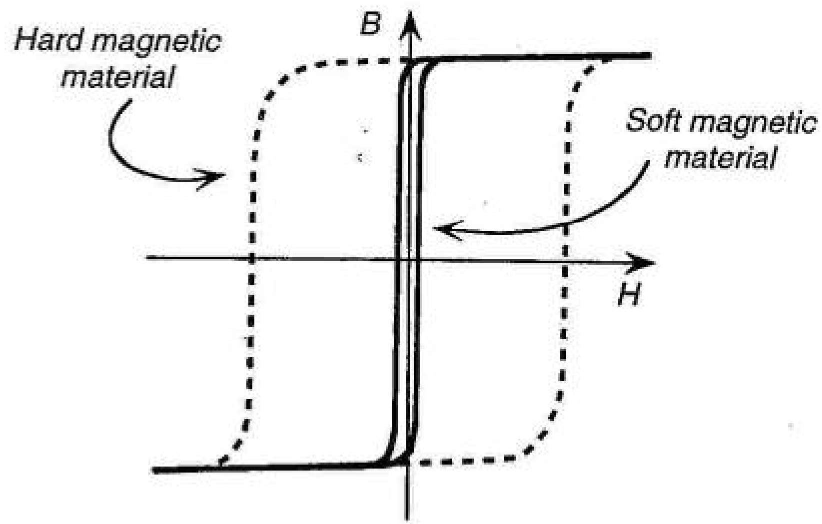


FIGURE 1.25 The *B-H* loops for soft and hard magnetic materials.

Reference: E. Furlani, *Permanent Magnet and Electromechanical Devices*, Academic Press, 2001, pp. 39

Cast Alnico

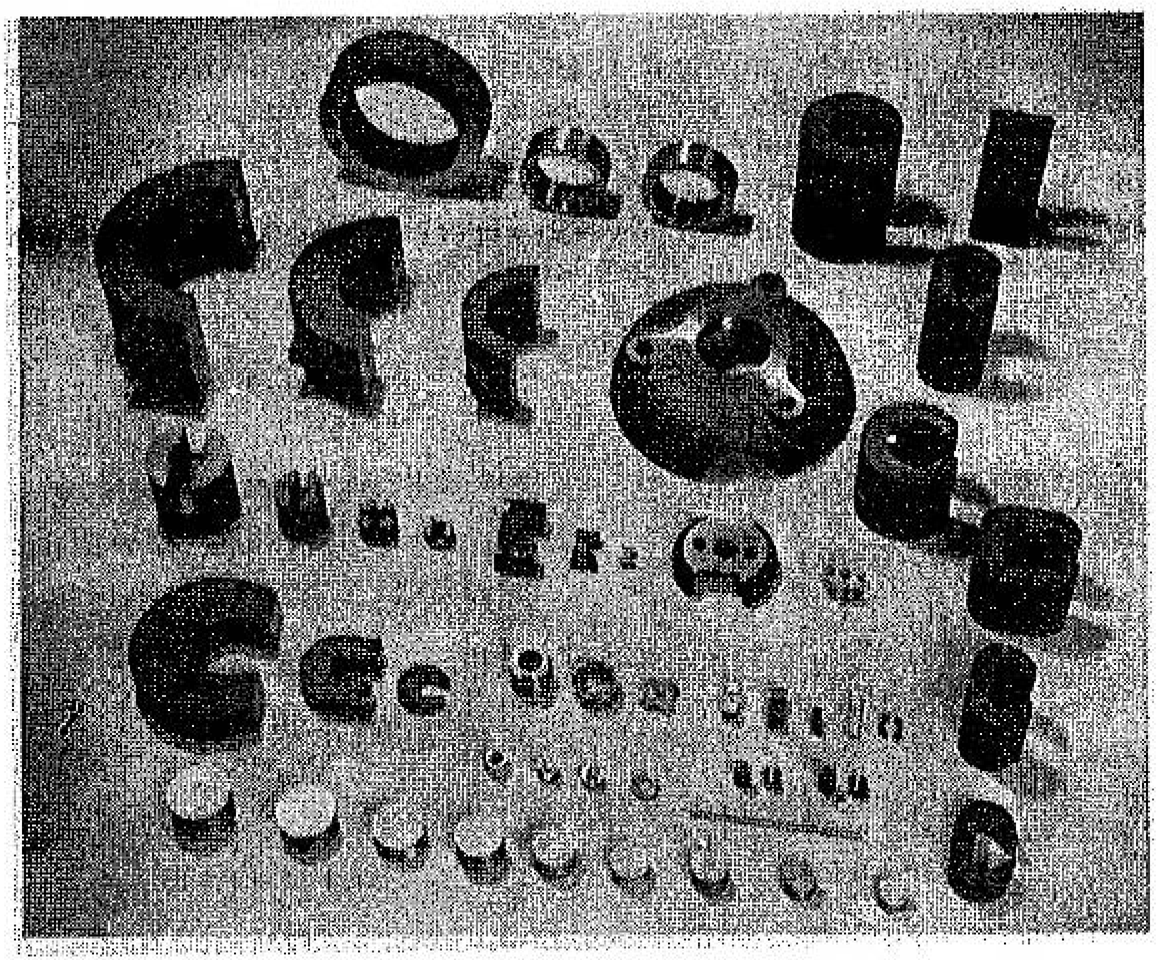
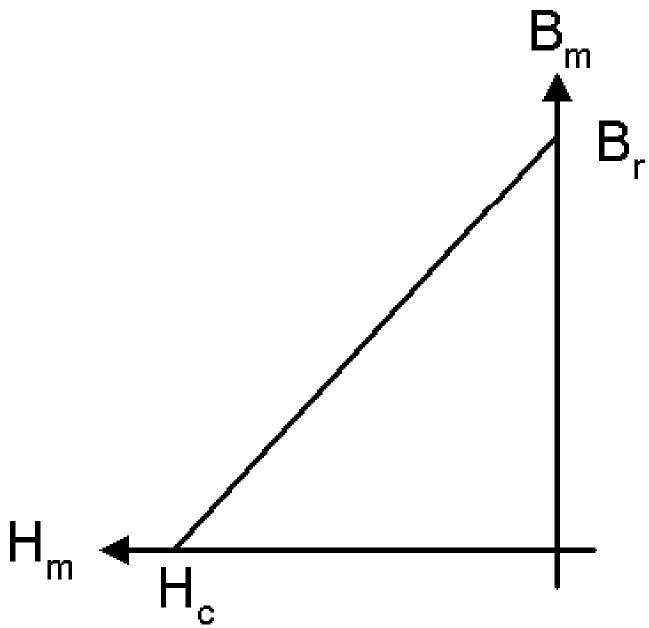


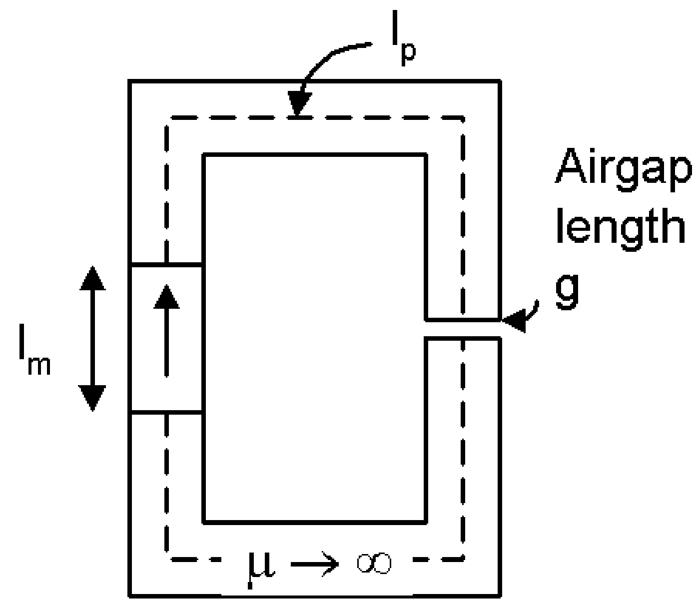
Figure 4.3 Typical cast Alnico magnet configurations.

Reference: R. Parker, *Advances in Permanent Magnetism*, John Wiley, 1990, pp. 65

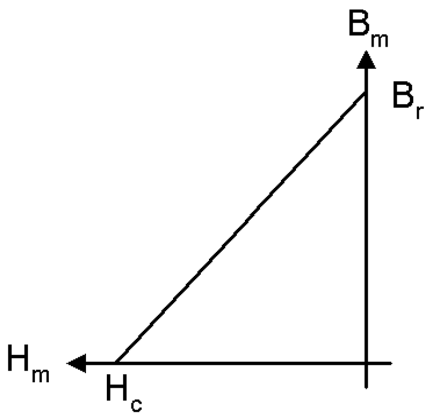
Example 1: Permanent Magnet in a Magnetic Ckt

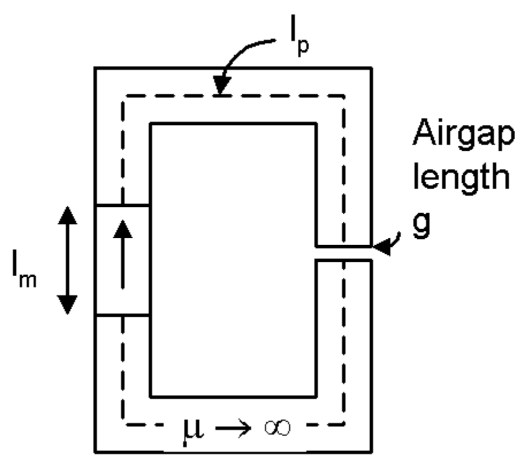
- This example uses NdFeB which has linear B/H curve in the 3rd quadrant





Example 1: Permanent Magnet in a Magnetic Ckt





Using Ampere's law (and noting that our approximation of infinite μ assures that H = 0 in the steel) we note:

$$H_m l_m + H_g g = 0$$

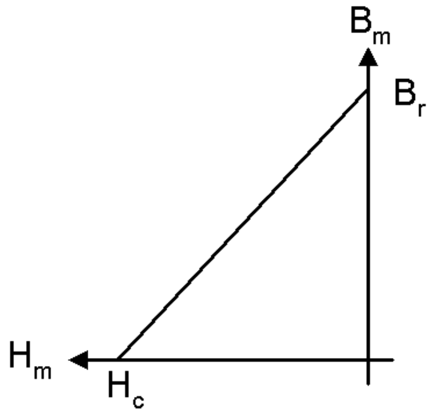
where H_g is the magnetic field in the airgap. Next we use Gauss' magnetic law which says that flux is continuous around a loop, to get:

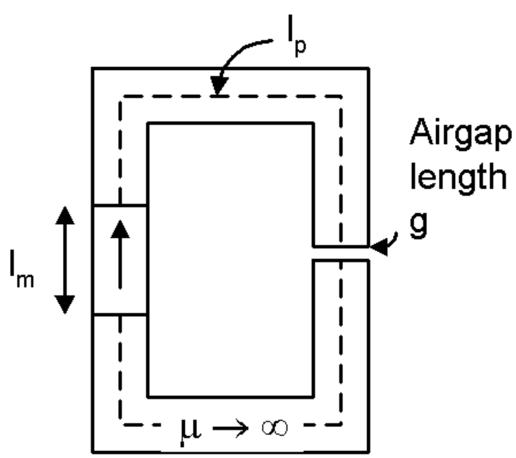
$$B_m A_m = B_g A_g$$

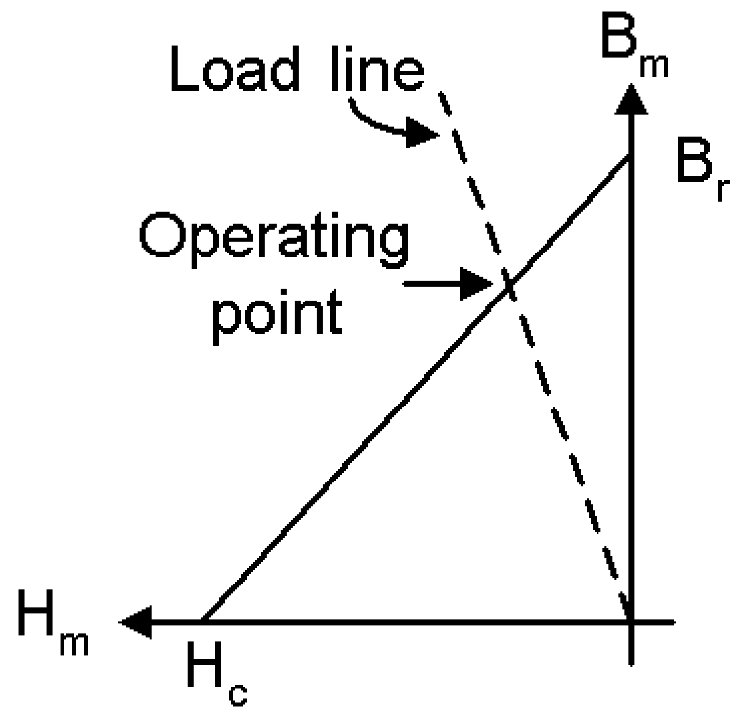
where A_m is the cross-sectional area of the permanent magnet and B_g is the cross-sectional area of the airgap. Noting that $B_g = \mu_o H_g$, we next solve for B_m, as a function of H_m, resulting in the load line equation:

$$B_m = -\mu_o \left(\frac{l_m A_g}{g A_m} \right) H_m$$

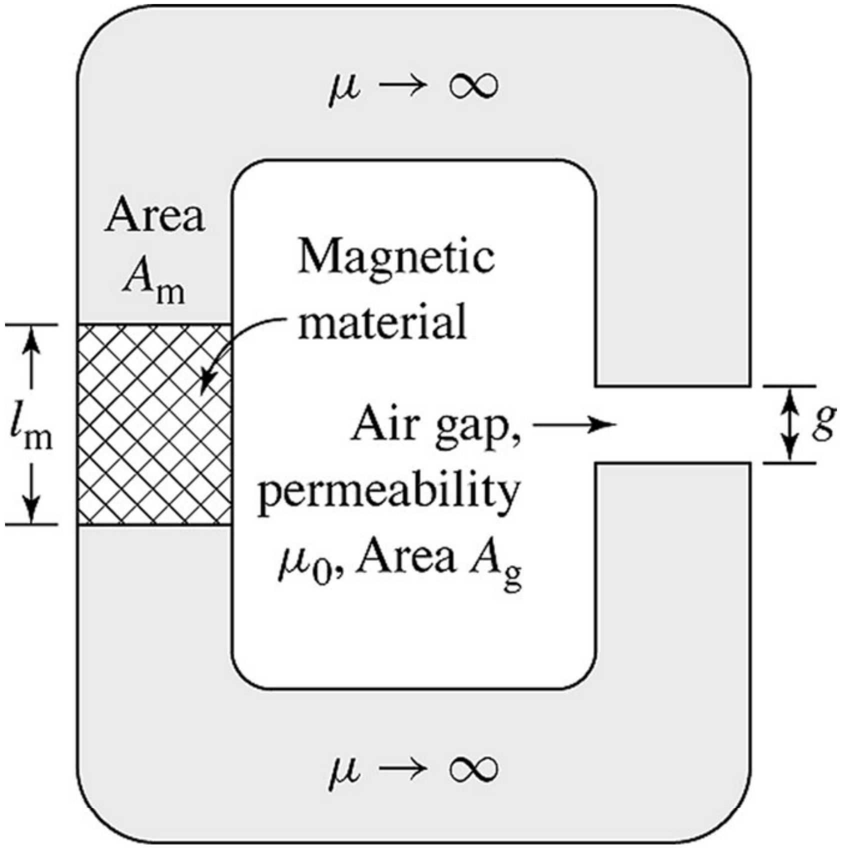
Example 1: Permanent Magnet in a Magnetic Ckt







Example 2: Permanent Magnet in a Magnetic Ckt



Example:

$g = 0.2$ cm

$l_m = 1.0$ cm

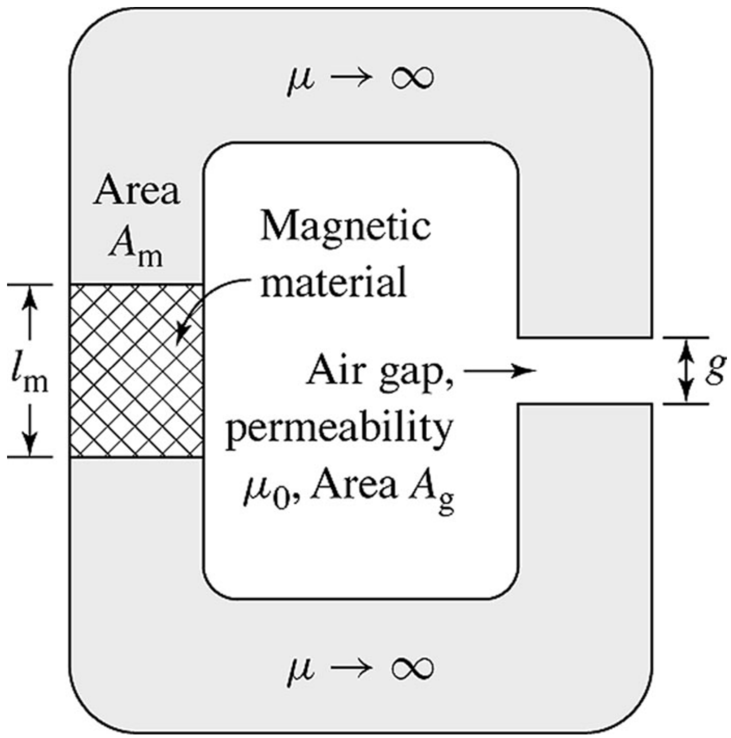
$A_m = A_g = 4$ cm^2

Find flux in airgap B_g for magnetic material

(a) Alnico 5

(b) M-5 steel

Example 2: With Alnico Permanent Magnet



$NI = 0$, so:

$$H_m l_m + H_g g = 0 \quad \text{(Ampere's Law)}$$

① $$H_g = -H_m \left(\frac{l_m}{g}\right)$$

Flux is continuous:

