

Power Electronics Notes 30B

Inductance Calculation Methods

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Methods of Inductance Calculation

- “Brute force method”
- Energy method
- Speed of light relationship
- Magnetic circuits
- Handbook methods

Summary of Magnetic Quantities

Item	Name	Units	Comment
H	Magnetic field	A/m	
J	Current density	A/m ²	
I	Current	A	
B	Magnetic flux density	Tesla, or <u>Wb/m²</u>	
μ_0	Magnetic permeability of free space	$4\pi \times 10^{-7}$ H/m	In magnetic steels, μ is higher
Φ	Flux	<u>Wb</u>	Integrate B (flux density) over an area to get Φ
N	Number of turns in an inductor or transformer	---	
λ	Flux linkage	<u>Wb</u> -turns	$\lambda = N\Phi$
L	Inductance	H	$L = \lambda/I$

Brute Force Method

- Calculate the magnetic flux density \mathbf{B} everywhere
- Use this value to calculate the flux Φ
- Once the flux is known, multiply by N to get flux linkage
 $\lambda = N\Phi$
- The inductance is the flux linkage divided by the coil current, or $L = \lambda/I$

Energy Method

- Everyone knows the lumped-circuit result for energy stored in an inductor:

$$E_m = \frac{1}{2} LI^2$$

- In many structures, the magnetic field over all space is easily found and the energy stored in the magnetic field can be directly calculated. The energy is found indirectly by integrating the magnetic flux density over all volume, as:

$$E_m = \frac{1}{2\mu_o} \iiint B^2 dV$$

Speed of Light

- In general, capacitance is a bit easier to calculate than inductance
- If the capacitance per unit length (C_o) of your structure is known, the inductance per unit length (L_o) is easily found by using the relationship:

$$c^2 = \frac{1}{L_o C_o}$$

- By the way, the speed of light is:

$$c = \frac{1}{\sqrt{\mu\epsilon}}$$

Magnetic Circuit Analogy

- Use Ohm's law analogy to model magnetic circuits

$$V \Leftrightarrow NI$$

$$I \Leftrightarrow \Phi$$

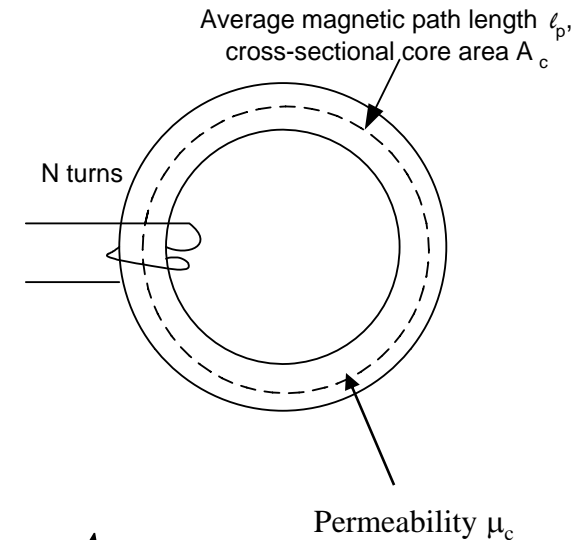
$$R \Leftrightarrow \mathcal{R}$$

- Use magnetic "reluctance" instead of resistance

$$R = \frac{l}{\sigma A} \Leftrightarrow \mathcal{R} = \frac{l}{\mu A}$$

Inductance of Toroid --- Brute Force Method

- Assume that the flux density is uniform inside the core and circulates around the coil axis
- Assume that the high- μ material guides all the flux, and that there is no leakage.



- Using Ampere's law:
$$B_\phi = \mu_c \frac{NI}{\ell_p}$$
- The total flux in the core is:
$$\Phi = B_\phi A_c = \mu_c NI \frac{A_c}{\ell_p}$$
- Total flux linkage is:
$$\lambda = N\Phi = \mu_c N^2 I \frac{A_c}{\ell_p}$$
- Inductance:
$$L = \frac{\lambda}{I} = \mu_c N^2 \frac{A_c}{\ell_p}$$

Note: inductance
always scales as
 N^2

Infinite Solenoid --- Using Energy Method

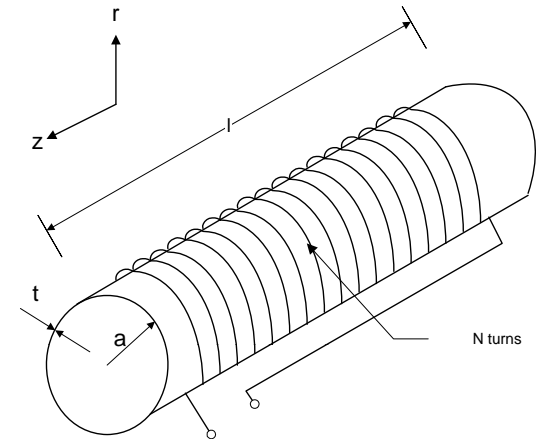
- Axial flux density B in the solenoid is:

$$B = \mu_o \frac{NI}{l}$$

- Energy storage is found by integrating B^2 over the volume of the solenoid bore:

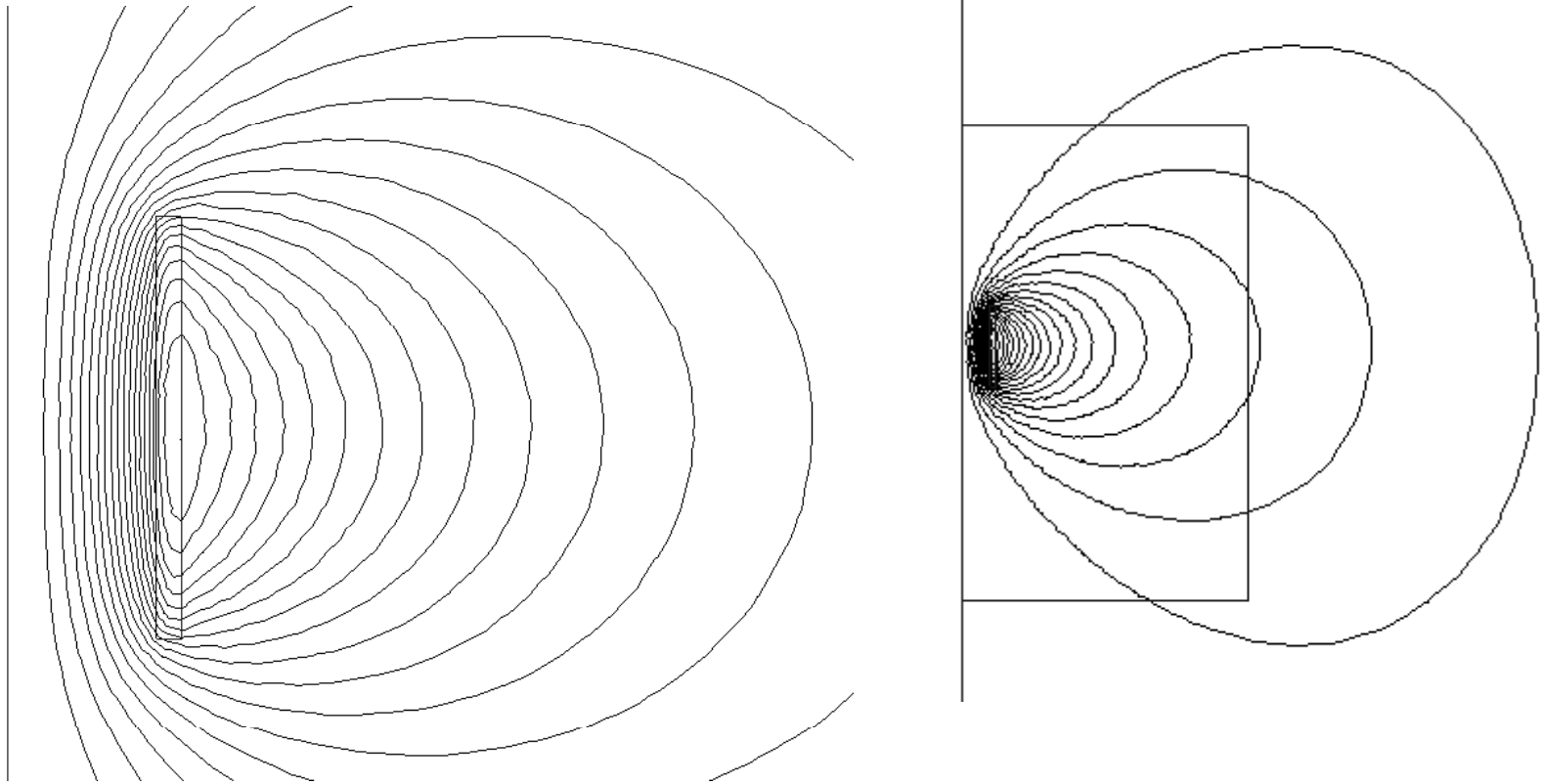
$$E_m = \frac{1}{2} \frac{B^2}{\mu_o} V = \frac{1}{2} LI^2$$

- Inductance: $L = \mu_o N^2 \frac{A}{l}$



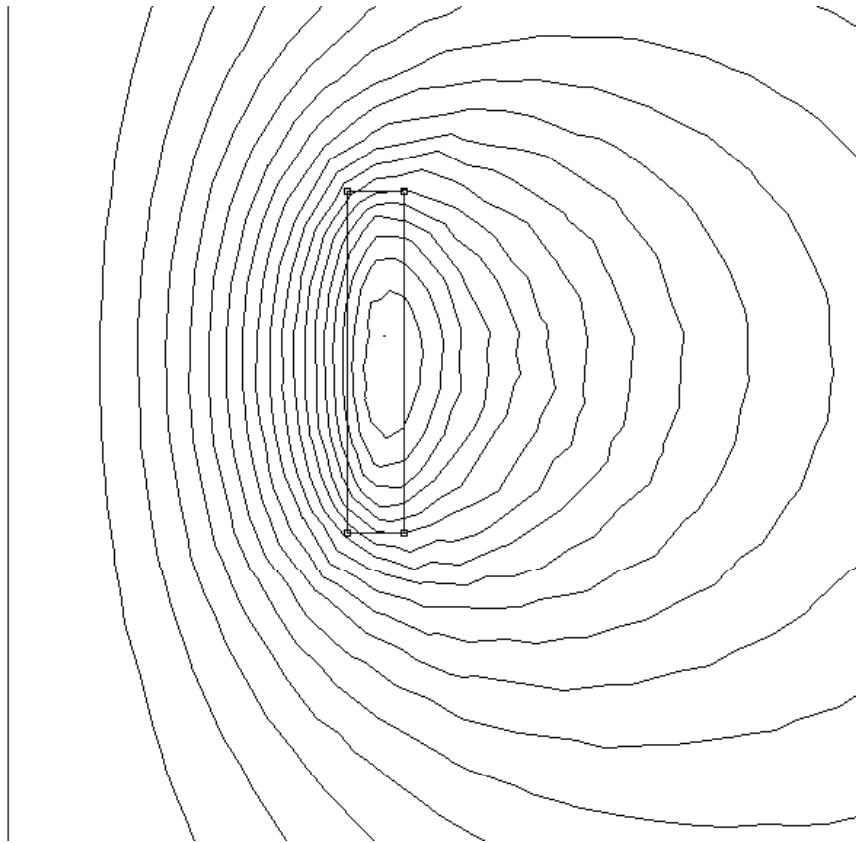
Solenoid --- FEA

- Note “end effects”; we assume that field inside solenoid is uniform and axially-directed; for finite length the field has curvature



Short Solenoid --- FEA

- For short solenoid, field inside is more curved
- This is a “less ideal” case



C-Core with Gap --- Using Magnetic Circuits

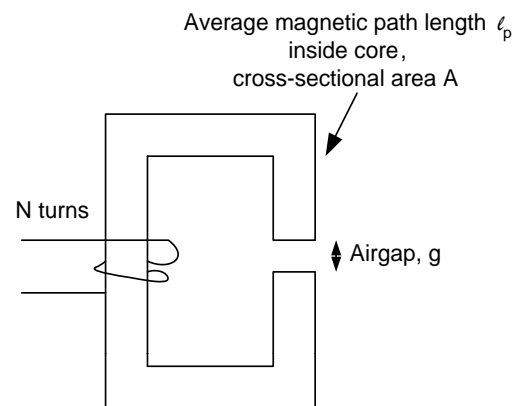
- Flux in the core is easily found by:

$$\Phi = \frac{NI}{\mathcal{R}_{core} + \mathcal{R}_{gap}} = \frac{NI}{\frac{l_p}{\mu_c A_c} + \frac{g}{\mu_o A_c}}$$

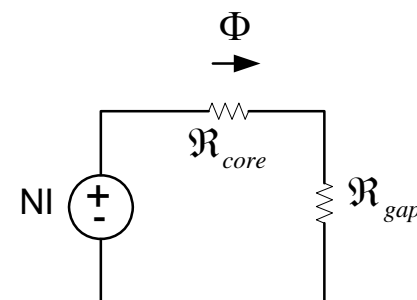
- Now, note what happens if $g/\mu_o \gg l_p/\mu_c$: The flux in the core is now approximately independent of the core permeability, as:

$$\Phi \approx \frac{NI}{\mathcal{R}_{gap}} \approx \frac{NI}{\frac{g}{\mu_o A_c}}$$

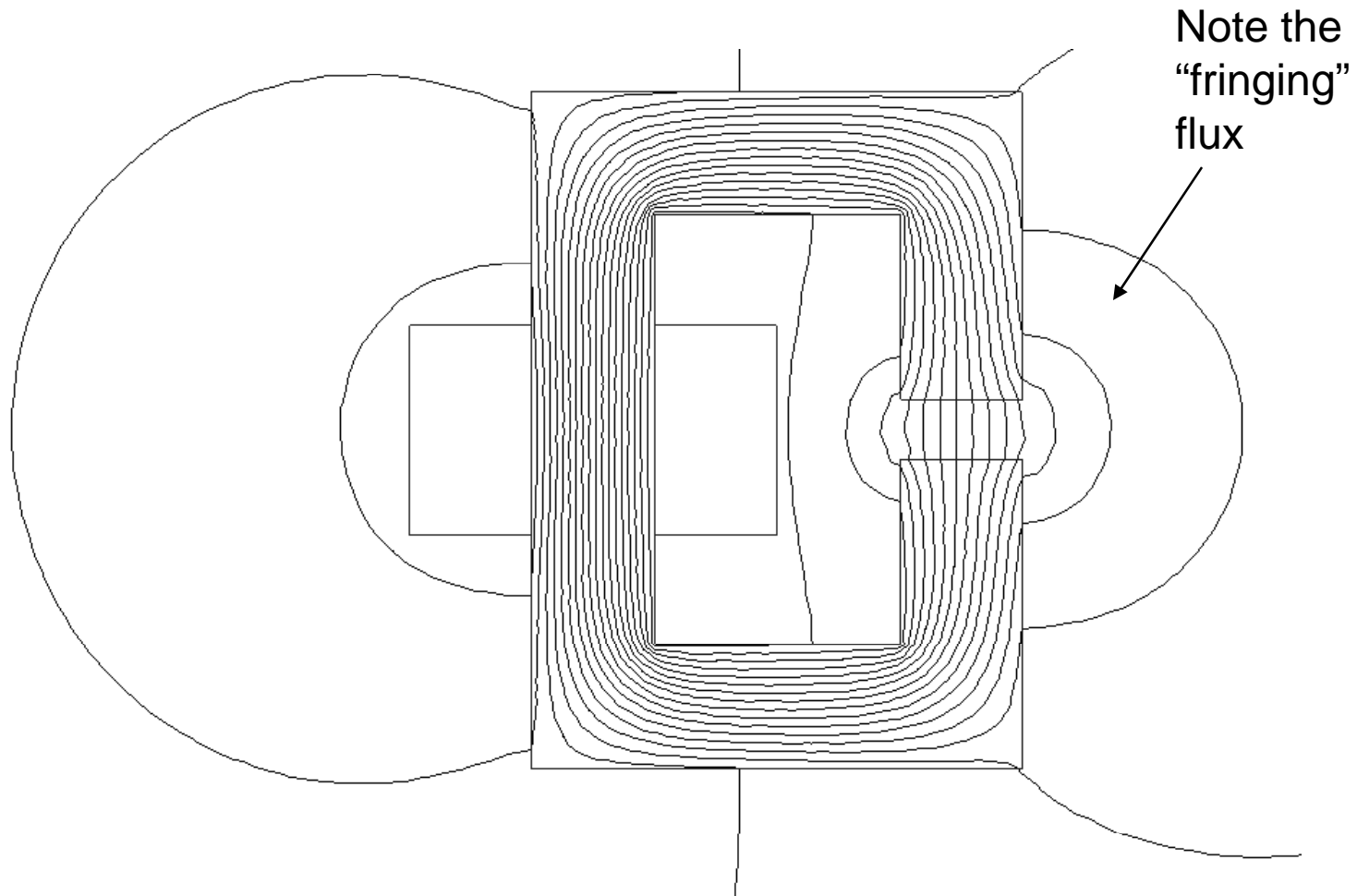
- Inductance:
$$L = \frac{N\Phi}{I} \approx \frac{N^2}{\mathcal{R}_{gap}} \approx \frac{N^2}{\frac{g}{\mu_o A_c}}$$