② $$A_g B_g = A_m B_m \rightarrow B_g = B_m \left(\frac{A_m}{A_g}\right)$$

Solve for B_m as a function of H_m

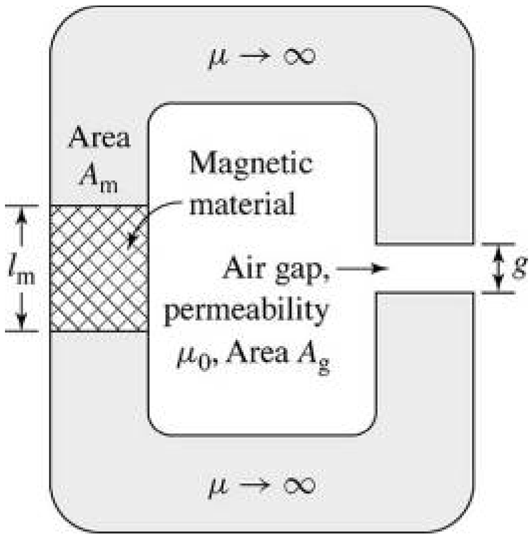
$$B_m = B_g \frac{A_g}{A_m} = \mu_0 H_g \frac{A_g}{A_m}$$

$$B_m = \mu_0 \left(-H_m \frac{l_m}{g}\right)\left(\frac{A_g}{A_m}\right) = -6.28 \times 10^{-6} H_m$$

Plot this load line on Alnico BH curve

Example 2: With Alnico Permanent Magnet

- Result: B_g = 0.3 Tesla



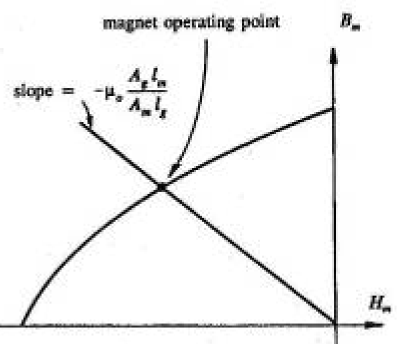
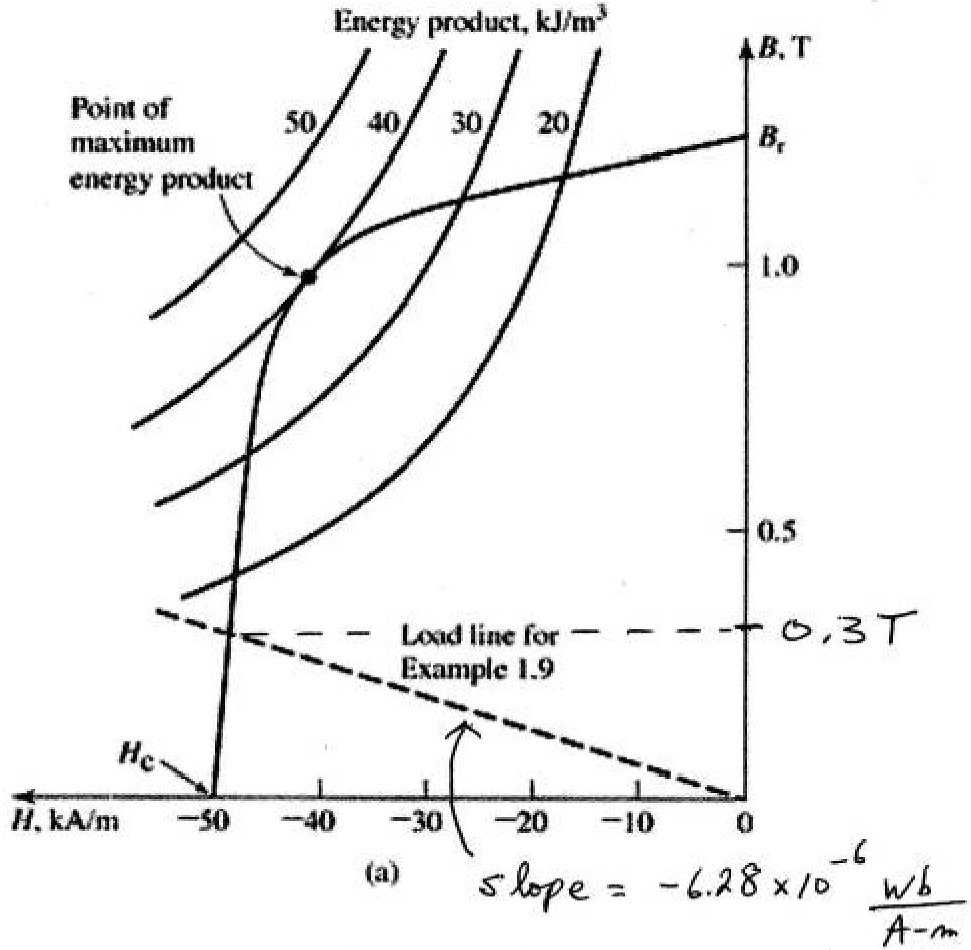


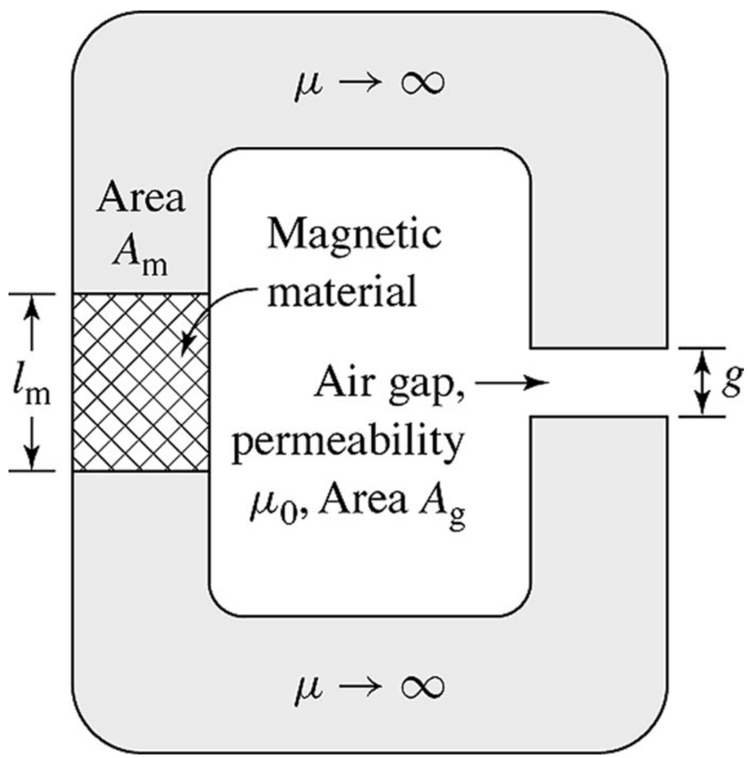
Figure 4.5. Demagnetization curve and load line for a magnet.

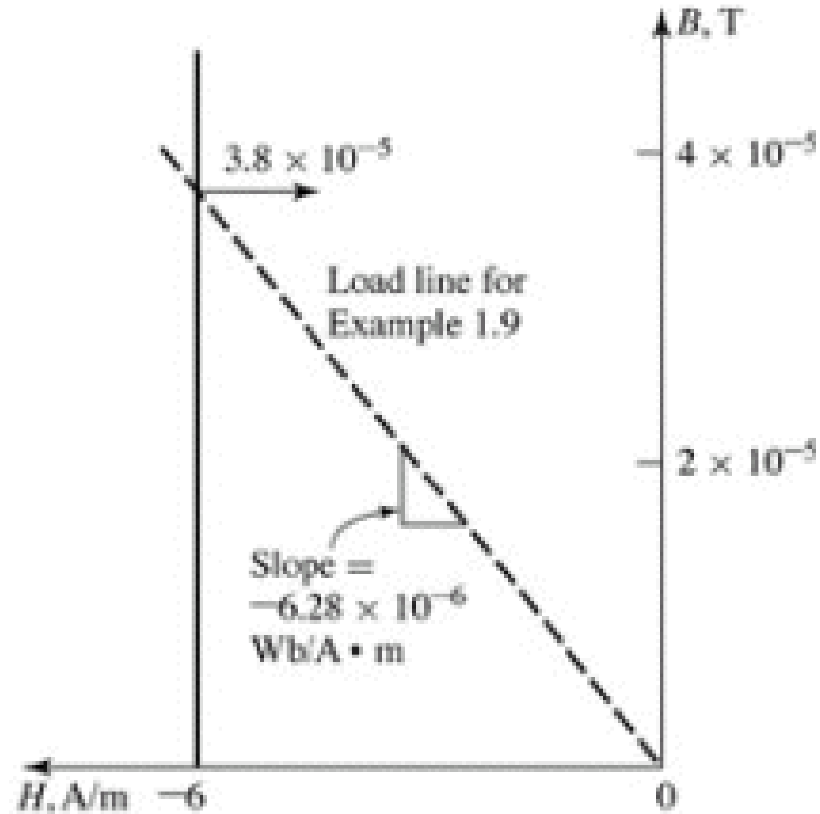


Reference: P. Campbell, *Permanent Magnet Materials and their Applications*, Cambridge University Press, 1994, pp. 89

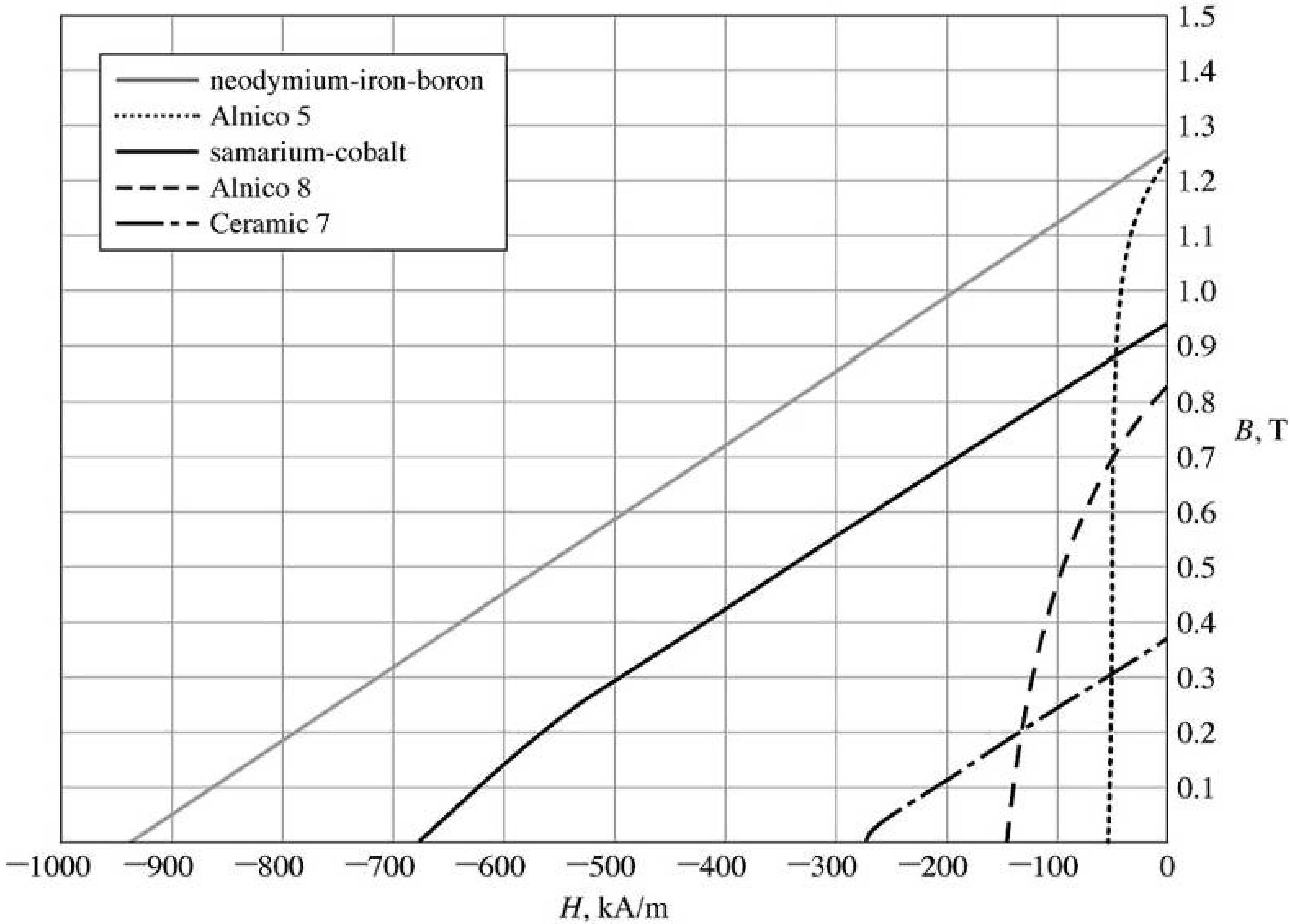
Example 2: Load Line Solution with M-5 Steel

- Use same load line; B_g = 0.38 Gauss (much lower than with Alnico)
- Note: Earth's magnetic field ~ 0.5 Gauss



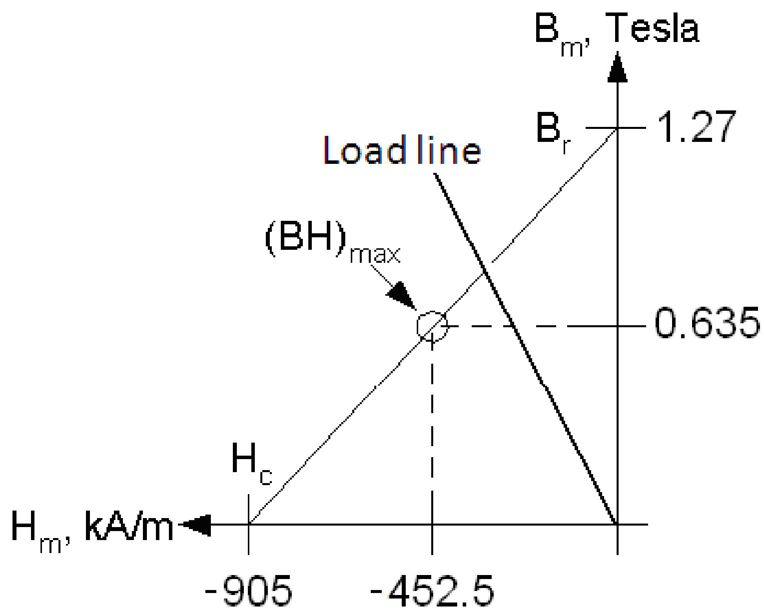


Some Common Permanent Magnet Materials



Typical NdFeB B-H Curve

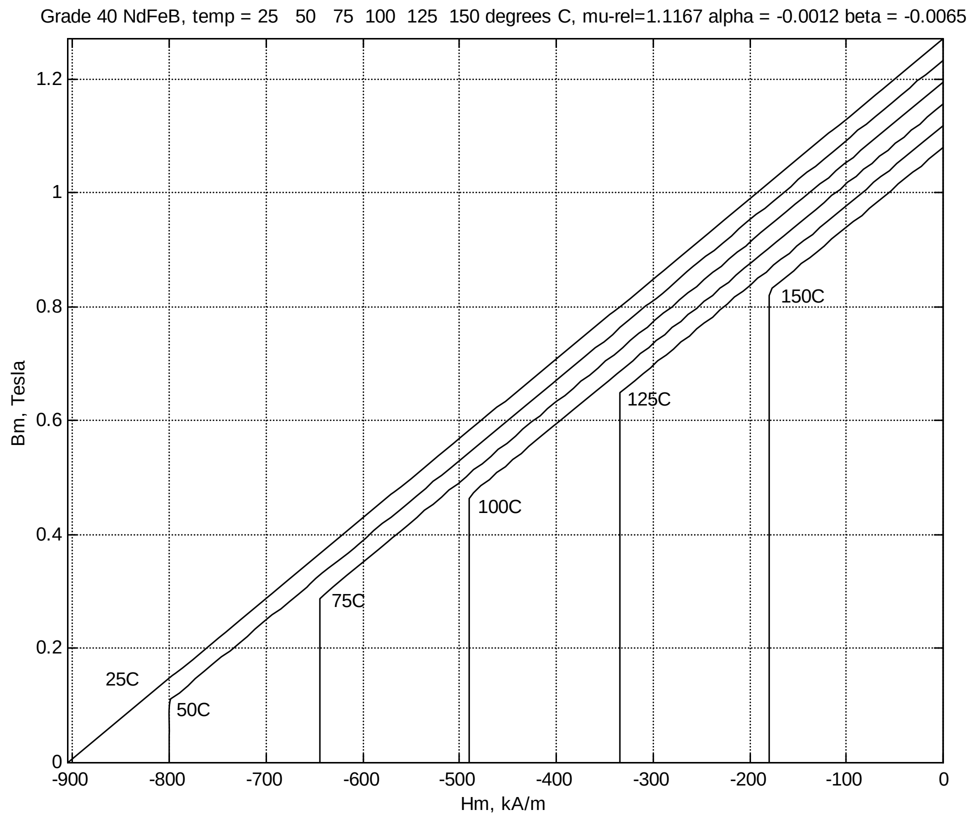
- Neodymium-iron-boron (NdFeB) is the highest strength permanent magnet material in common use today
- Good material for applications with temperature less than approximately 80 - 150C
- Cost per pound has reduced greatly in the past few years
- Curve below for “grade 40” or 40 MGOe material



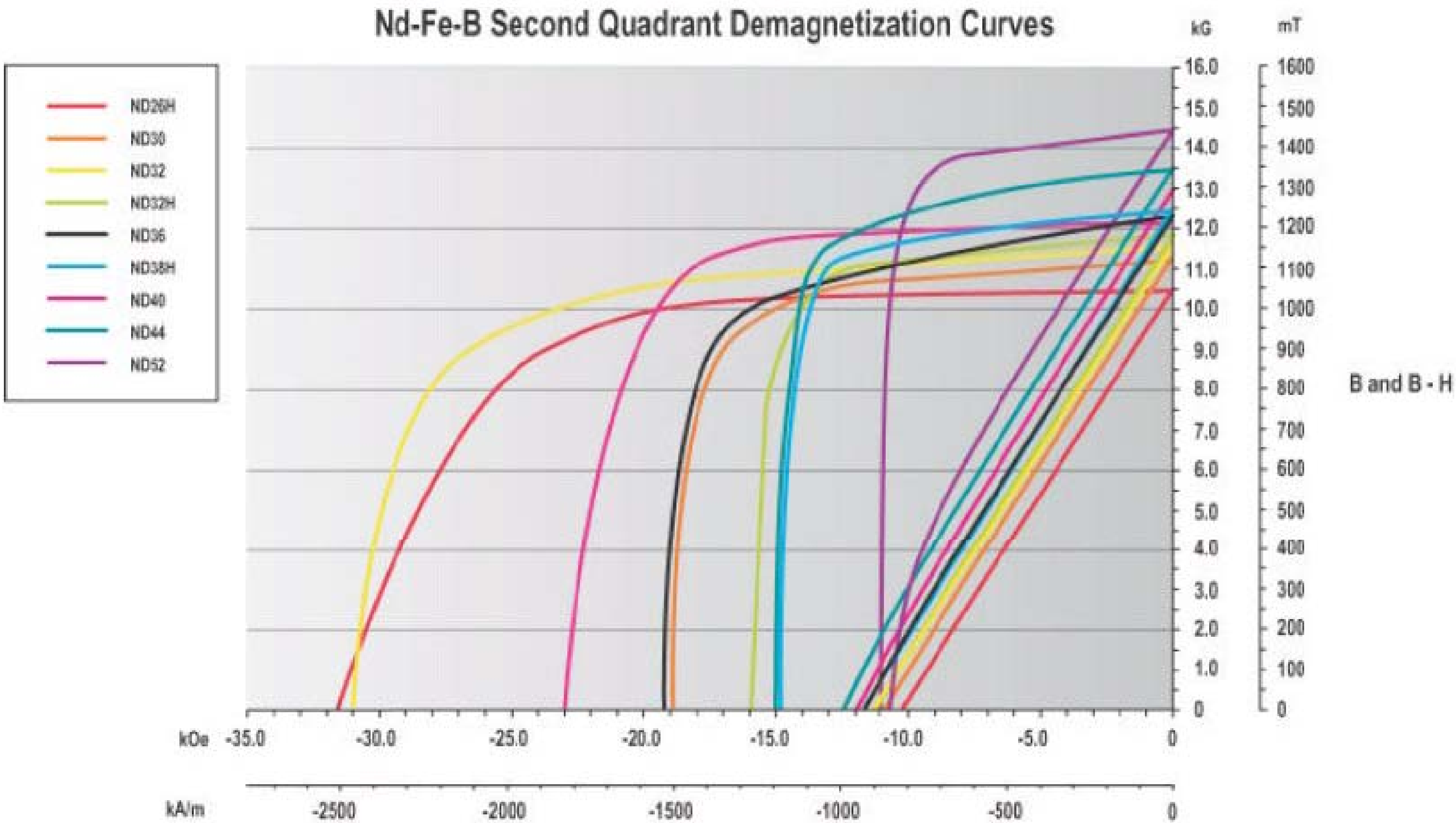
Typical NdFeB B-H Curve --- Elevated Temp.