Magnetic circuit analogy



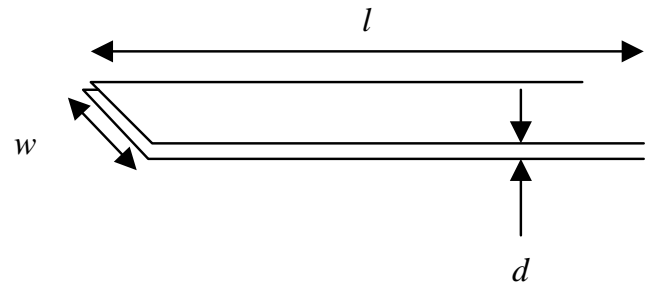
C-Core with Gap --- FEA



Microstrip --- Using Speed of Light

- Capacitance per unit length:

$$C_o = \frac{\epsilon w}{d}$$

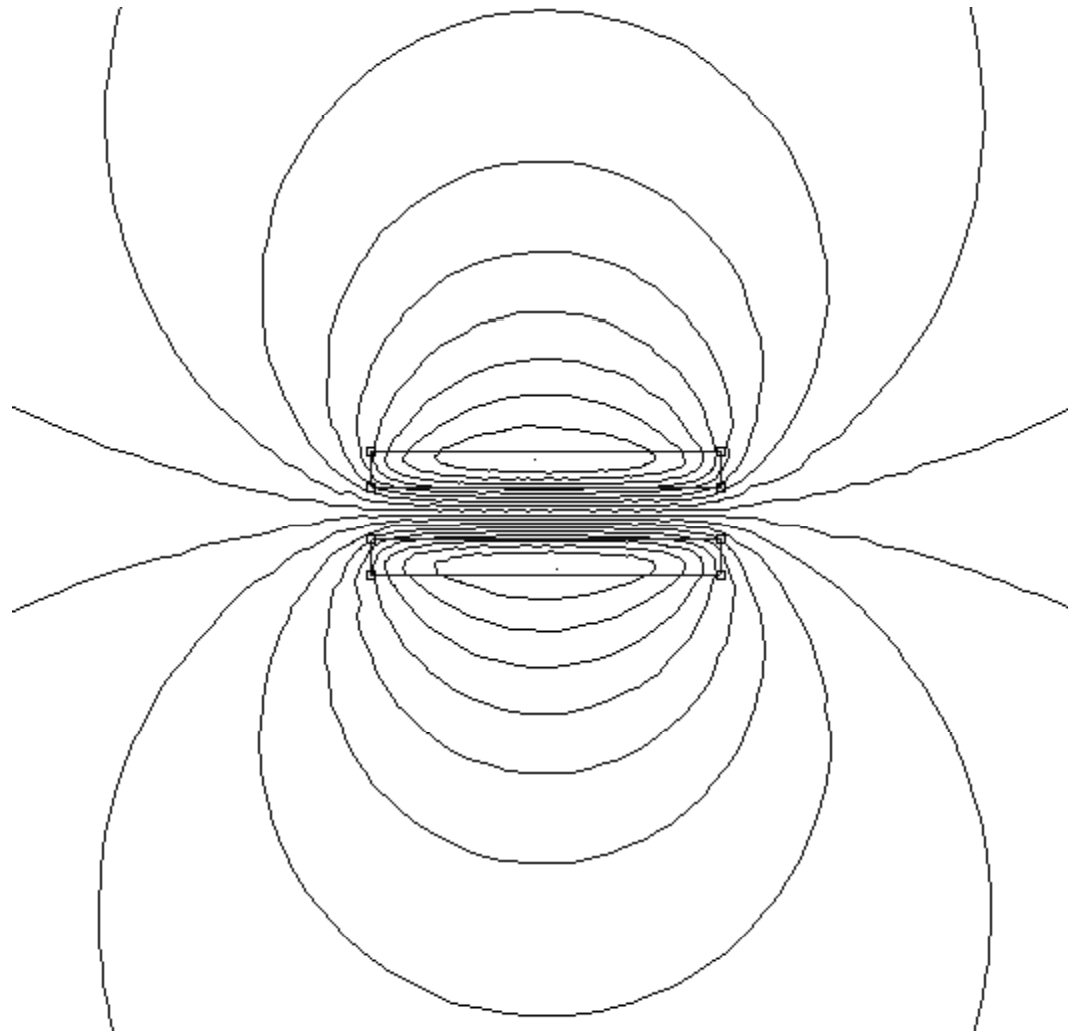


- Inductance per unit length:

$$L_o = \frac{1}{c^2 C_o} = \frac{\mu \epsilon}{\left(\frac{\epsilon w}{d} \right)} = \mu \frac{d}{w}$$

- Inductance: $L = \mu \frac{ld}{w}$

Microstrip --- FEA



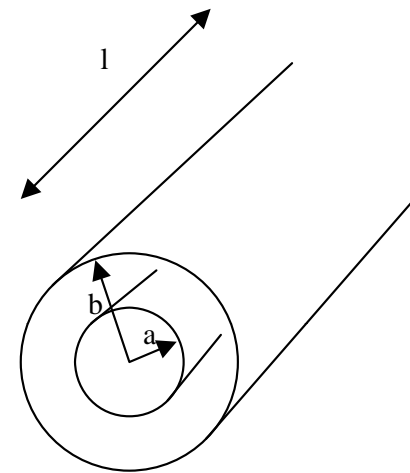
Coax --- Using Speed of Light

- Capacitance per unit length:

$$C_o = \frac{2\pi\epsilon_o}{\ln(b/a)}$$

- Inductance per unit length:

$$L_o = \frac{\mu}{2\pi} \ln(b/a)$$



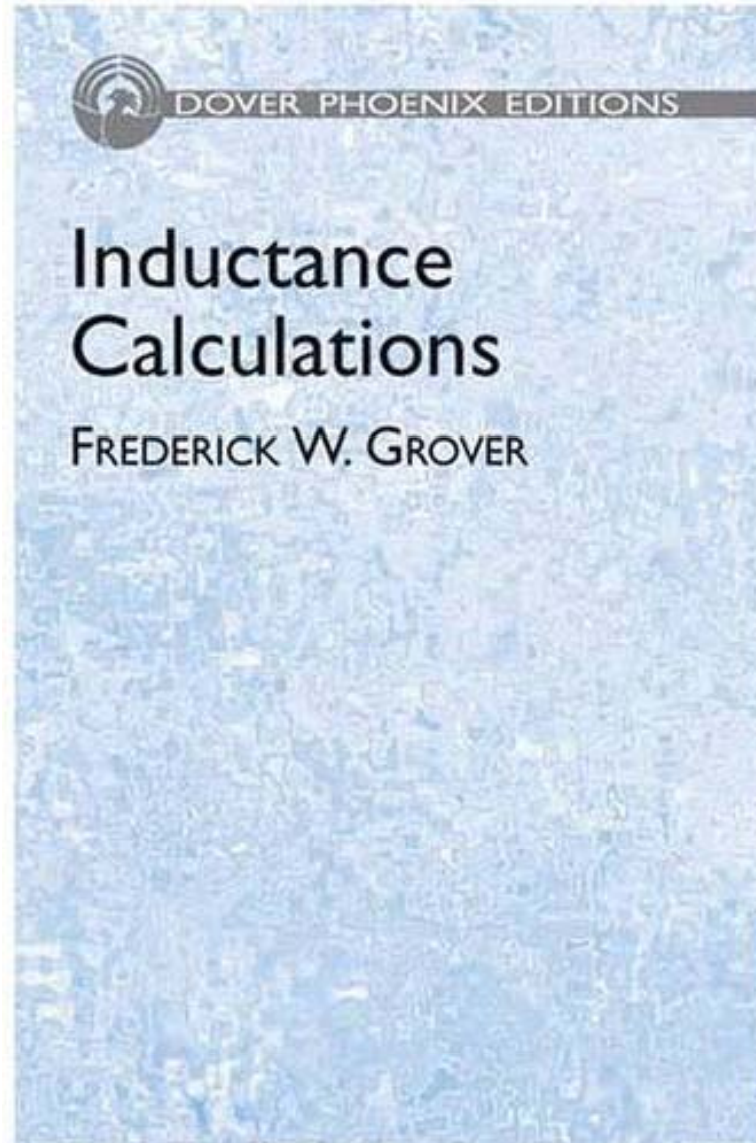
Summary of Closed-Form Methods

- Each method will give the same answer.
- The key is to decide which method gives the correct answer with a minimum of calculation
 - Energy methods and speed of light method are very useful for simple structures where fields are known
 - Magnetic circuits are very useful for complicated circuits
 - Airgaps
 - Multiple windings
 - Can use PSPICE to model magnetic circuits
 - If all else fails, use Maxwell's equations and brute force
 - In last resort (or to check your answers) use finite-element analysis (FEA) or.....
 - Handbook methods (next section)
 - Tabulated answers to known problems that are not easily solved

Handbook Methods

- People have been calculating inductances since the 1800s
- Much of this is tabulated
- Great references:
 - James C. Maxwell, *Electricity and Magnetism*, vols. 1 and 2, reprinted in Dover Publications, 1954
 - Frederick W. Grover, *Inductance Calculations: Working Formulas and Tables*, Dover Publications, Inc., New York, 1946
 - Frederick E. Terman, *Radio Engineers' Handbook*, McGraw-Hill, New York, 1943

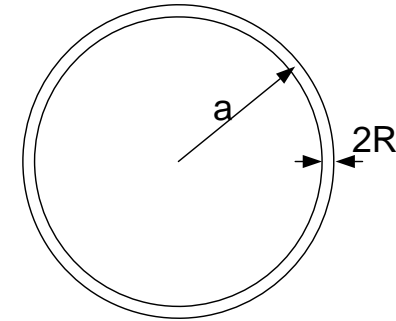
The “Bible” for Handbook Methods



Loop of Round Wire

- A circular loop of round wire with loop radius a and wire radius R has the approximate low frequency inductance

$$L = \mu_0 a \left[\ln \left(\frac{8a}{R} \right) - 1.75 \right]$$



Grover, pp. 143

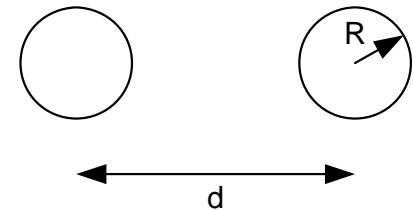
- Weak function of wire radius R
- The inductance of a 1 meter circumference loop of 14 gauge wire is $1.12 \mu\text{H}$; for 16 gauge wire it's $1.17 \mu\text{H}$; and for 18 gauge wire it's $1.21 \mu\text{H}$
- A crude approximation for circular loops is $L \approx \mu_0 \pi a$

Reference: Harold A. Wheeler, "Formulas for the Skin Effect," *Proceedings of the I.R.E.*, September 1942, pp. 412-424

Parallel-Wire Line

- For two parallel wires whose length l is great compared to their distance d apart, the inductance of the loop is

$$L = \frac{\mu_o l}{\pi} \ln \left[\frac{d}{R} + \frac{1}{4} - \frac{d}{l} \right]$$



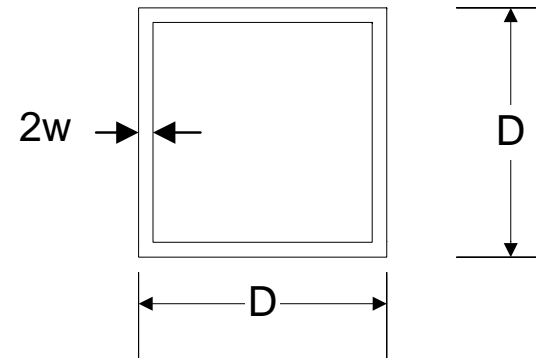
Grover, pp. 39

- For $l = 0.5$ meter and a wire-wire spacing $d = 1$ cm, results are: $L = 0.505$ μH for 14 gauge; $L = 0.551$ μH for 16 gauge and $L = 0.598$ μH for 18 gauge.
- For the parallel-wire line with closely-spaced conductors, the inductance is approximately 0.5 microHenries per meter of total wire length.

Square Loop

- Loop with side length D and wire width w , with $w \ll D$

$$L \approx \frac{2\mu_o D}{\pi} \left[\sinh^{-1} \left(\frac{D}{w} \right) - 1 \right]$$



Zahn, pp. 343

Polygon of Round Wire

- It is found in practice that the inductance of a multi-faced polygon may be approximated by replacing a polygon by a simpler plane figure of either equal area or equal perimeter
- If the perimeter of the coil is p and the area enclosed is A then the inductance takes the general form:

$$L \approx \frac{\mu_0 p}{2\pi} \left[\ln\left(\frac{2p}{R}\right) + 0.25 - \ln\left(\frac{p^2}{A}\right) \right]$$