- Note that the B/H curve degrades at elevated temperatures
- If you operate the magnet below the "knee" irreversible demagnetization may result

Item	Grade 40
B_r	1.27 T
H_c	905 kA/m
H_{ci}	955 kA/m
α	-0.0012
β	-0.0065
Max. working temperature T_{max} as reported by manufacturer	80 °C



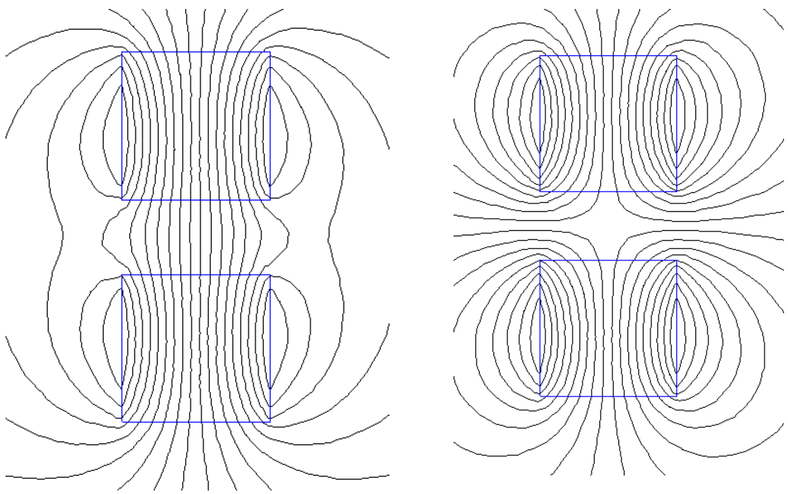
NdFeB B-H Curves for Different Grades

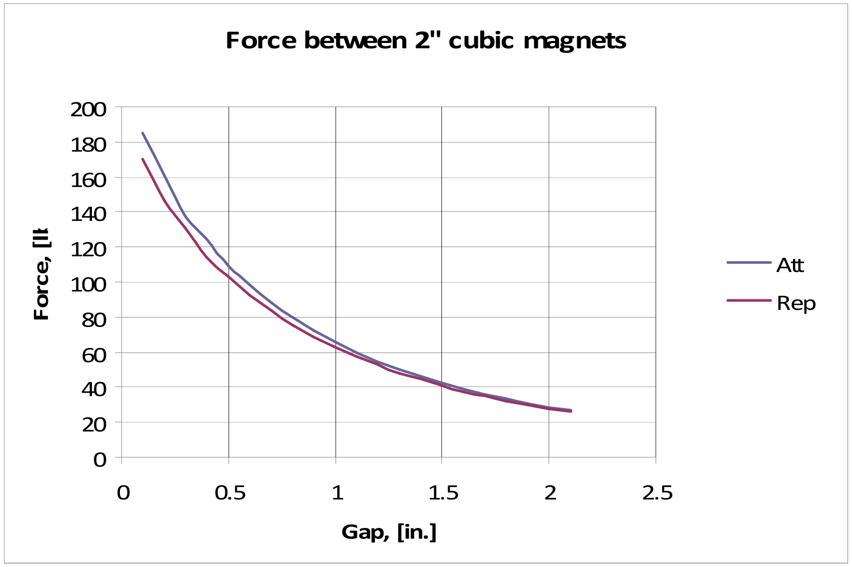


Reference: Dexter Magnetics, Inc. http://www.dextermag.com

Example 3: Force Between 2” NdFeB Magnet Cubes vs. Airgap

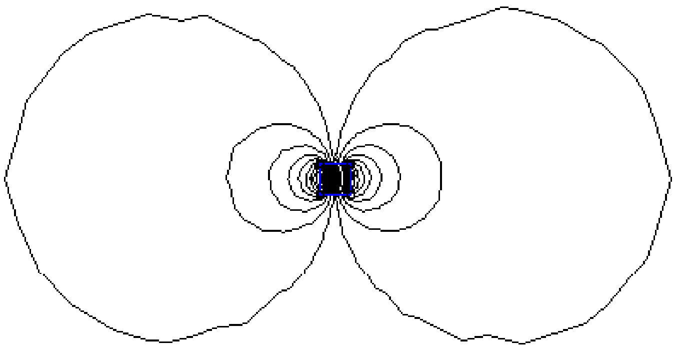
Attraction Repulsion

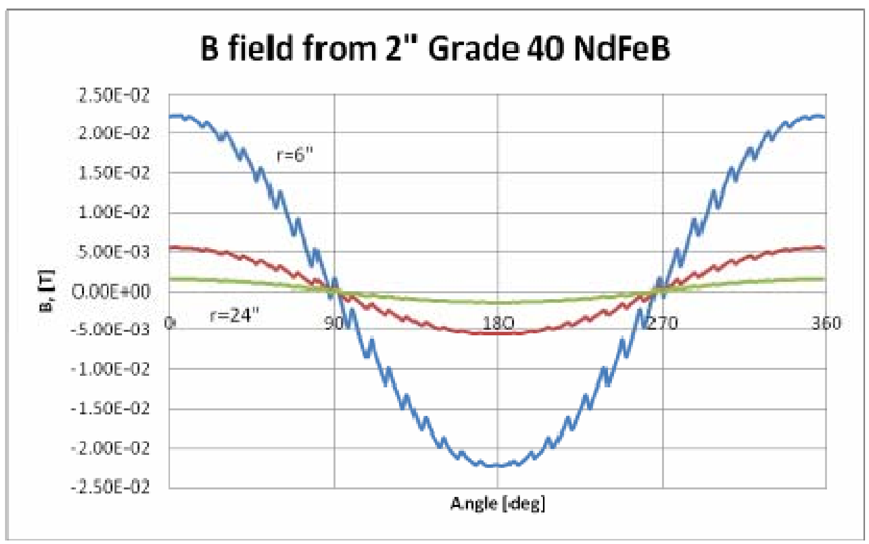




Example 4: Stray Fields from 2” Grade 40 Magnet Cube

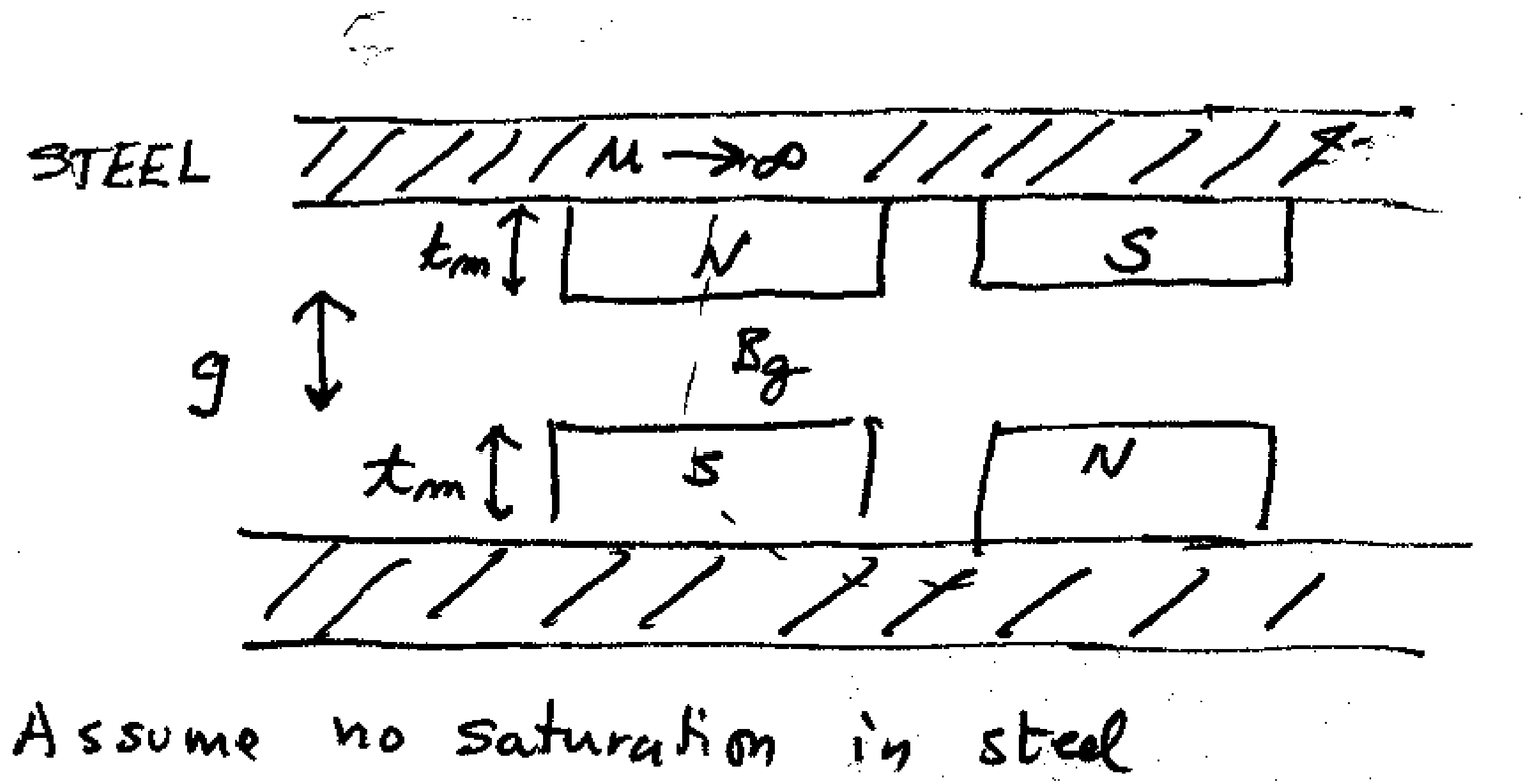
- At distances r = 6”, 12” and 24”





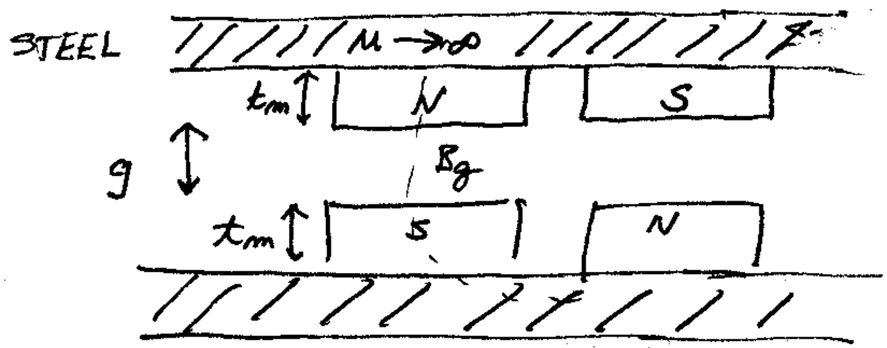
Example 5: Magnetic Circuit With Steel

- Estimate airgap field B_g assuming grade 42 NdFeB
- Airgap g, magnet thickness t_m



Example 5: Magnetic Circuit With Steel

- This analysis also ignores leakage



Assume no saturation in steel

① $$2 H_m t_m + H_g g = 0$$
(Ampere's law)

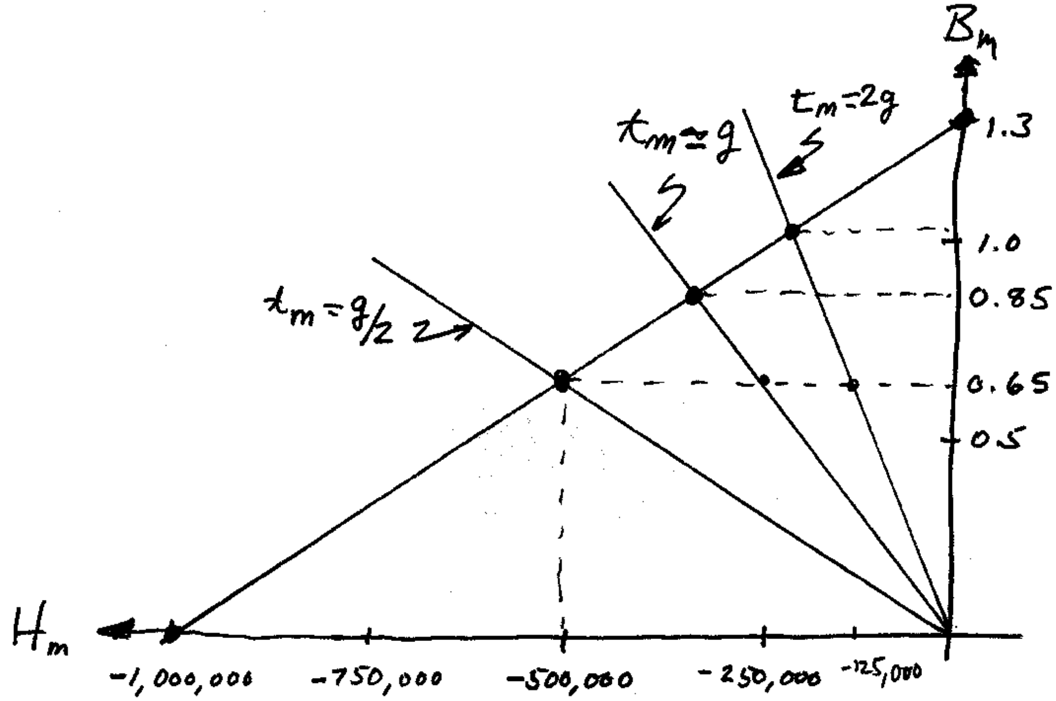
② $$H_g = \frac{B_g}{\mu_0} = \frac{B_m}{\mu_0}$$
(constitutive relation)

$$2 H_m t_m + \frac{B_m g}{\mu_0} = 0$$

$$\frac{B_m g}{\mu_0} = -2 H_m t_m$$

$$\therefore \frac{B_m}{H_m} = \frac{-2 t_m \mu_0}{g}$$

Example 5: Magnetic Circuit With Steel --- Operating Point vs. Magnet Thickness t_m