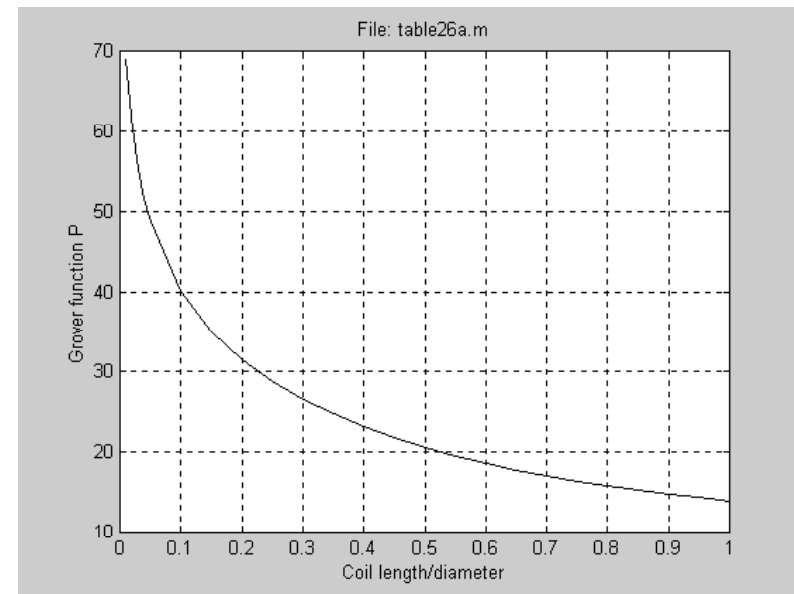
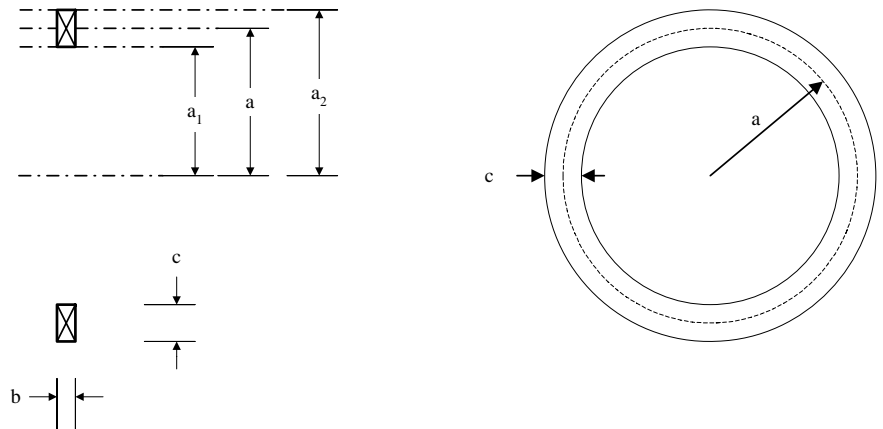
- In general, coils enclosing the same perimeter with similar shapes will have approximately the same inductance.
- This is a very useful result for strange-shaped coils

Disk Coil

- Round loop with rectangular cross section, with mean radius a , axial thickness b , and trace width c
- From *Grover* :

$$L = \frac{25\mu_o}{\pi} aPF$$

- P and F are unitless constants
 - P is a function of the coil normalized radial thickness $c/2a$ and applies to a coil of zero axial thickness ($b = 0$),
 - F accounts for the finite axial length of the coil. For $b \ll c$ and $c \ll a$ (coils resembling thin disks) the factor $F \approx 1$, an important limiting case.



Scaling Law from Disk Coil Result

- Magnetic scaling laws show that large magnetic elements are more efficient in energy conversion than smaller ones
- For this disk coil geometry, this effect can be quantified by considering the ratio of inductance to resistance
- The inductance L is approximately proportional to a as shown above
- The resistance of the coil is proportional to a/bc , the ratio of current path length to coil cross-sectional area.
- Therefore, the ratio of inductance to resistance is proportional to bc , or the cross-sectional area of the coil.
- If all coil lengths are scaled up by the same factor l , this ratio increases by the factor l^2 , or the length squared

Brooks' Coil

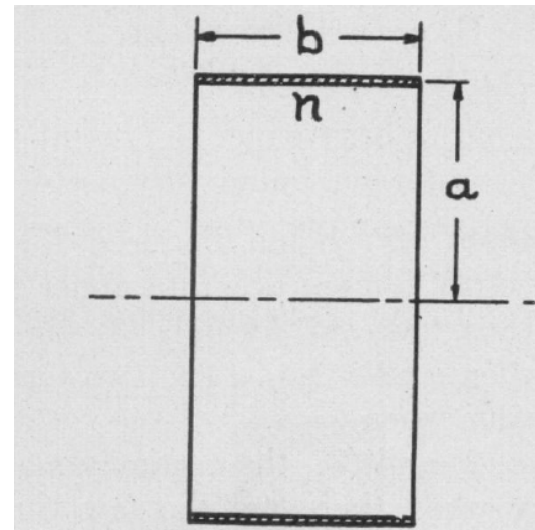
- An interesting problem is to maximize the inductance with a given length of wire
- Maxwell found that the optimal coil has a square cross section with mean diameter 3.7 times the dimension of the square cross section, or $2a = 3.7c$
- Brooks and others later refined this estimate and recommend $2a/c = 3$ as the optimum shape, with $b = c$. The result for the Brooks coil is:

$$L = 1.353\mu_0 aN^2$$

Finite Length Solenoid --- Wheeler Method

- The inductance of a finite-length solenoid of radius a and length b made of round wire can be calculated with good accuracy by using results from H. Wheeler formulas
- If the length b of the solenoid is larger $0.8a$, the accuracy is better than 1%:

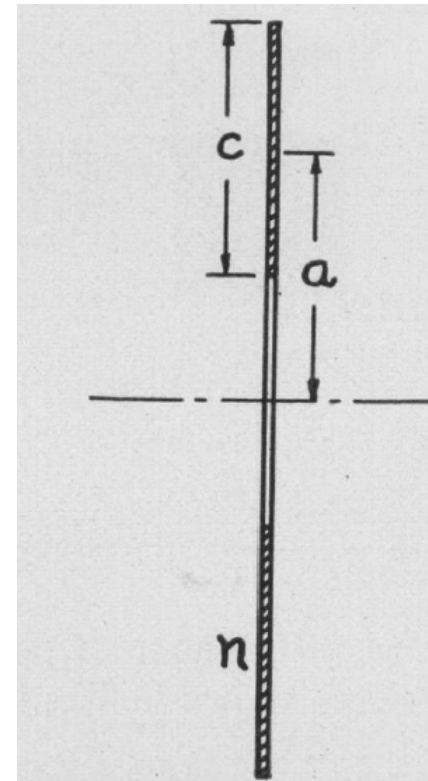
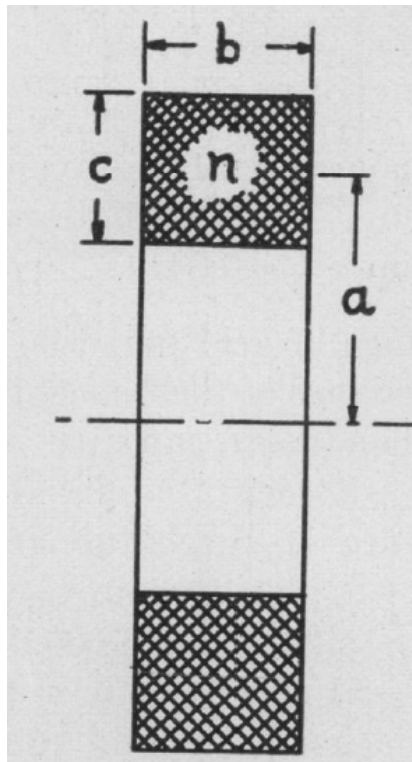
$$L \approx \frac{10\pi\mu_o N^2 a^2}{9a + 10b}$$



Reference: Harold A. Wheeler, "Simple Inductance Formulas for Radio Coils," *Proceedings of the I.R.E.*, October 1928, pp. 1398-1400.

More Wheeler Calculations

- Wheeler also gives approximations for the generic and flat pancake-type coils

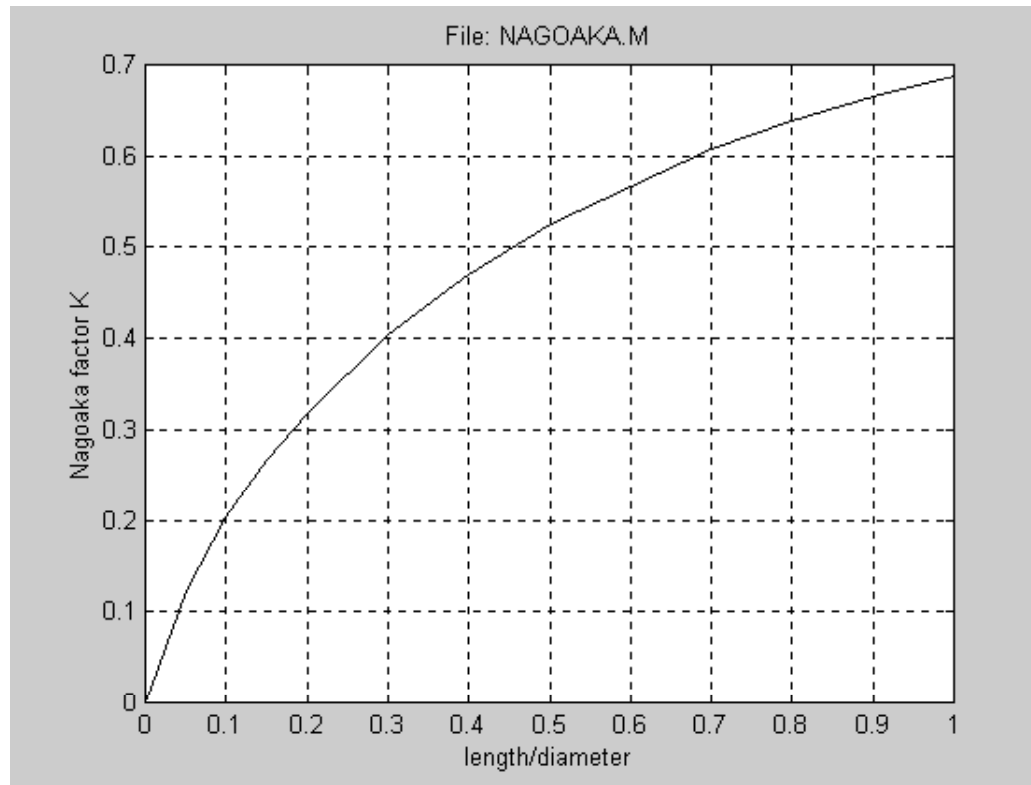


Reference: Harold A. Wheeler, "Simple Inductance Formulas for Radio Coils," *Proceedings of the I.R.E.*, October 1928, pp. 1398-1400.

Finite Length Solenoid --- Nagaoka Method

- For the short solenoid with $l < a$, a handbook method may be applied by using the Nagaoka formula where (in MKS units):

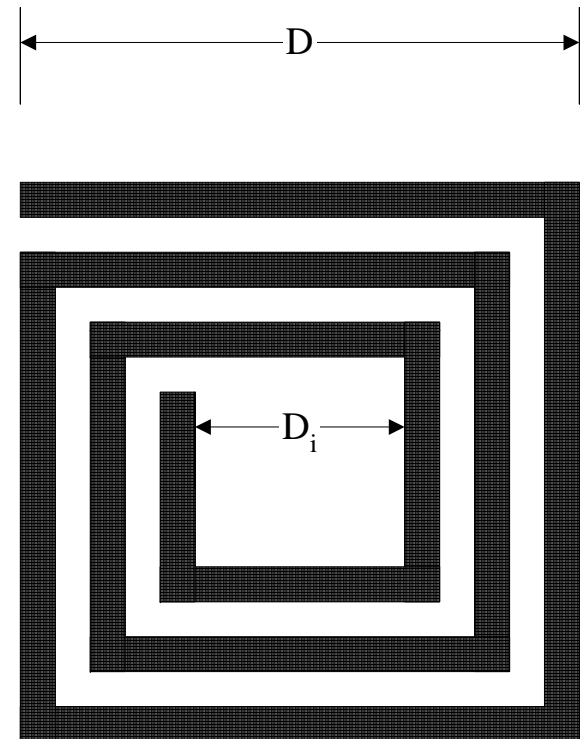
$$L = \pi\mu_0 a^2 l N^2 K$$



Planar Spirals

- Widely used on-chip inductor
- Used in power electronics (planar transformers)

$$L \approx 8.5 \times 10^{-10} DN^{5/3}$$



Reference: H. Dill, "Designing Inductors for Thin-Film Applications?" *Electronic Design*, Feb. 17, 1964, pp. 52-59

Planar Spirals in Practice

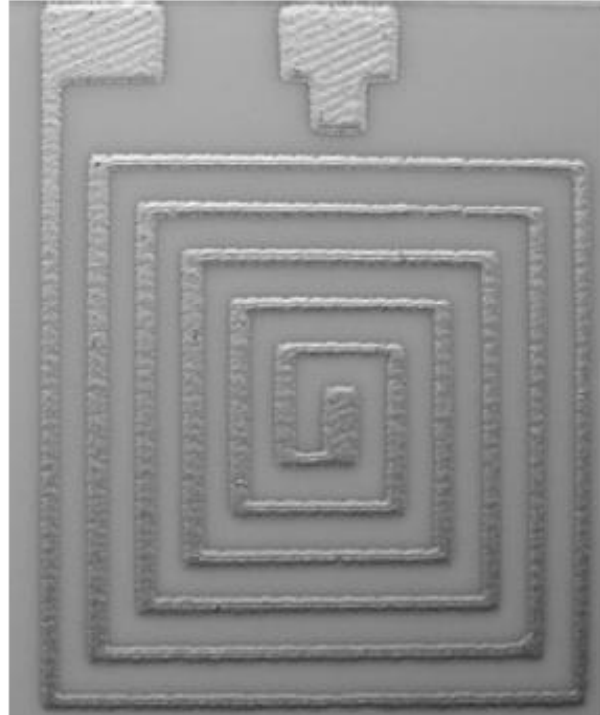


Fig. 3. The screen printed base of 5.5 coils of inductor made on ceramic substrate.

Reference: J. Vanek, et. al., "Optimization of Properties of Planar Spiral Inductors," *IEEE 30th International Spring Seminar on Electronics Technology*, May 9-13, 2007, pp. 235-238

Summary of Magnetic Design Issues

1. There are a variety of methods available for approximately analyzing magnetic structures
2. Ampere's Law shows how a current creates a magnetic field
3. Faraday's Law shows how eddy currents are produced. By extension, Faraday's law results in skin effect and proximity effect
4. Airgaps are often used in magnetic structures to make the magnetic properties less variable with gap, temperature, material properties, etc.
5. The skin and proximity effects can result in much higher than expected AC resistance to current flow, and hence higher losses
6. Use FEA when necessary

Some FEA Examples

- Using FEMM (Finite Element Method Magnetics), a 2D magnetic, electrostatic and thermal solver from Dave Meeker at Foster-Miller

Example 1: Air-Core Coil

- Air-core coil, mean radius $a = 2.5''$, axial length $b = 1''$, radial width $c = 1''$, $N = 1$