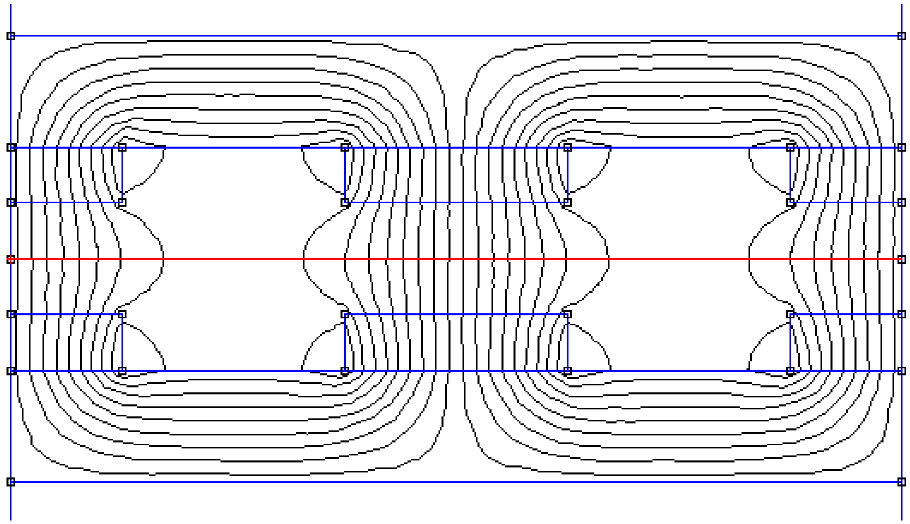


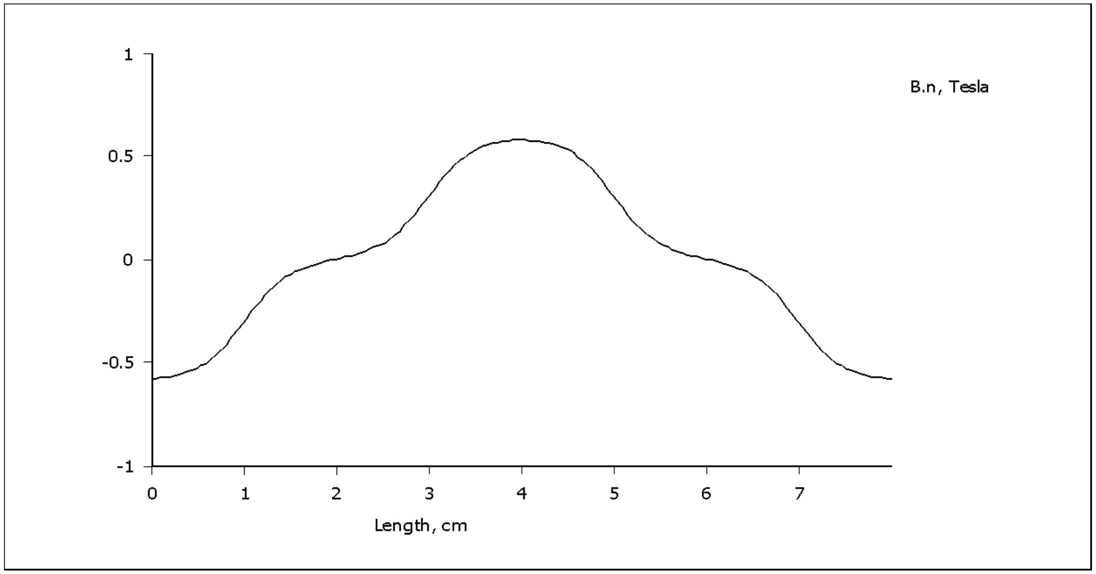
$t_m = g/2 \rightarrow B_g \sim 0.65\,T$

$t_m \approx g \rightarrow B_g \sim 0.85\,T$

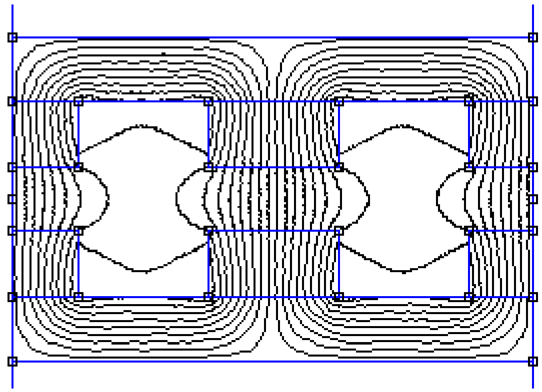
$t_m = 2g \rightarrow B_g \sim 1.05\,T$

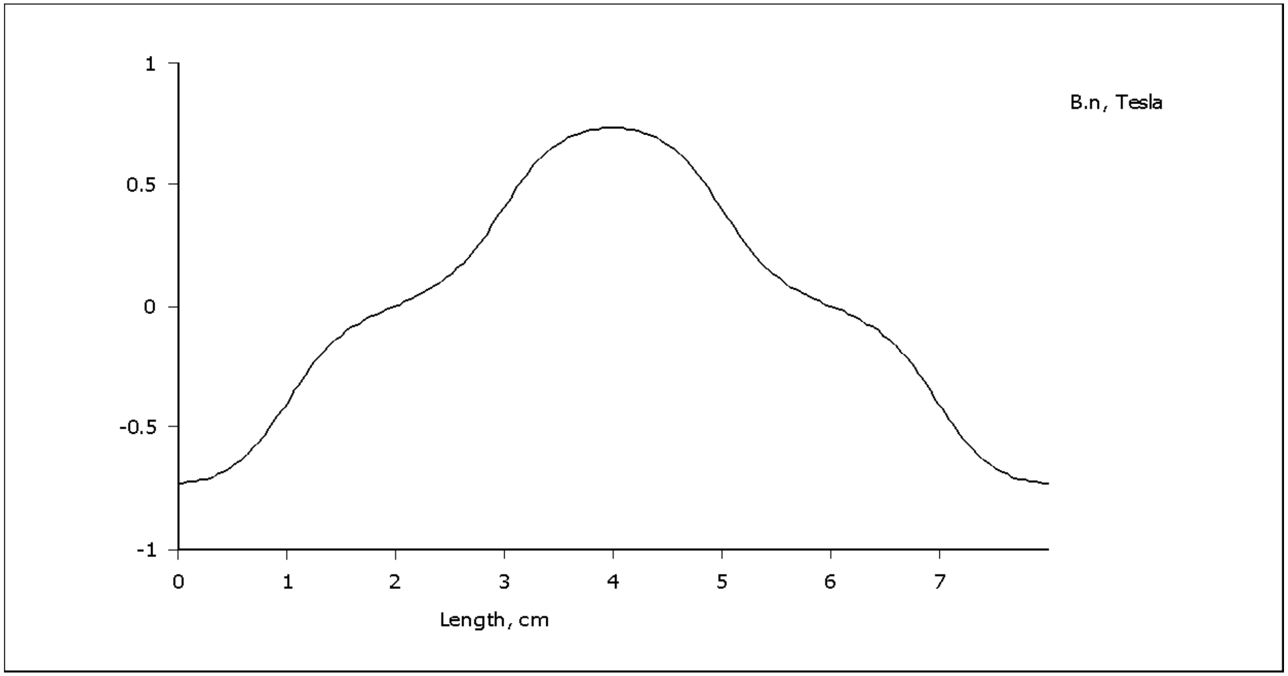
Example 5: 2D FEA, Magnet Thickness $t_m = g/2$



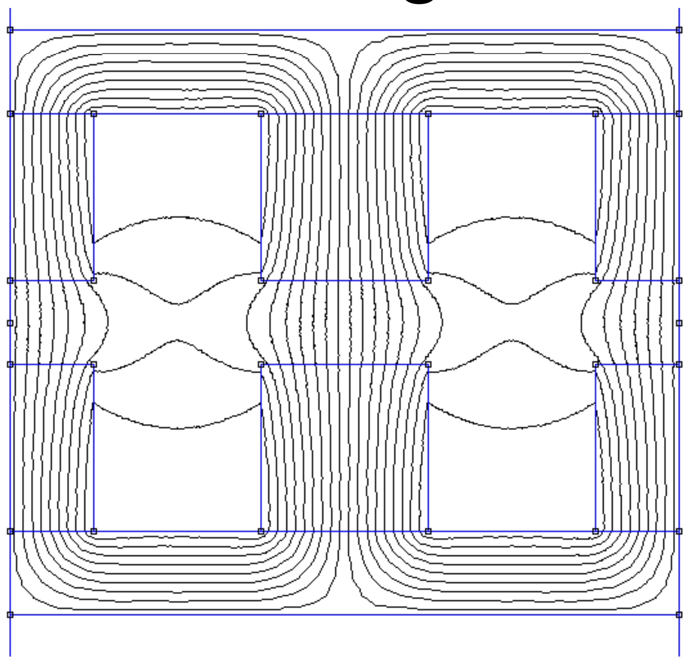


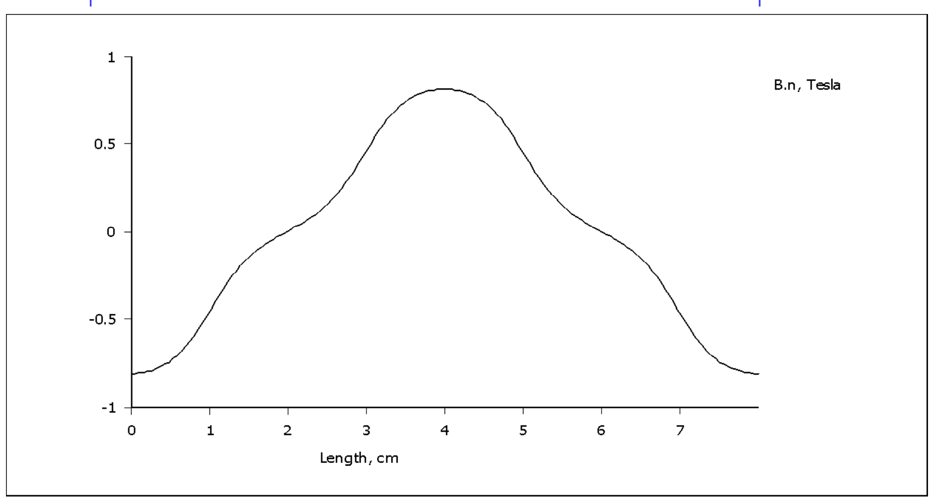
Example 5: 2D FEA, Magnet Thickness $t_m = g$





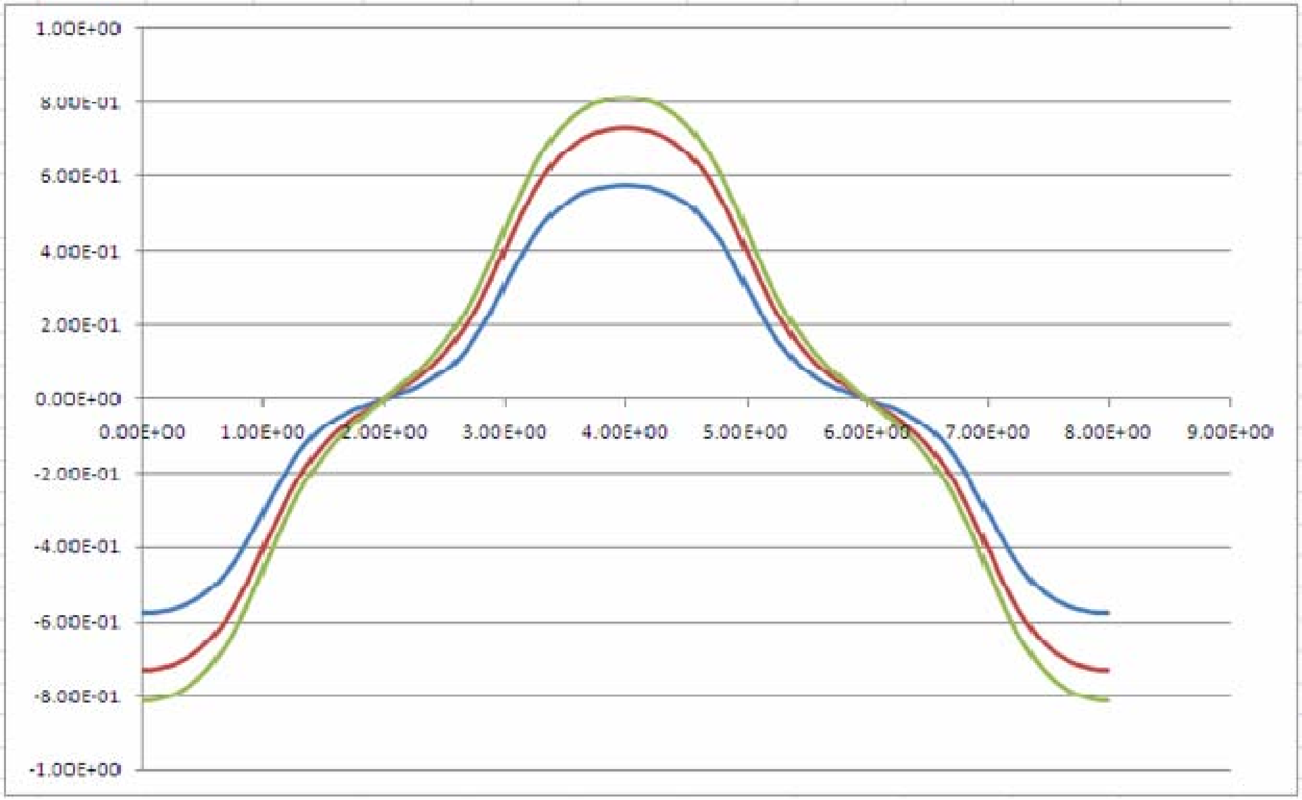
Example 5: 2D FEA, Magnet Thickness $t_m = 2g$





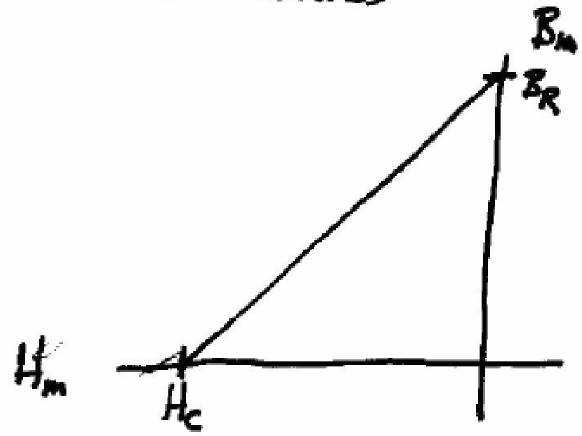
Example 5: Comparison of Different Magnet Thicknesses

	Bpk, 2D analytic	Bpk, 2D FEA
tm = g/2	0.65T	0.58T
tm = g	0.85T	0.73T
tm=2g	1.05T	0.82T



Circuit Modeling of Permanent Magnets

SOME MAGNETS (I.E. ND-FE-B) HAVE LINEAR 2ND QUADRANT DEMAG CURVES



① $B_m = B_R + \mu_m H_m$

and

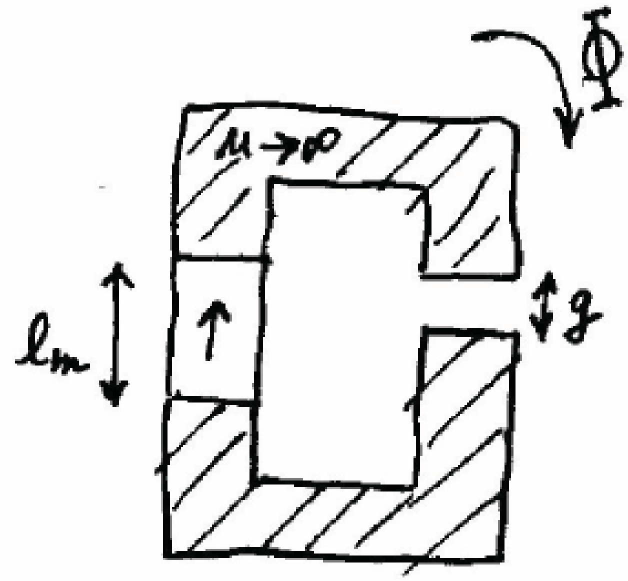
② $\mu_m = \dfrac{B_R}{H_c}$

③ $H_m = \dfrac{B_m}{\mu_m} - H_c$

Reference: E. P. Furlani, *Permanent Magnet and Electromechanical Devices*, Academic Press, 2001

Circuit Modeling of Permanent Magnets

LET'S USE THIS IN A MAGNETIC CIRCUIT WITH AIRGAP:



AMPERE'S LAW

(4) $H_m l_m + H_g g = 0$

Circuit Modeling of Permanent Magnets

PUT ③ INTO ④

$$\left(\frac{B_m}{\mu_m} - H_c\right) l_m + H_g g = 0$$

$$\Downarrow$$

$$⑤ \quad H_g g + \frac{B_m}{\mu_m} l_m = H_c l_m$$

OR $$\frac{B_g g}{\mu_o} + \frac{B_m l_m}{\mu_o} = H_c l_m$$

TOTAL FLUX:

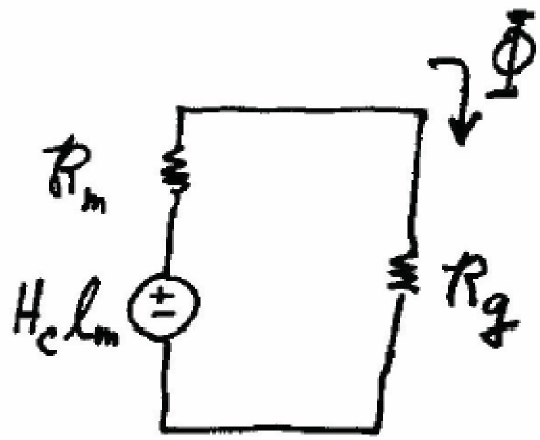
$$\Phi = B_g A_g = B_m A_m$$

SO, ⑤ REDUCES TO:

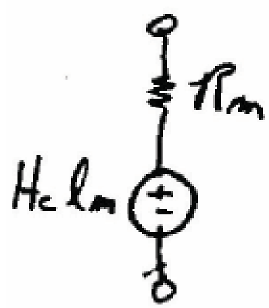
$$\Phi (\mathcal{R}_g + \mathcal{R}_m) = H_c l_m$$

Circuit Modeling of Permanent Magnets

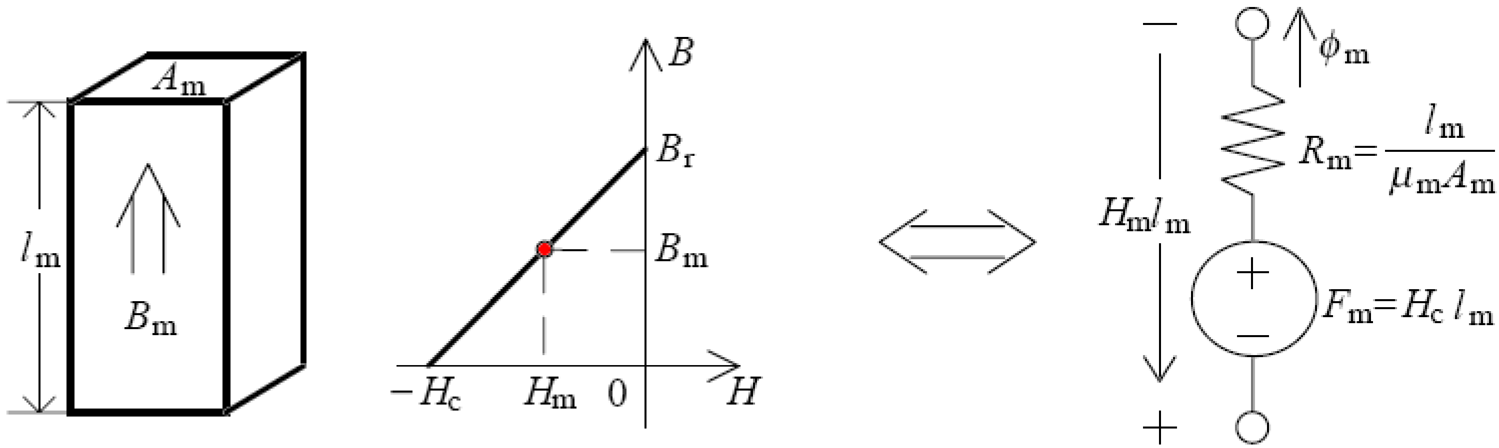
WE CAN MODEL THIS AS FOLLOWS:



PM MODEL:



Circuit Modeling of Permanent Magnets



Magnetic circuit model of a magnet with linear demagnetization curve

Example 6: Circuit Modeling of Permanent Magnets

Example:

Grade 37 NdFeB

$H_c = 950{,}000$ A/m

$\mu_m = 1.048\,\mu_o$

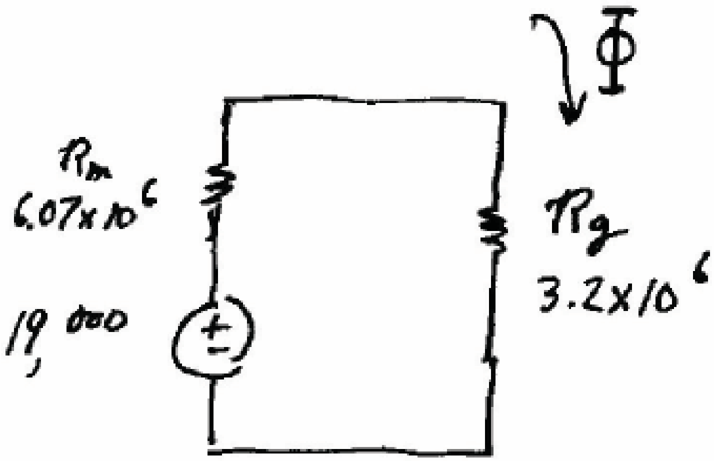
$l_m = 2$ cm

$g = 1$ cm

$A_m = A_g = 25\,cm^2$

Find B_g

Example 6: Circuit Modeling of Permanent Magnets



This example is particularly simple because $A_m = A_g$

Solution:

$$H_c l_m = (950{,}000)(0.02) = 19{,}000$$

$$\mathcal{R}_m = \frac{l_m}{\mu_R A_m} = \frac{0.02}{(1.048)(4\pi\times10^{-7})(25\times10^{-4})} = 6.07\times10^{6}$$

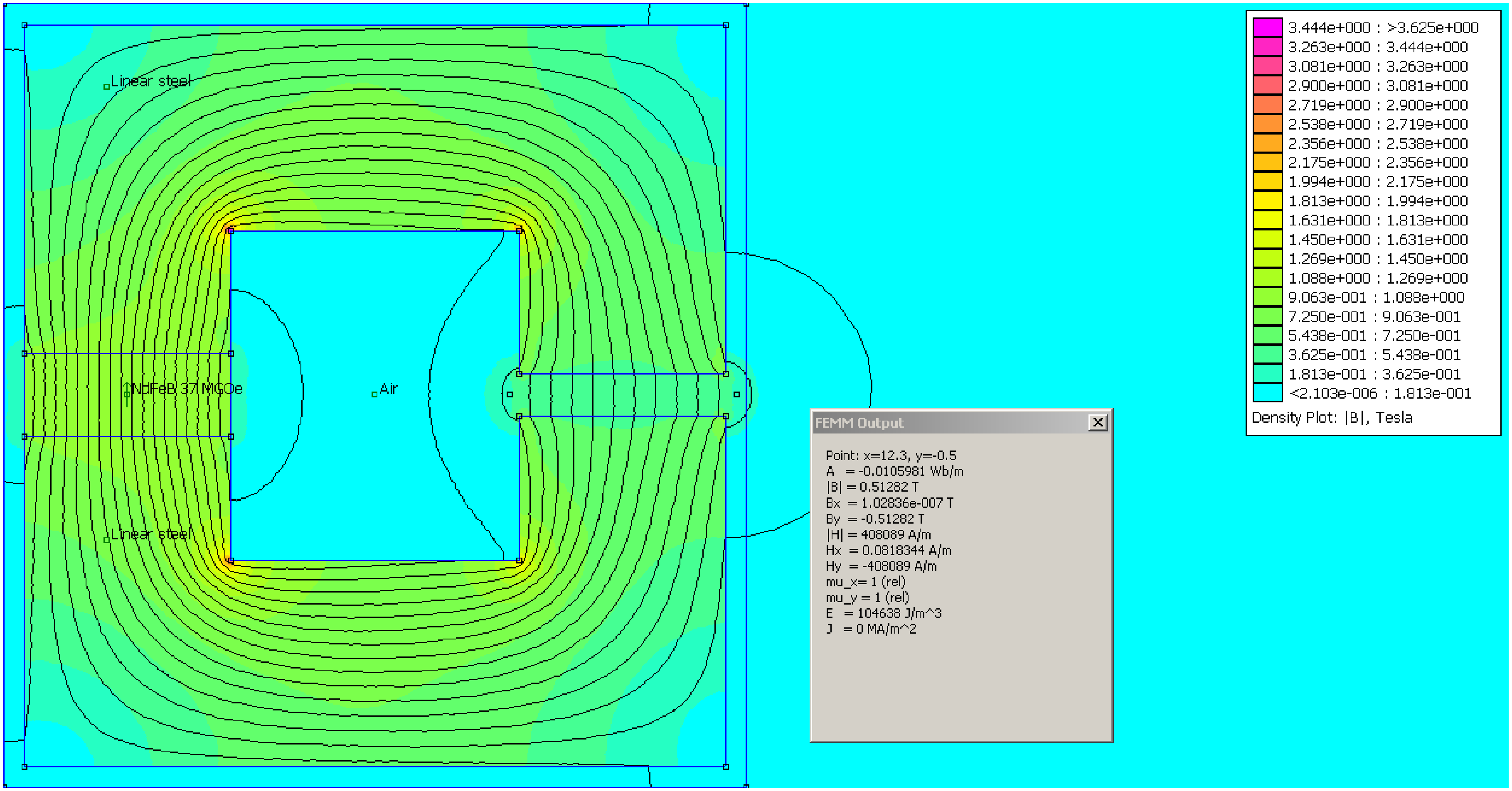
$$\mathcal{R}_g = \frac{g}{\mu_0 A_g} = \frac{0.01}{(4\pi\times10^{-7})(25\times10^{-4})} = 3.2\times10^{6}$$

$$\Phi = \frac{H_c l_m}{\mathcal{R}_m + \mathcal{R}_g} = 2.04\times10^{-3}\ Wb$$

$$B_g = \frac{\Phi}{A_g} = \frac{2.04\times10^{-3}}{25\times10^{-4}}$$

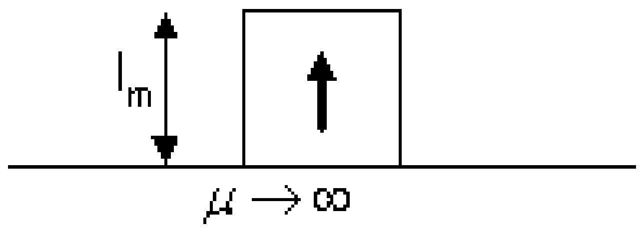
$\therefore B_g = 0.82\ Wb/m^2$
0.82 Tesla

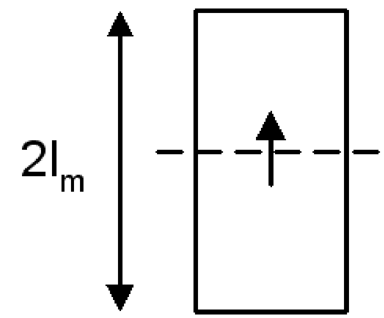
Example 6: Circuit Modeling of Permanent Magnets---FEA



Method of Images

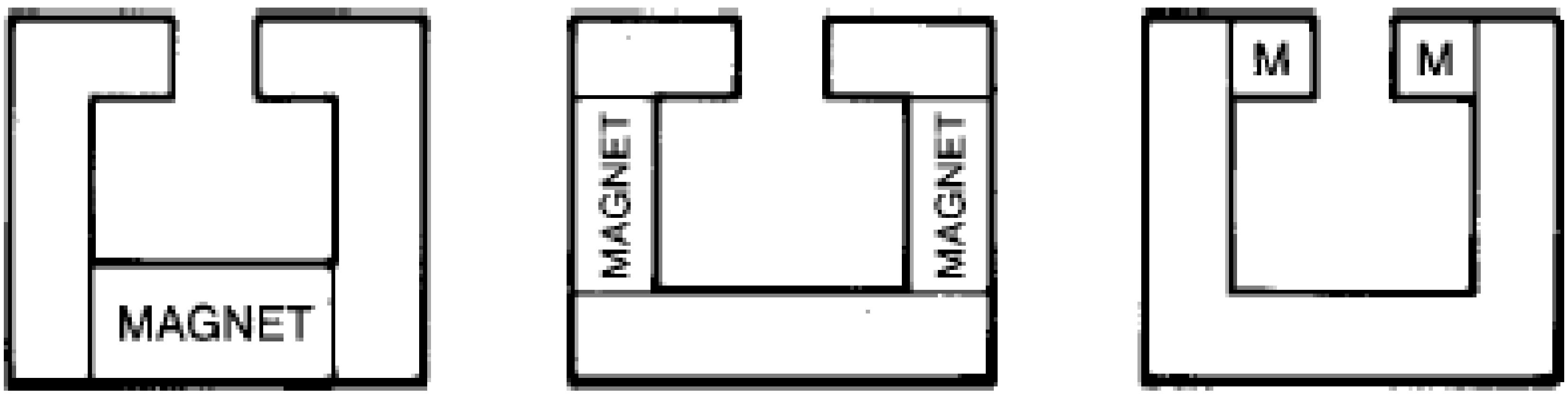
- These two are equivalent in the upper half-plane





Another Example: Magnetic Circuit With Steel

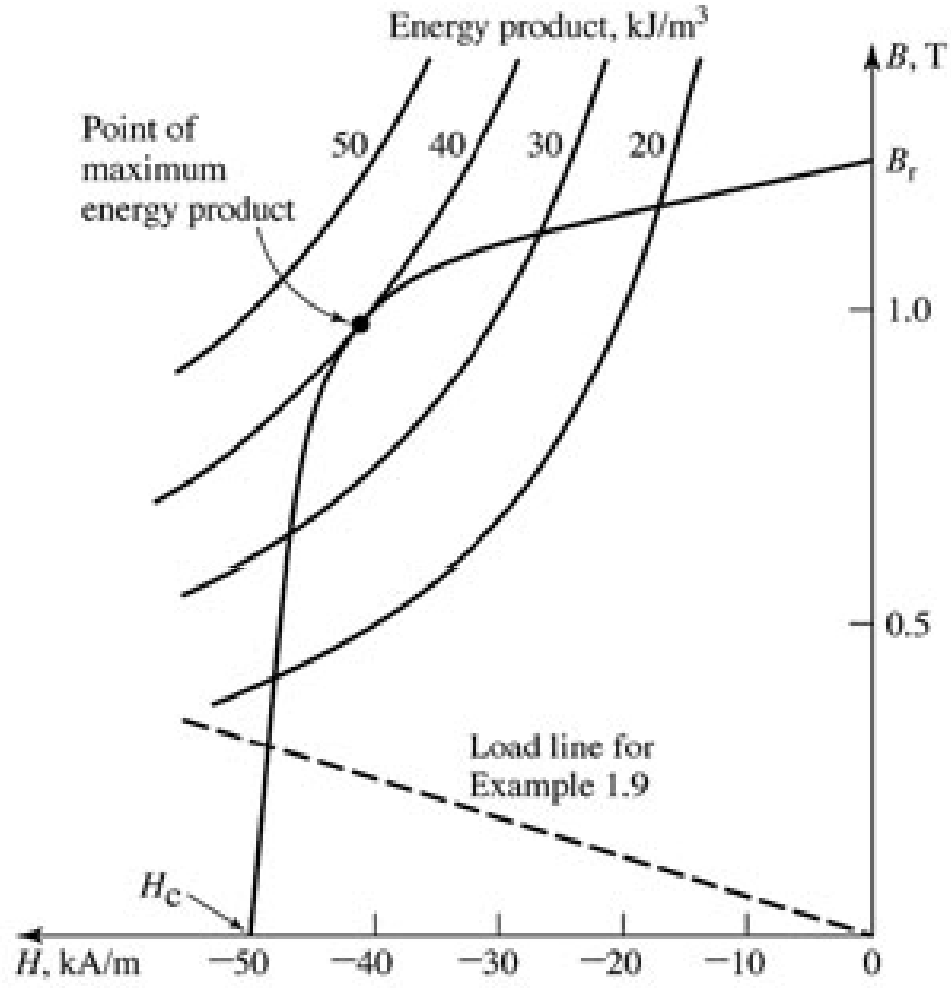
- Which scenario has the lowest leakage flux?



Reference: www.mmpa.org

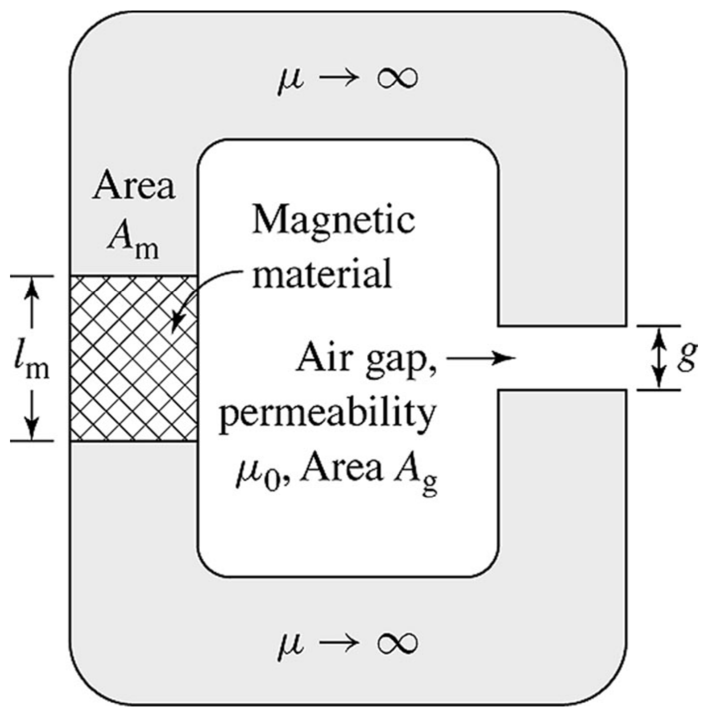
Maximum Energy Product

- BH has units of Joules per unit volume



(a)

Maximum Energy Product



Why is maximum energy product important?

① $B_g = B_m \left(\frac{A_m}{A_g}\right)$

② $\frac{H_m l_m}{H_g g} = -1$

Let's find B_g^2

$$B_g^2 = B_m\left(\frac{A_m}{A_g}\right) \times \mu_0 H_g$$

$$= -B_m\left(\frac{A_m}{A_g}\right)\mu_0 \frac{H_m l_m}{g}$$

$$= \mu_0\left(\frac{Vol_{mag}}{Vol_{gap}}\right)\left(-B_m H_m\right)$$

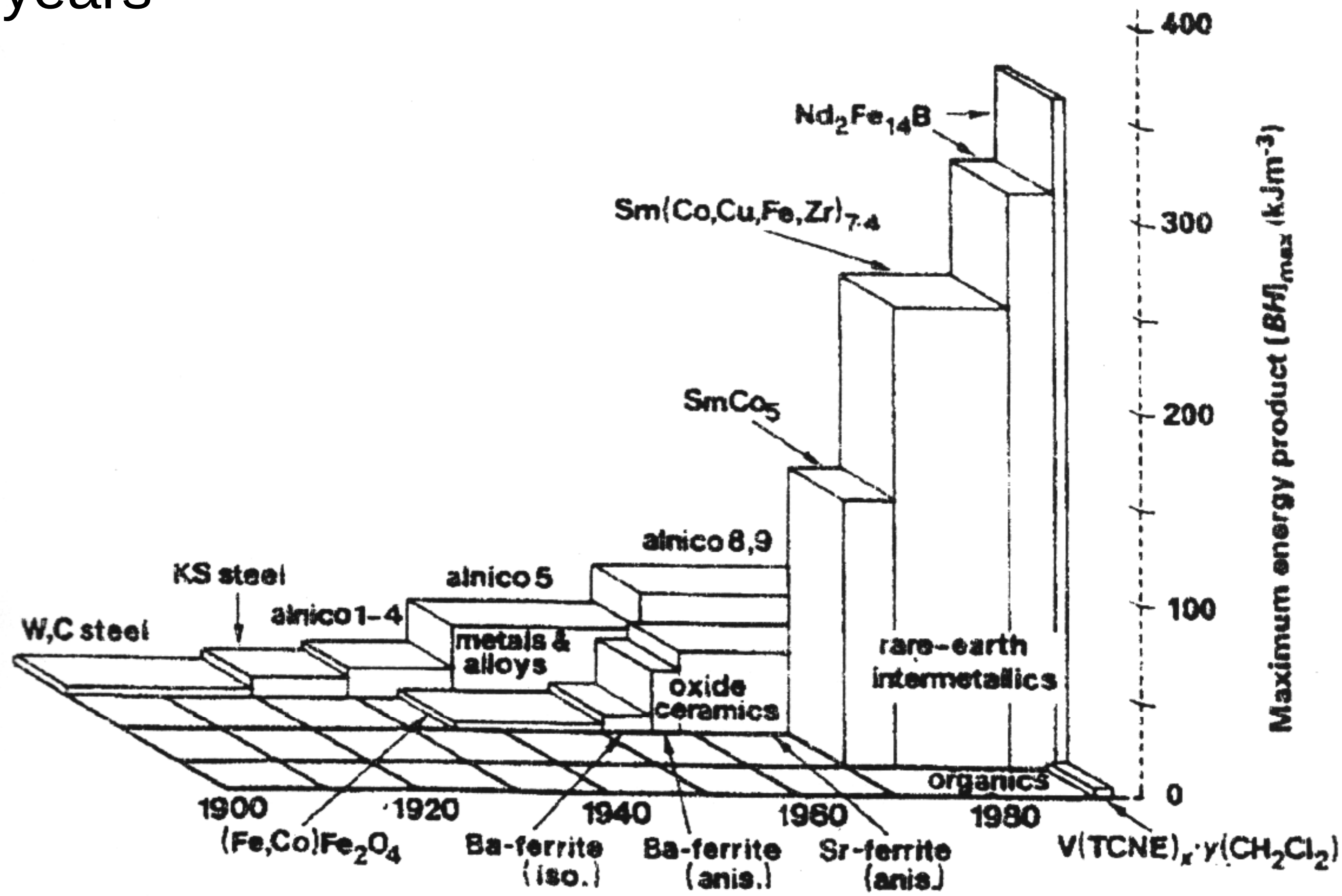
Solve for Vol_{mag}

$$Vol_{mag} = B_g^2 \frac{Vol_{gap}}{(-B_m H_m)}$$

To use minimum volume of magnetic material at a given B_g, operate magnet at $(BH)_{max}$ point.