2D FEA model

Using Grover, pp. 95

$$N = 1$$

$$a = 2.5'' \text{ (6.35 cm)}$$

$$b = 1'' \text{ (2.54 cm)}$$

$$c = 1'' \text{ (2.54 cm)}$$

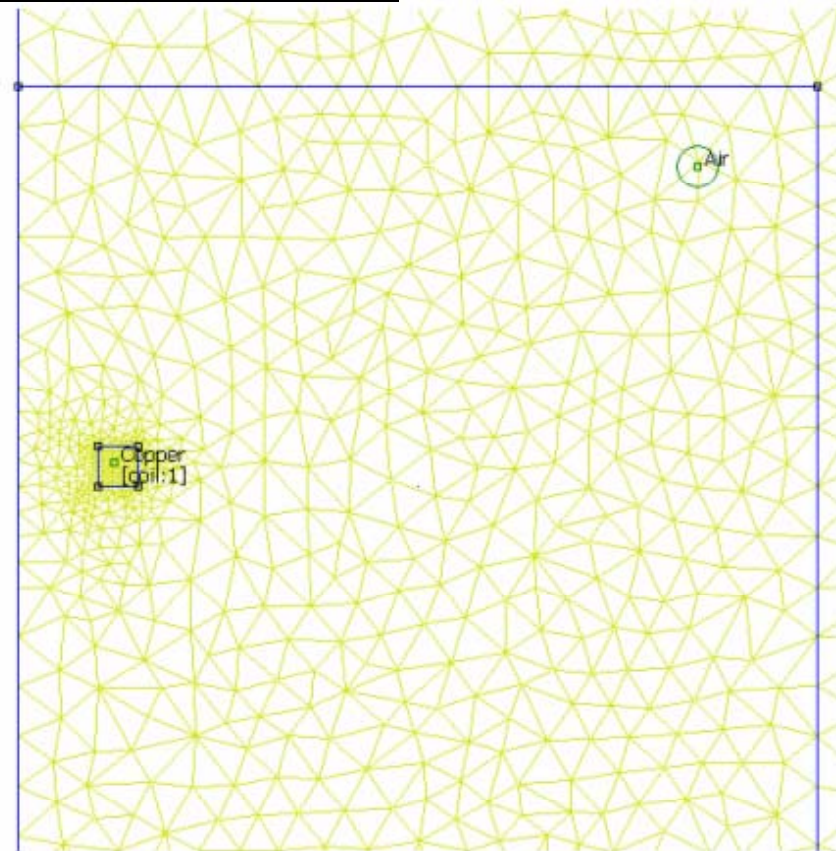
$$c/2a = 0.2$$

$$b/c = 1$$

$$L = 0.001aN^2P_o \text{ (L in } \mu\text{H, a in cm)}$$

$$P_o = 22.954 \text{ from Table 21, pp. 96}$$

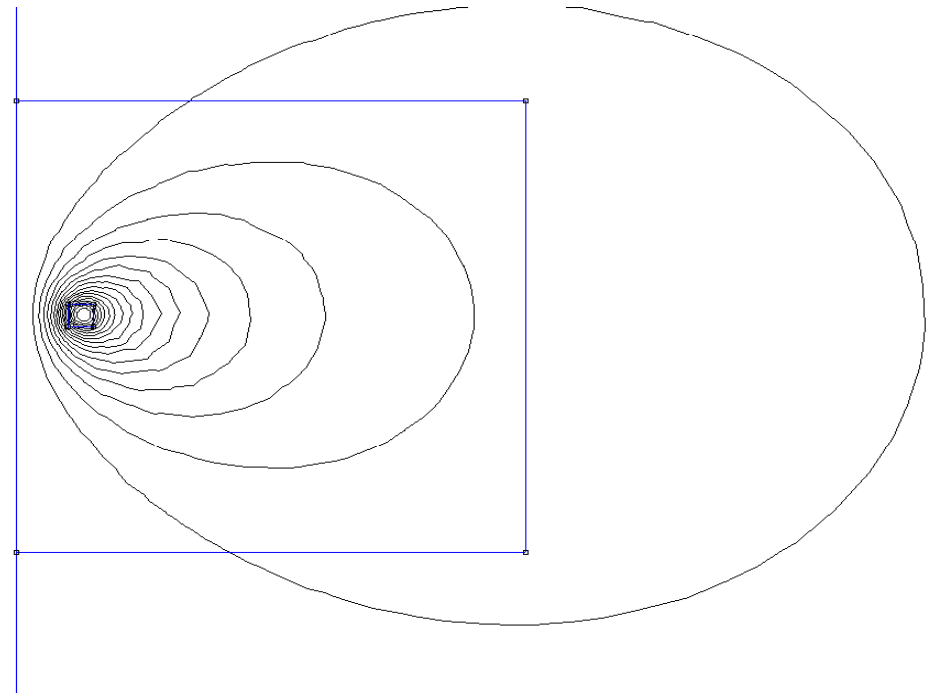
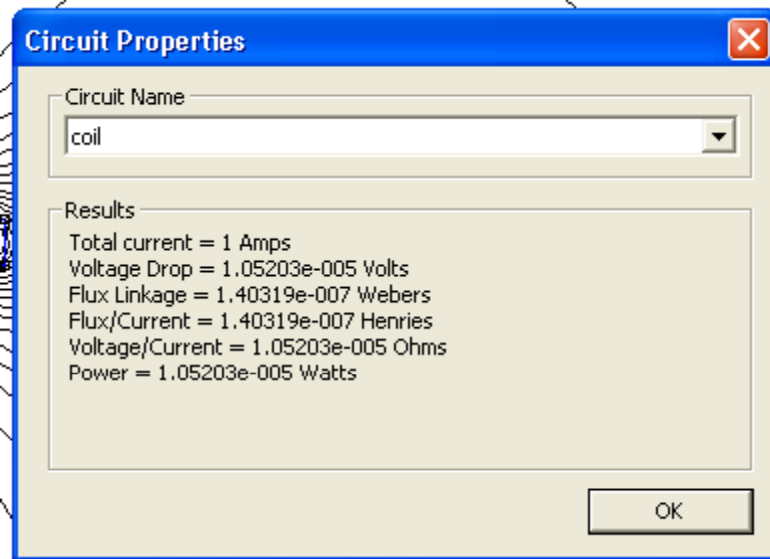
$$L = 145.7 \text{ nH}$$