Progress in PM Specs

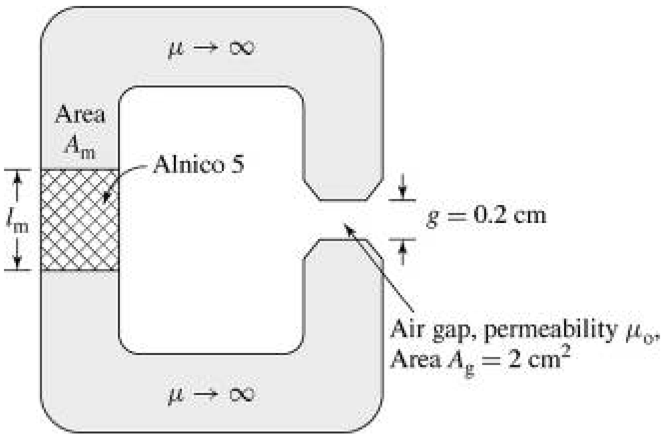
- Maximum $(BH)_{max}$ product has gone up a lot in the past 20 years

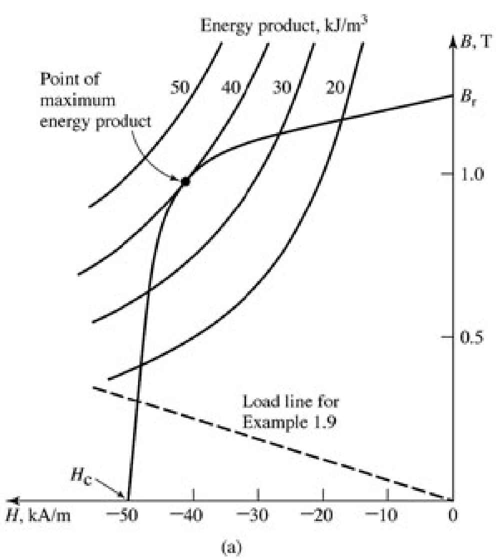


Reference: J. Evetts, *Concise Encyclopedia of Magnetic and Superconducting Materials*, Pergamon Press, Oxford, 1992

Example 7: Use of Maximum Energy Product

- Find magnet dimensions for desired B_g = 0.8 Tesla operating at $(BH)_{max}$





(a)

- At max (BH), $B_m = 1.0$ T and $H_m = -40\ \frac{kA}{m}$
- $A_m = A_g \left(\frac{B_g}{B_m}\right) = (2\ cm^2)\left(\frac{0.8}{1.0}\right) = 1.6\ cm^2$
- $l_m = -g\left(\frac{H_g}{H_m}\right) = -g\left(\frac{B_g}{\mu_o H_m}\right)$

$$= -(0.2\ cm)\left(\frac{0.8}{4\pi\times10^{-7}}\right)(-40{,}000)$$

$$= 3.18\ cm$$

So, magnet is 3.18 cm long and 1.6 cm² in area

Comparison of Different PM Types

Table 2.1 Magnet Material Comparisons

Material	Grade	Br	Hc	Hci	BHmax	T_{max} (Deg C)*
NdFeB	39H	12,800	12,300	21,000	40	150
SmCo	26	10,500	9,200	10,000	26	300
NdFeB	B10N	6,800	5,780	10,300	10	150
Alnico	5	12,500	640	640	5.5	540
Ceramic	8	3,900	3,200	3,250	3.5	300
Flexible	1	1,600	1,370	1,380	0.6	100

* T_{max} (maximum practical operating temperature) is for reference only. The maximum practical operating temperature of any magnet is dependent on the circuit the magnet is operating in.

Reference: www.magnetsales.com

Magnetic Conversion Factors

Table 3.2 Conversion Factors

Multiply	By	To obtain
inches	2.54	centimeters
$lines/in^2$	0.155	Gauss
$lines/in^2$	1.55×10^{-5}	Tesla
Gauss	6.45	$lines/in^2$
Gauss	0^{-4}	Tesla
Gilberts	0.79577	ampere turns
Oersteds	79.577	ampere turns /m
ampere turns	0.4π	Gilberts
ampere turns/in	0.495	Oersteds
ampere turns/in	39.37	ampere turns/m

Reference: www.magnetsales.com

Magnetic Field Estimates

a. Cylindrical Magnets

$$B_x = \frac{B_r}{2}\left(\frac{(L+X)}{\sqrt{R^2+(L+X)^2}} - \frac{X}{\sqrt{R^2+X^2}}\right)$$

Equation 4

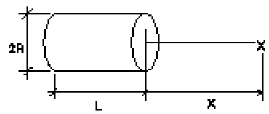


Table 4.1 shows flux density calculations for a magnet 0.500" in diameter by 0.250" long at a distance of 0.050" from the pole surface, for various materials. Note that you may use any unit of measure for dimensions; since the equation is a ratio of dimensions, the result is the same using any unit system. The resultant flux density is in units of gauss.

Table 4.1 Flux Density vs. Material

Material and Grade	Residual Flux Density, Br	Flux at distance of 0.050" from surface of magnet
Ceramic 1	2,200	629
Ceramic 5	3,950	1,130
SmCo 18	8,600	2,460
SmCo 26	10,500	3,004
NdFeB 35	12,300	3,518
NdFeB 42H	13,300	3,804

Reference: www.magnetsales.com

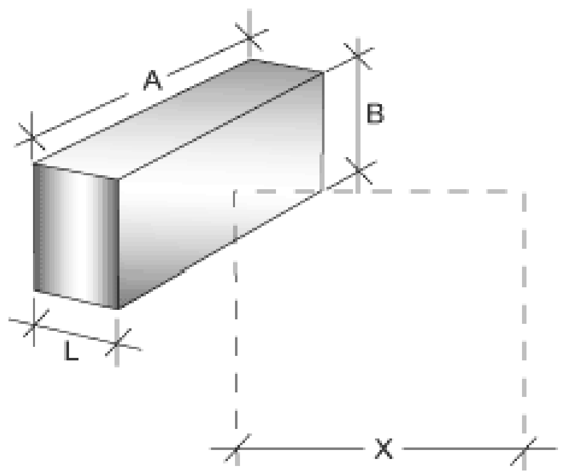
Magnetic Field Estimates

b. Rectangular Magnets

$$B_x = \frac{B_r}{\pi}\left(\tan^{-1}\frac{AB}{2X\sqrt{4X^2 + A^2 + B^2}} - \tan^{-1}\frac{AB}{2(L+X)\sqrt{4(L+X)^2 + A^2 + B^2}}\right)$$

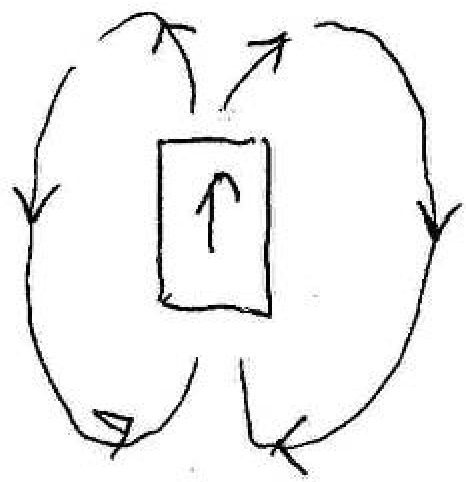
Equation 5

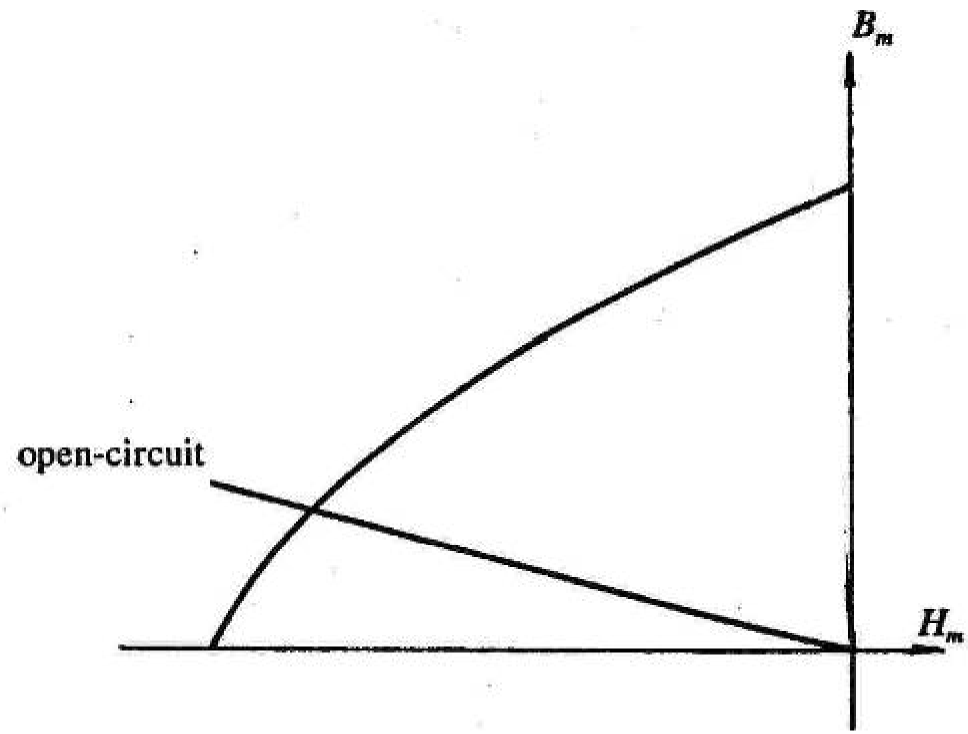
(where all angles are in radians)



Reference: www.magnetsales.com

Open Circuited Permanent Magnet

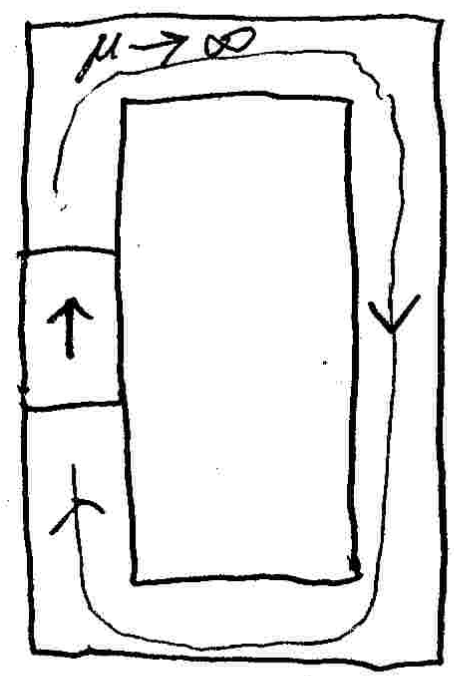




Reference: P. Campbell, *Permanent Magnet Materials and their Applications*, Cambridge University Press, 1994, pp. 89

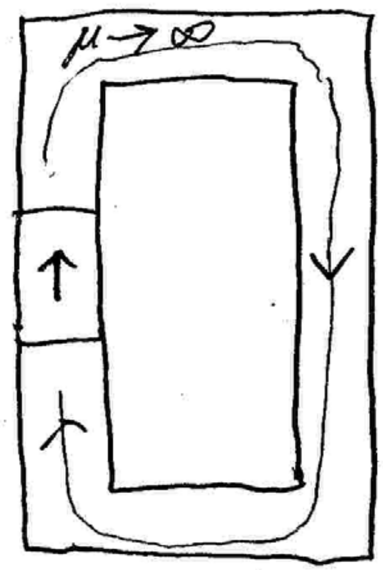
Short-Circuited Permanent Magnet

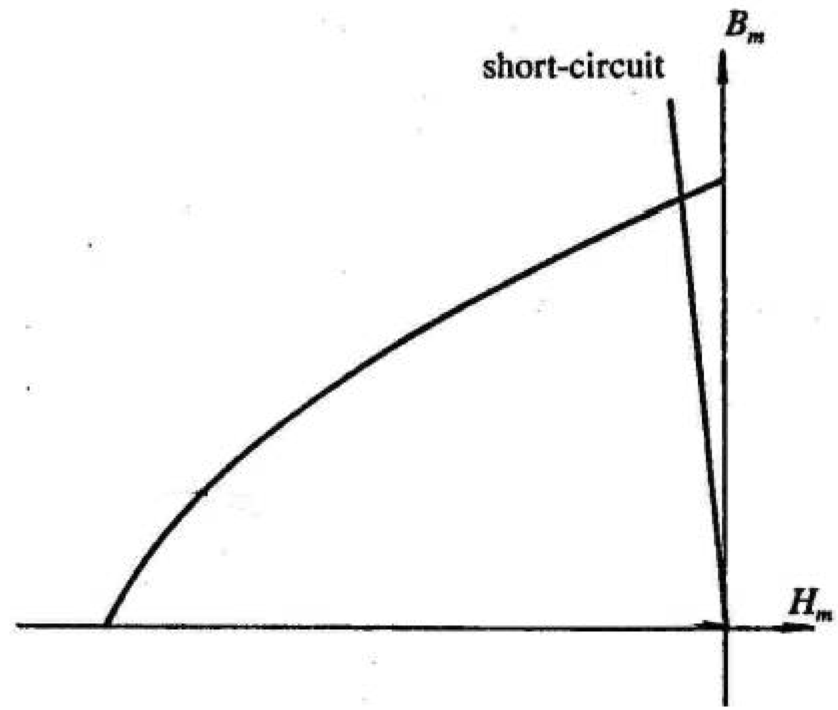
- Find B inside core, ignoring any leakage and assuming infinite permeability in core



Short Circuited Permanent Magnet

- For infinite permeability, load line is vertical
- Intersection of load lines occurs at $B \approx B_r$

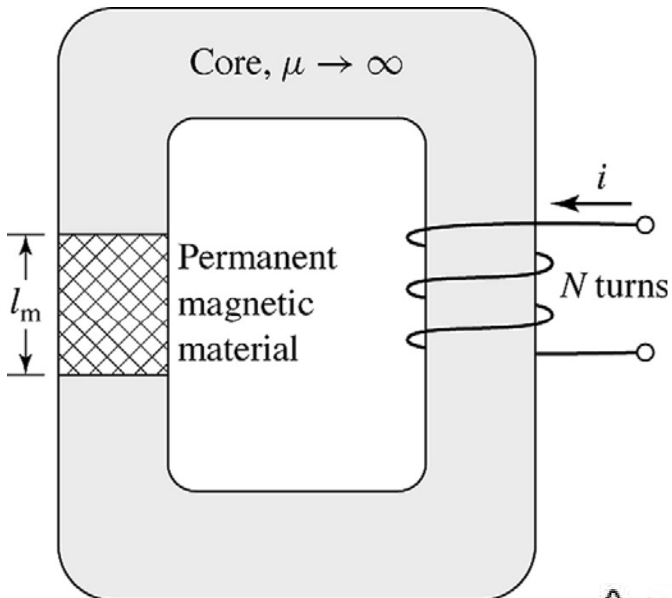




Reference: P. Campbell, *Permanent Magnet Materials and their Applications*, Cambridge University Press, 1994, pp. 89

PM and a Winding

- Many motors have permanent magnets, steel and windings



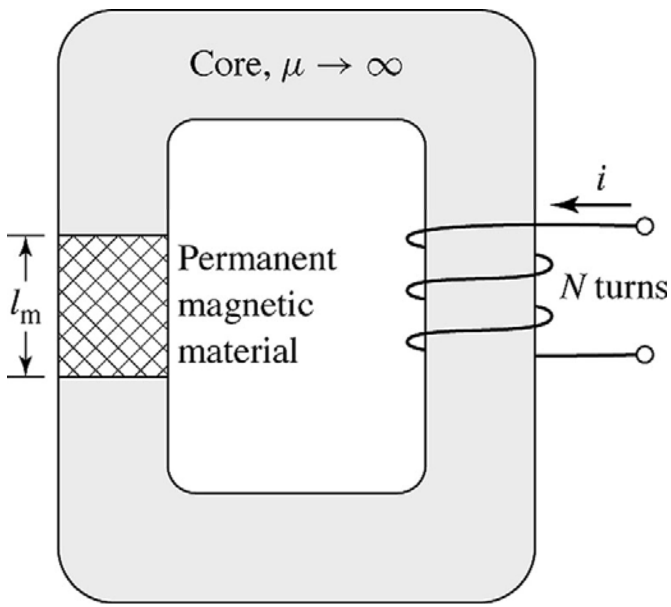
ANALYSIS OF PM IN CLOSED CORE, WITH EXCITATION

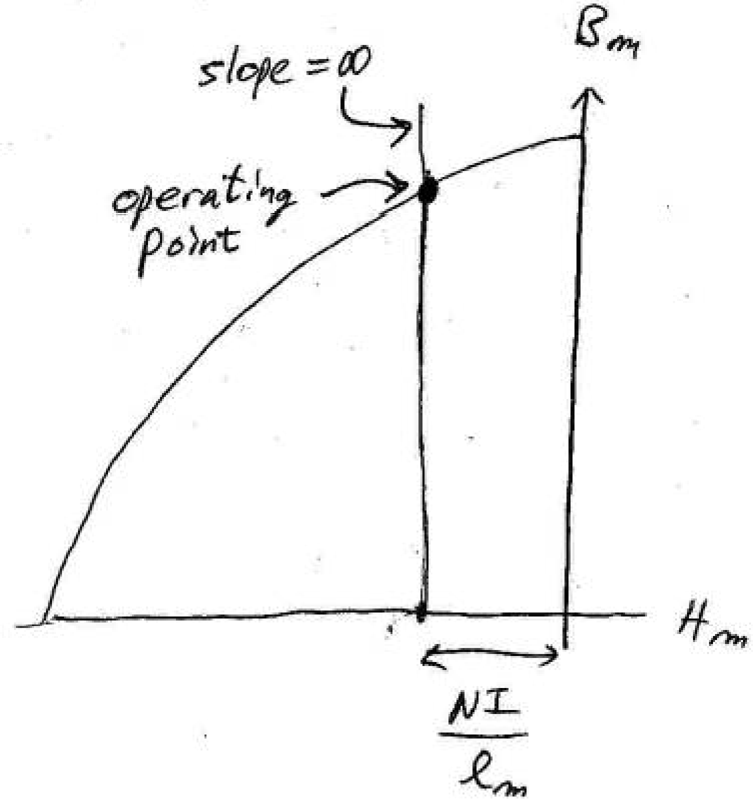
$$H_m l_m = NI \quad \text{(AMPERE'S LAW)}$$

$$\therefore H_m = \frac{NI}{l_m}$$

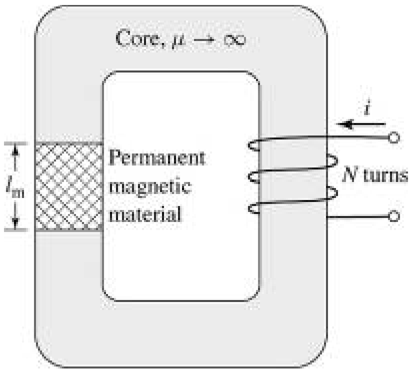
PM and a Winding --- Load Line

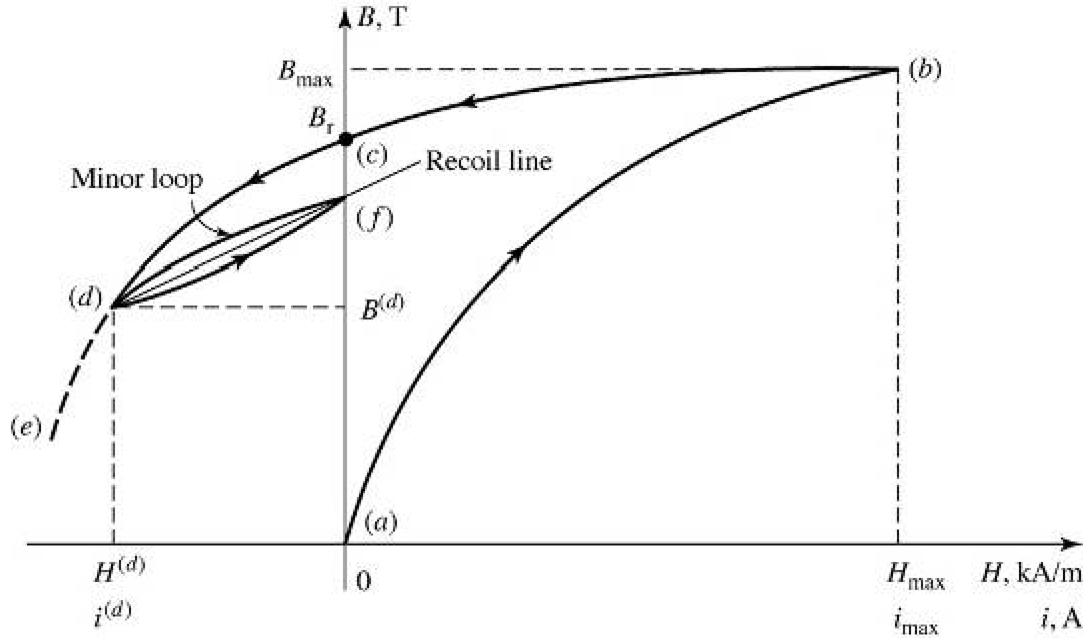
- Note that demagnetization can occur if current is sufficiently high





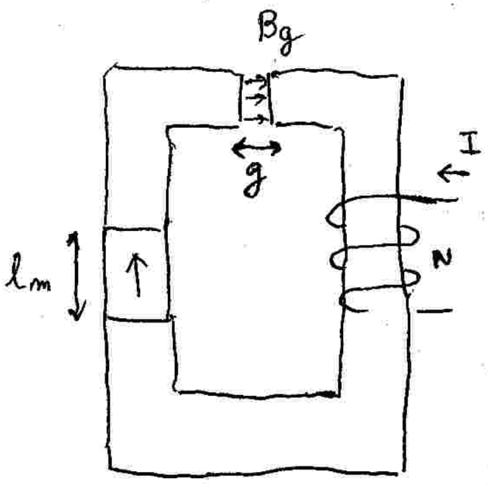
PM and a Winding --- Graphical Analysis





- At point (a), steel is initially unmagnetized
- As current increases, B and H follow the locus from (a) to (b)
- From (b) to (c), current reduces to zero, and flux density reduces to B_r at $i = 0$
- As current goes negative from (c) to (d), curve traces hysteresis loop. Note that operating point (d) is the same operating point we'd get if there was zero current and an airgap.
- If current goes further negative, locus traces from (d) to (e)
- But, if current is reduced to zero at point (d), locus traces minor loop from (d) to (f). The "recoil line" is an approximation to this minor loop

Another Example --- Excitation and Airgap



① $H_m l_m + H_g g = NI$ (AMPERE'S LAW)

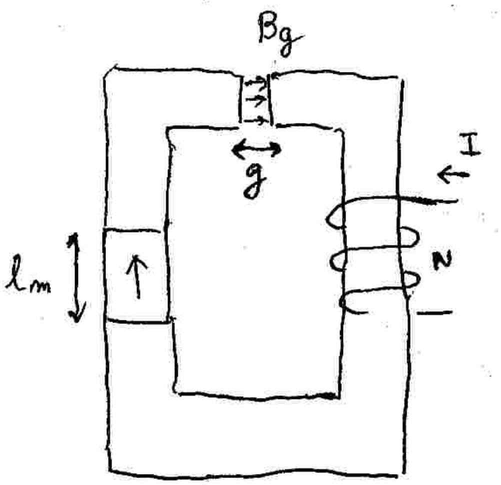
② $B_g = \mu_0 H_g$ (CONSTITUTIVE)

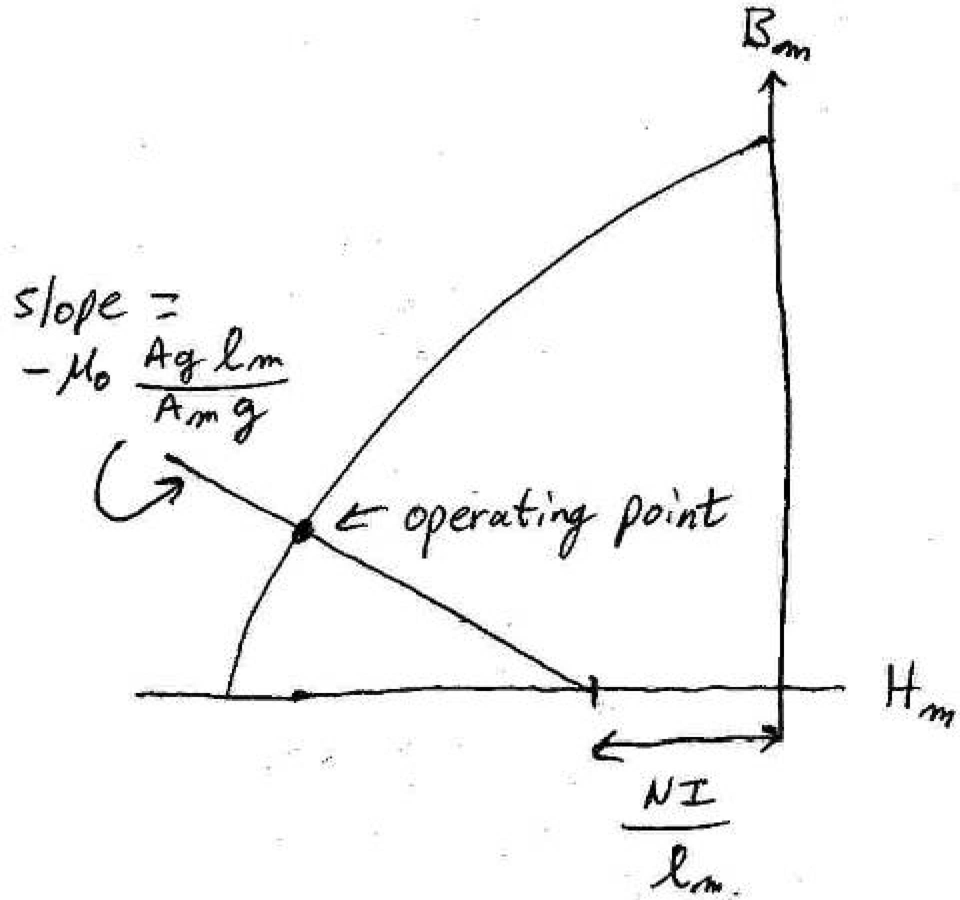
③ $B_m A_m = B_g A_g$ (GAUSS' MAGNETIC LAW)

SOLVE FOR $B_m - H_m$ LOAD LINE

$$\therefore \; B_m = -\mu_0 \left(\frac{A_g l_m}{A_m g} \right) \left(H_m - \frac{NI}{l_m} \right)$$

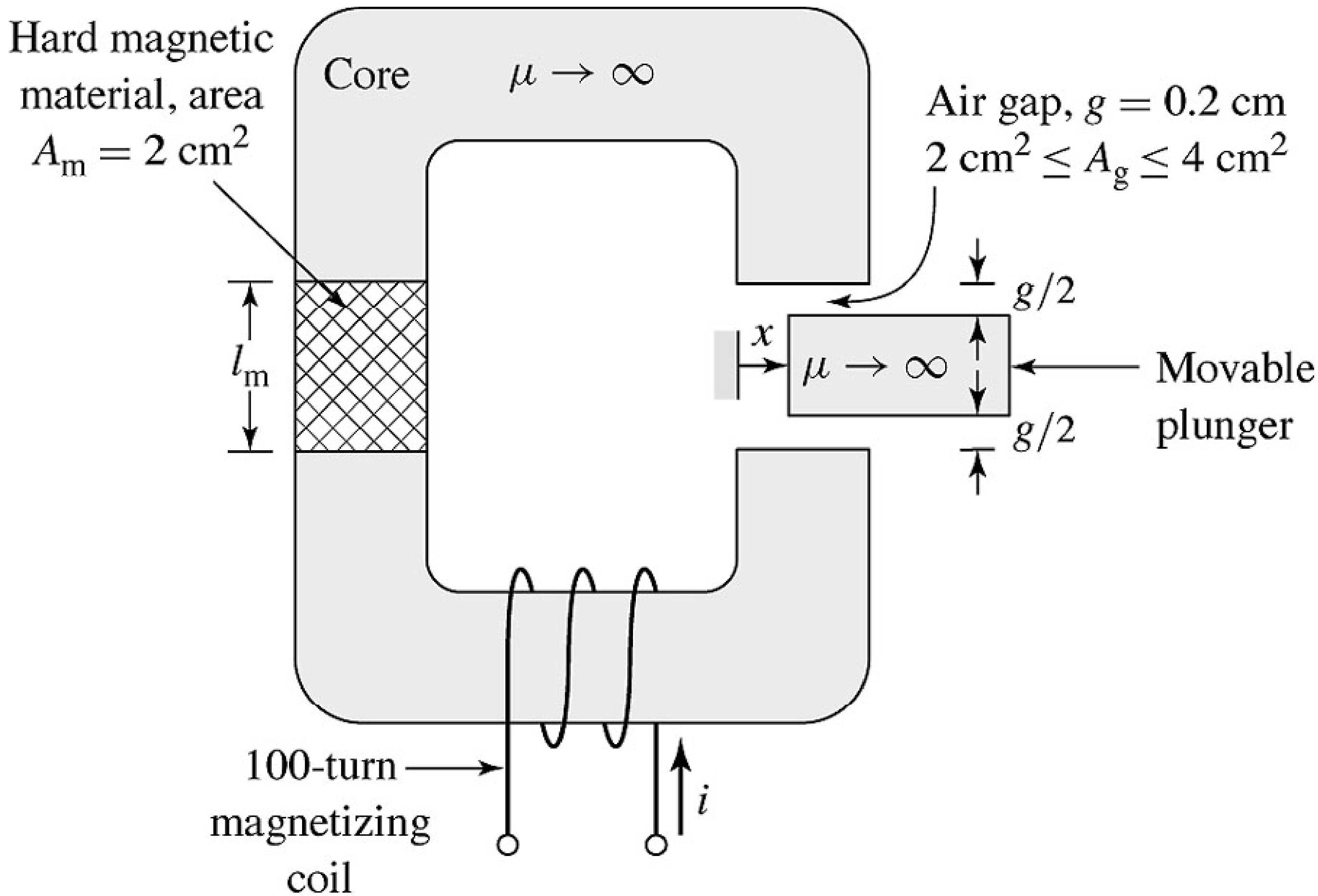
Another Example --- Excitation and Airgap --- Load Line



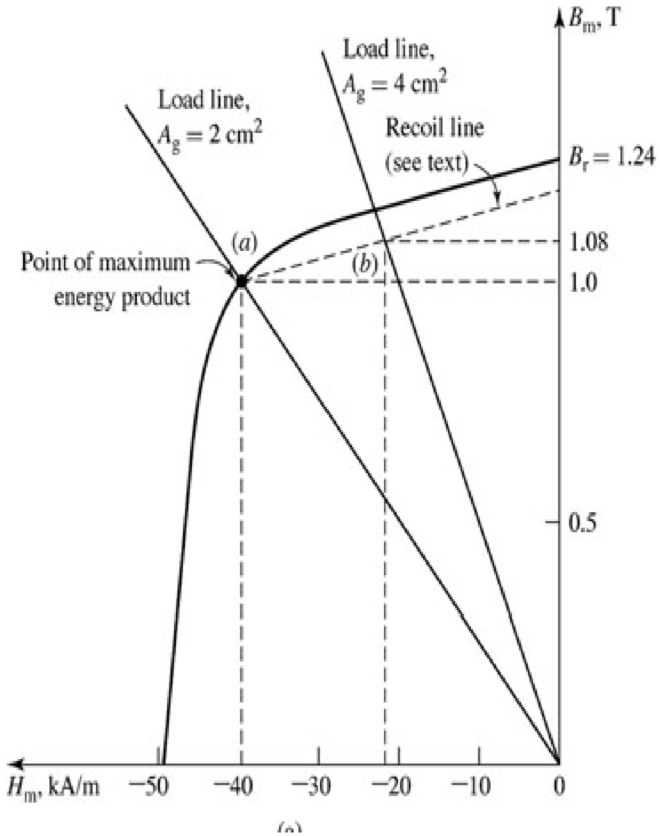


Another Example

- Let's figure out how to magnetize



Another Example



(a) FIND MAGNET LENGTH l_m SO MAGNET OPERATES ON RECOIL LINE INTERSECTING BH_{max} AT $A_g = 2cm^2$

SOLN:

$B_{max} = 1.0T, \quad H_{max} = -40\ kA/m$

FROM GAUSS' LAW $\quad B_g = \frac{A_m}{A_g} B_m \qquad (1)$

FROM AMPERE'S LAW $\quad \frac{H_m l_m}{H_g g} = -1 \qquad (2)$

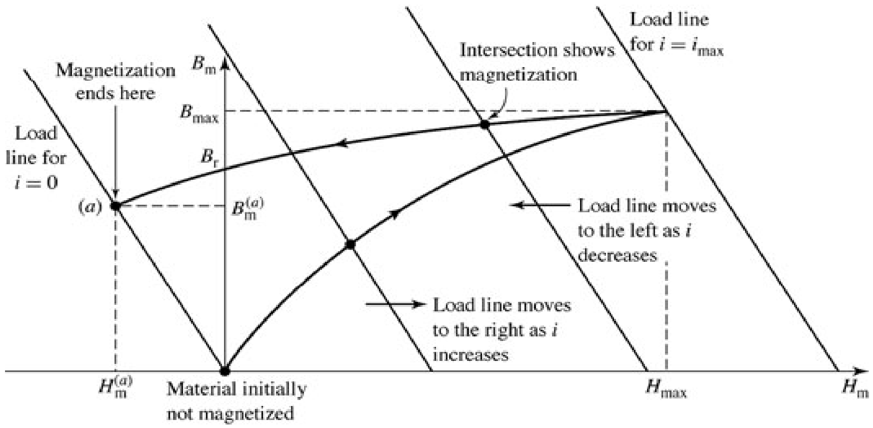
From (2) $\quad B_g = \frac{-\mu_0 H_m l_m}{g}$

Put this into (1): $\quad \frac{-\mu_0 H_m l_m}{g} = \frac{A_m B_m}{A_g}$

$$l_m = g \left(\frac{A_m}{A_g}\right)\left(\frac{B_m}{-\mu_0 H_m}\right)$$

$$l_m = 0.2cm \left(\frac{2cm^2}{2cm^2}\right)\left(\frac{1.0\ T}{(4\pi \times 10^{-7})(4 \times 10^4)}\right) = \boxed{4cm}$$

Another Example



(b) HOW TO MAGNETIZE FULLY ?

① $NI = H_m l_m + H_g g$

② $B_m A_m = B_g A_g = \mu_o H_g A_g$

Solve for B_m as a function of H_m and I

$$B_m = -\mu_o \left(\frac{A_g}{A_m}\right)\left(\frac{l_m}{g}\right) H_m + \frac{\mu_o N}{g}\left(\frac{A_g}{A_m}\right) I$$

$$= -2.5 \times 10^{-5} H_m + 6.28 \times 10^{-2} I$$

ESTIMATE B_{max} and H_{max}

$B_{max} \approx 2.1 T$

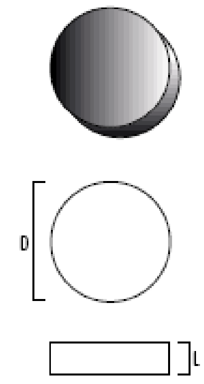
$H_{max} \approx 200 kA/m$

$\therefore I = 45 A$

What Can You Buy?

NdFeB Rounds

Part Number	Material	Diameter	Length	Orientation
PN35C1250B-N	N36	0.123	0.250	L
PN45C0140B	N45	0.140	0.500	D
PN45C0189B	N45	0.188	0.057	L
94C5668B	94EB	0.250	0.100	L
CM40685	4014	0.250	0.125	L
PN36C0250B	N36	0.250	0.250	L
97C5513B	97CB	0.250	2.000	L
97C5613B	97CB	0.251	0.200	L
PN45C1200B	N45	0.328	1.200	D
PN36SHC0330B	N36SH	0.330	0.330	L
94C5536C	94EB	0.370	0.250	L
97C5446C	97CB	0.375	0.125	L
94C5562B	94EB	0.375	2.100	L
94C4822B	94EA	0.499	0.188	L
NA67A460	34B	0.500	0.190	L
CM41085-2630	2630	0.500	0.250	L
CM41086-3220	3520	0.500	0.250	L
CM40713-3714	4014	0.500	0.250	L
CM41087-3714	4014	0.625	0.250	L
CM41088-2630	2630	0.625	0.250	L
CM40884-3714	4014	0.866	0.393	L
CM41089-3714	4014	0.875	0.500	L
CM41090-2630	2630	0.875	0.500	L
CM41091-2630	2630	1.000	0.500	L
CM41092-3220	3520	1.000	0.500	L
CM41093-3714	4014	1.000	0.500	L

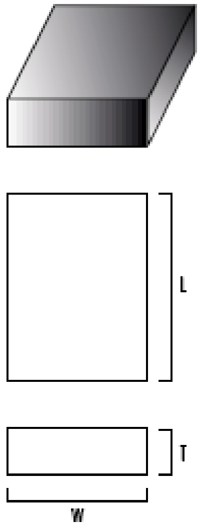


Reference: www.dextermag.com

What Can You Buy?

NdFeB Blocks

Part Number	Material	Thickness	Width	Length	Orientation
CM41030-3714	4014	0.250	0.325	0.500	T
CM41094-2630	2630	0.250	0.500	0.500	T
CM41095-3220	3520	0.250	0.500	0.500	T
CM41096-3714	4014	0.250	0.500	0.500	T
PN36HR2933B	N36H	0.293	0.560	1.044	T
PN38HR0990B	N38H	0.315	0.750	0.990	T
PN38HR0375B	N38H	0.375	0.866	2.362	T
CM40325-3714	4014	0.500	0.750	0.750	T
CM40729-3714	4014	0.500	1.000	1.000	T
CM41097-2630	2630	0.500	1.000	1.000	T
CM41098-3220	3520	0.500	1.000	1.000	T
CM40975-3714	4014	0.590	0.750	0.750	T
PN38HR0122B	N38H	1.000	2.000	2.000	T
CM41099-2630	2630	1.000	2.000	2.000	T
CM40680-3714	4014	1.000	2.000	2.000	T
PN48R0100B	N48	1.000	2.000	2.000	T



Reference: www.dextermag.com

What Can You Buy?

Rectangular Magnets - Finished Sizes sorted in ascending order of Length, Width, and then Thickness.

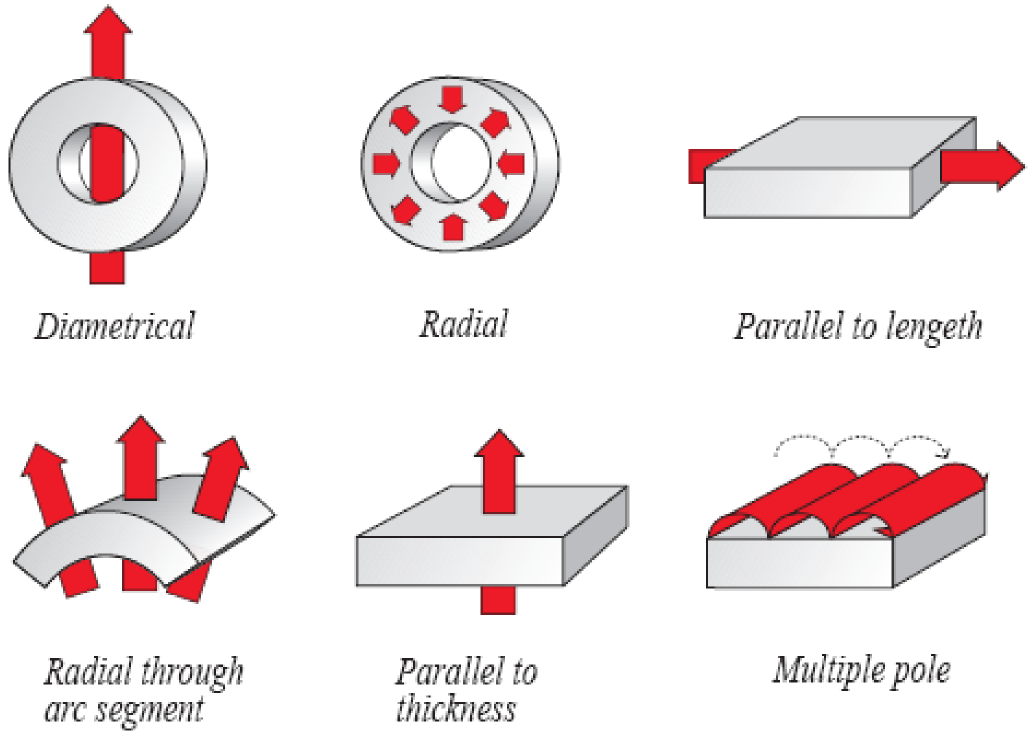
Item Number	Grade	L	W	T*	Condition
35NE111111	35	0.175	0.175	0.175	Die Pressed, Machined
35NE111111-NI	35	0.175	0.175	0.175	Die Pressed, Machined, Nickel Plated
30NE261916-NI	30	0.400	0.290	0.250	Die Pressed, Machined, Nickel Plated
35NE301412	35	0.475	0.230	0.190	Die Pressed, Machined
35NE323232-NI	35	0.500	0.500	0.500	Die Pressed, Machined, Nickel Plated
35NE501212	35	0.790	0.195	0.195	Die Pressed, Machined
35NE642424	35	1.000	0.375	0.375	Die Pressed, Machined
35NE646416	35	1.000	1.000	0.250	Die Pressed, Machined
35NE646432	35	1.000	1.000	0.500	Die Pressed, Machined
35NE646432-NI	35	1.000	1.000	0.500	Die Pressed, Machined, Nickel Plated
35NE209620	35	1.500	0.305	0.305	Die Pressed, Machined
35NE281107	35	2.000	0.170	0.107	Die Pressed, Machined
35NE2812832	35	2.000	2.000	0.500	Die Pressed, Machined
35NE2812832	35	2.000	2.000	0.500	Die Pressed, Machined
35NE2812864	35	2.000	2.000	1.000	Die Pressed, Machined

Tolerances on "machined" blocks are the greater of ±1.5% of the dimension or ±0.015" on cross sectional dimensions, and ±0.005" on the orientation direction.

The T* dimension represents the orientation direction.

Reference: www.magnetsales.com

Magnetization Patterns



Different Magnetizing Patterns Give Different Results

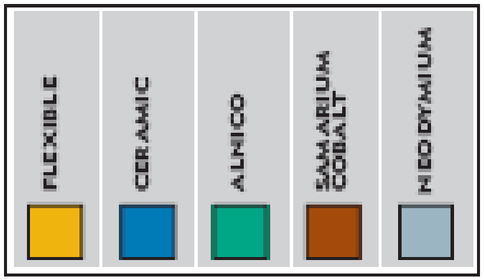
Except for the regular Flexible and Ceramic 1 materials, all magnet materials are "pre-oriented" and can only be magnetized in a particular direction.

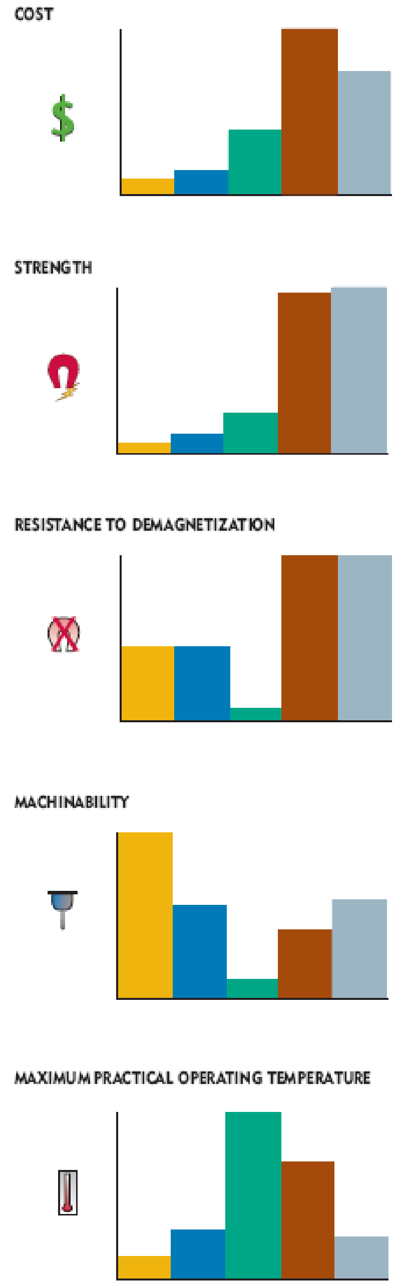
Standard (or "conventional") magnetization is straight through the orientation direction, and produces one North pole and one South pole. The Rare Earth magnets are extremely difficult to magnetize in non-standard ways. However, the Flexible and Ceramic types can be magnetized in many non-standard ways to give special results.

Reference: www.magnetsales.com

Comparison

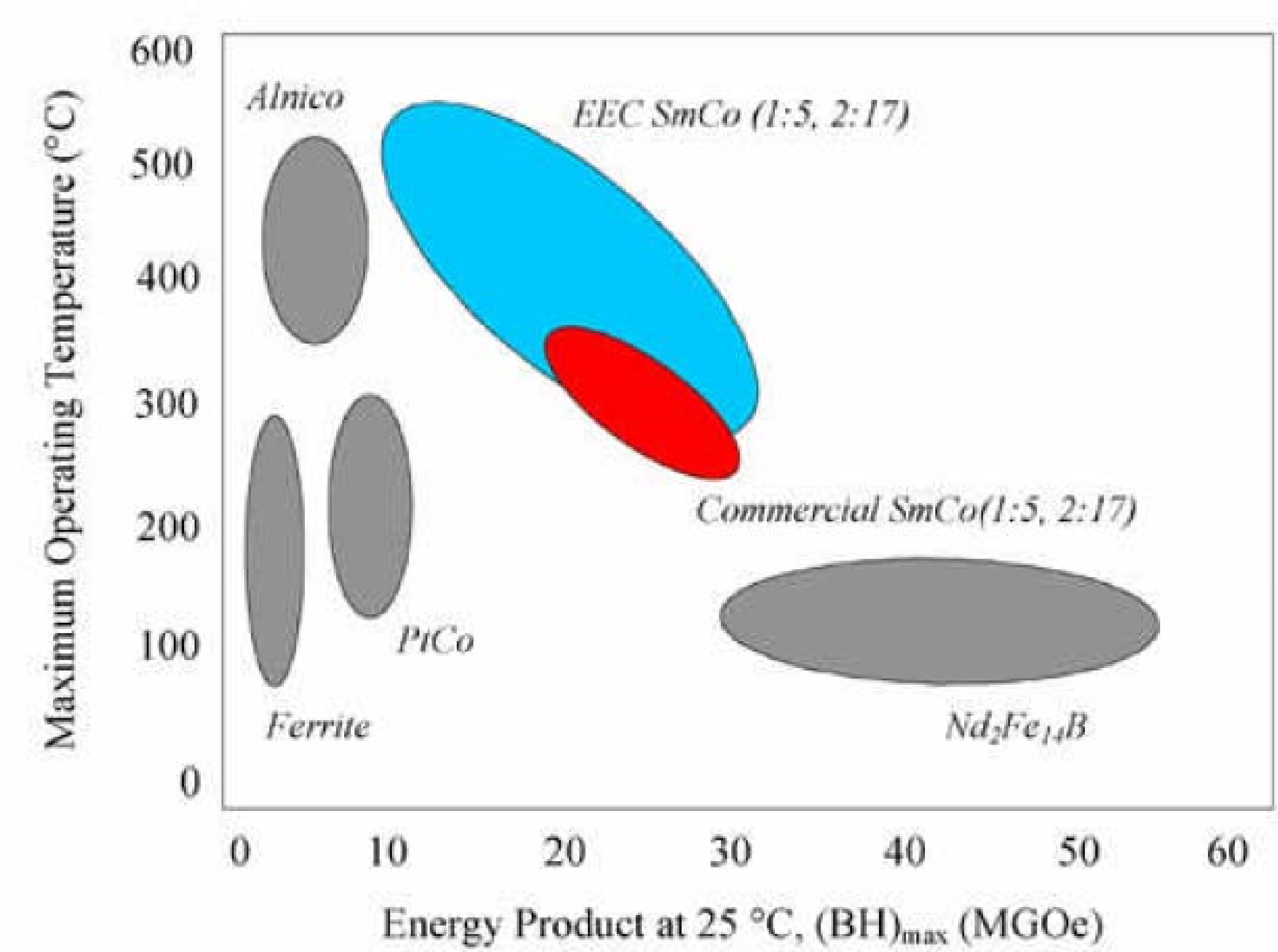
Magnet Materials Color Index





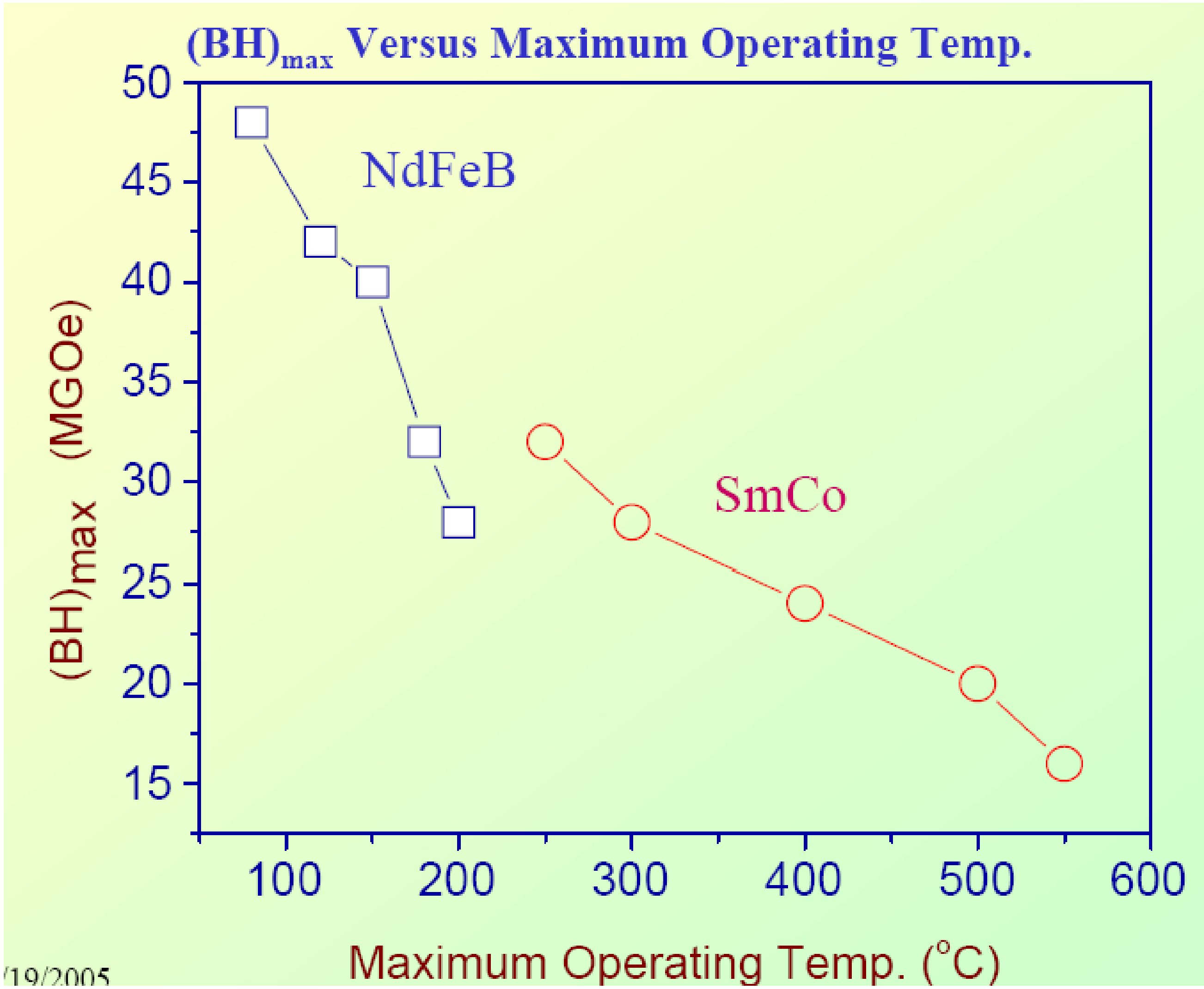
Reference: www.magnetsales.com

Comparison of Maximum Operating Temperatures



Reference: http://www.electronenergy.com/media/Magnetics%202005.pdf

Comparison of Maximum Operating Temperatures



Reference: http://www.electronenergy.com/media/Magnetics%202005.pdf

Quotes

It is well to observe the force and virtue and consequence of discoveries, and these are to be seen nowhere more conspicuously than in printing, gunpowder, and the magnet.
--- Sir Francis Bacon

The mystery of magnetism, explain that to me! No greater mystery, except love and hate.
---John Wolfgang von Goethe

References

- Peter Campbell, *Permanent Magnet Materials and Their Applications*, Cambridge University Press, 1994
- Edward Furlani, *Permanent Magnet and Electromechanical Devices*, Academic Press, 2001
- Rollin J. Parker, *Advances in Permanent Magnetism*, John Wiley, 1990
- Lester Moskowitz, *Permanent Magnet Design and Application Handbook*, 2nd edition, Krieger, 1995
- Vacuumschmelze, *Soft Magnetic Materials*, 1979
- Dexter Magnetics, www.dextermag.com
- Magnet Sales, Inc., www.magnetsales.com
- Magnetic Materials Producers Association, Std. PMG-88, "Permanent Magnet Guidelines"