Symmetry axis

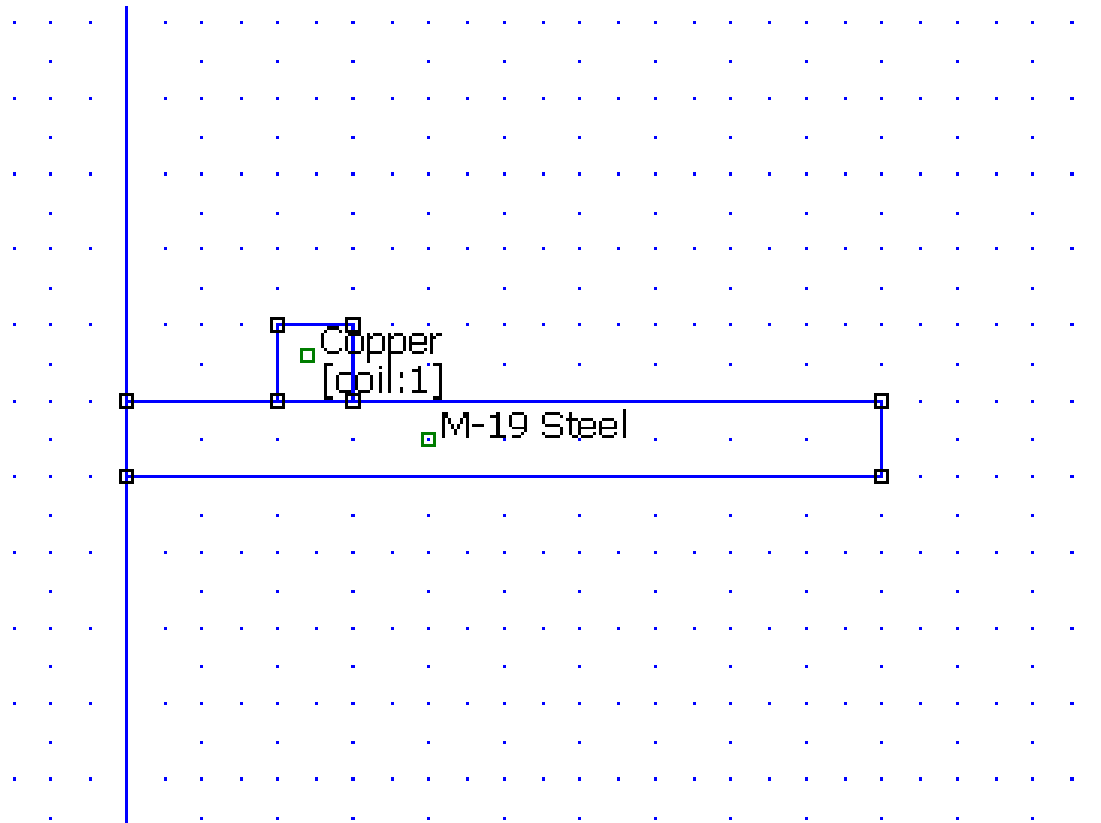
Example 1: Air-Core Coil

- DC inductance $L = 140.3 \text{ nH}$ from FEA



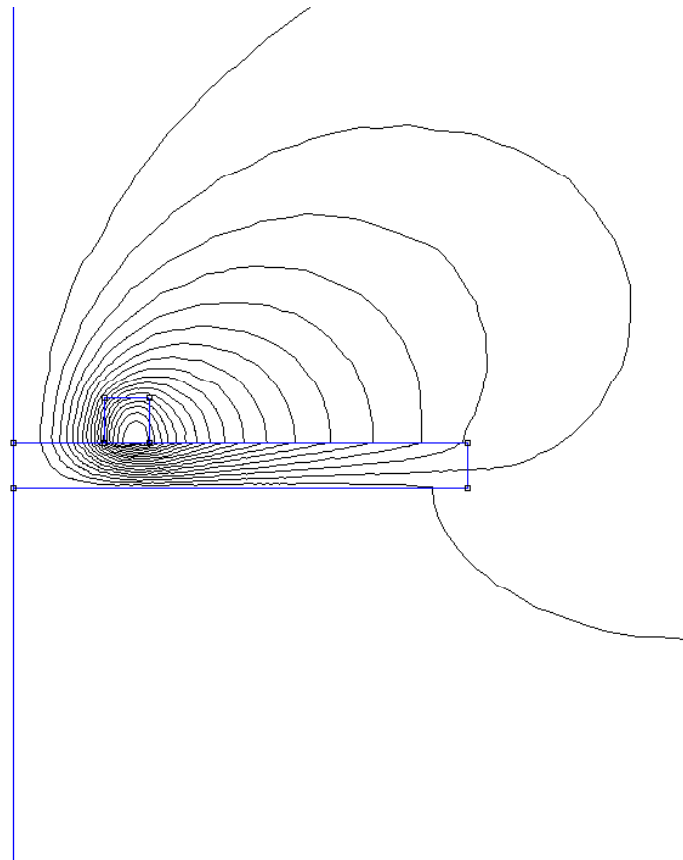
Example 2: Air Core Coil Above Steel

- What is inductance if I put this coil on top of high- μ M-19 steel?



Example 2: Air Core Coil Above Steel

- DC FEA result showing field lines



Example 2 : Air Core Coil Above Steel

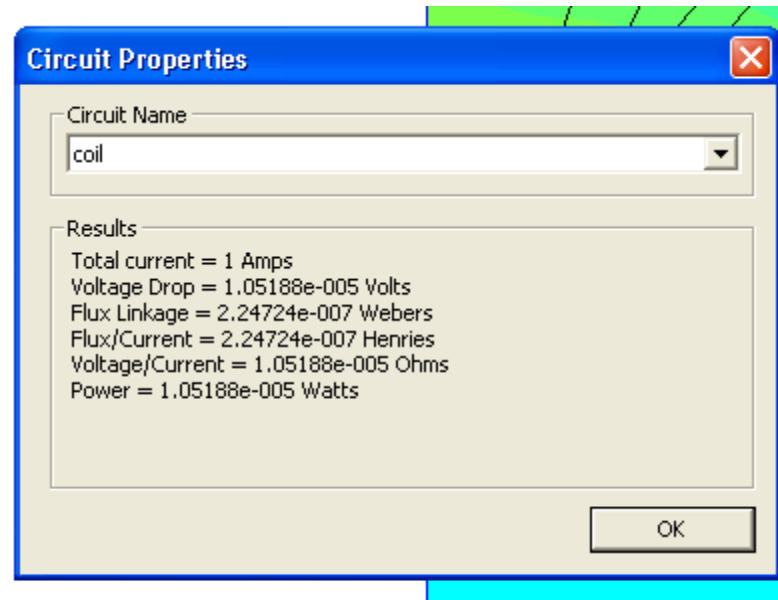
- We know the inductance goes up with the steel underneath the coil.
- Does it double, triple, or quadruple?

Example 2 : Air Core Coil Above Steel

- By the method of images, we get $\sim 2x$ the flux through the coil
- So, the inductance goes up by a factor of $\sim 2x$

Example 2 : Air Core Coil Above Steel

- FEA shows that $L = 224 \text{ nH}$, a factor of ~ 1.57
- Why not a factor of 2 exactly?



- The “image” coil is spaced some distance away from the original coil, as it’s a relatively thick coil

Example 3: Thin Disk Coil

- Compare FEA to Grover result

Using Grover, pp. 105

$N = 1$

$a = 2.5''$ (6.35 cm)

$b = 0.1''$ (0.254 cm)

$c = 1''$ (2.54 cm)

$c/2a = 0.2$

$b/c = 0.1$

$L = 0.001aN^2PF$ (uH)

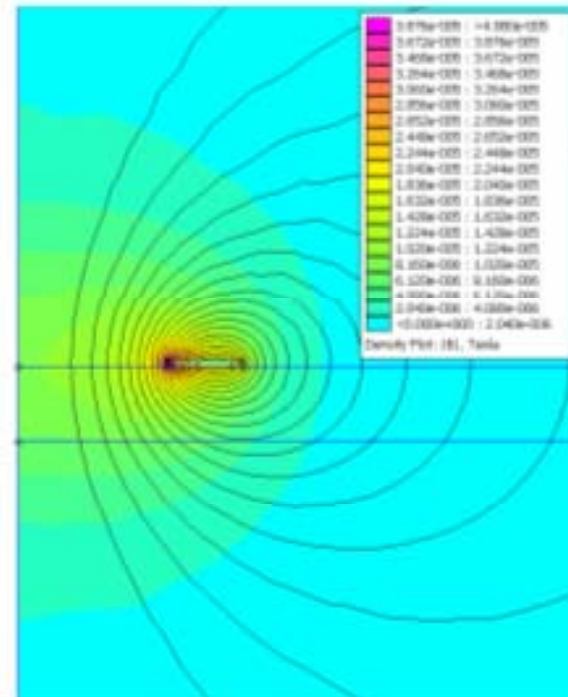
$P = 31.5$ from Table 26

$F = 0.9612$ from Table 24

$L = 192$ nH

Disk coil in air

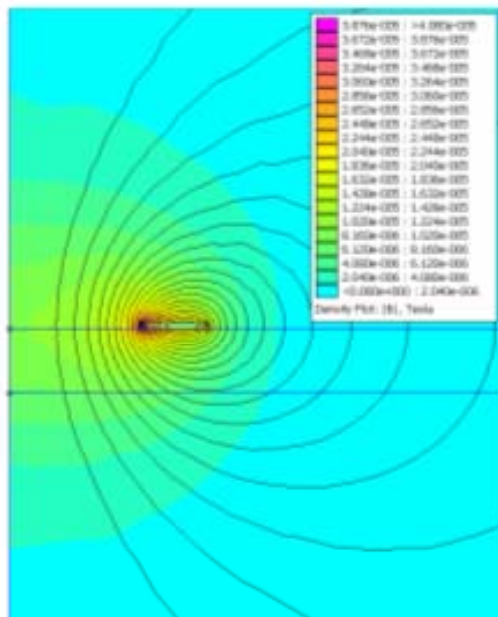
$L = 187$ nH at DC



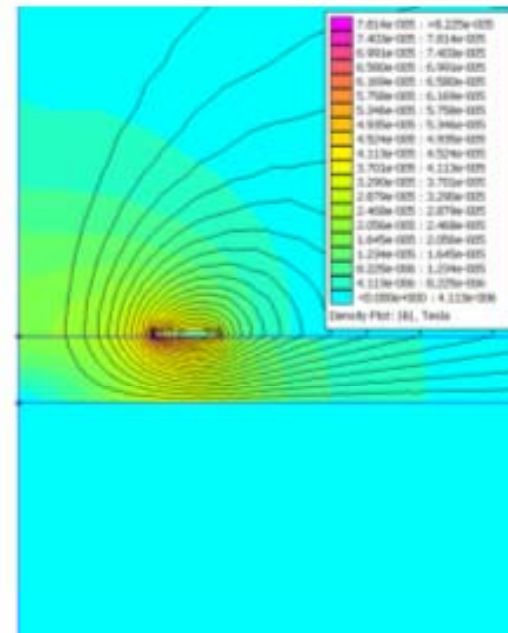
Example 3: Thin Coil With and Without Steel

- We'll get a better fit to a factor of 2, if the axial length of the coil is smaller, since the image coil is closer to the original coil

Disk coil in air
 $L = 187 \text{ nH at DC}$



Disk coil on steel
 $L = 360 \text{ nH at DC}$

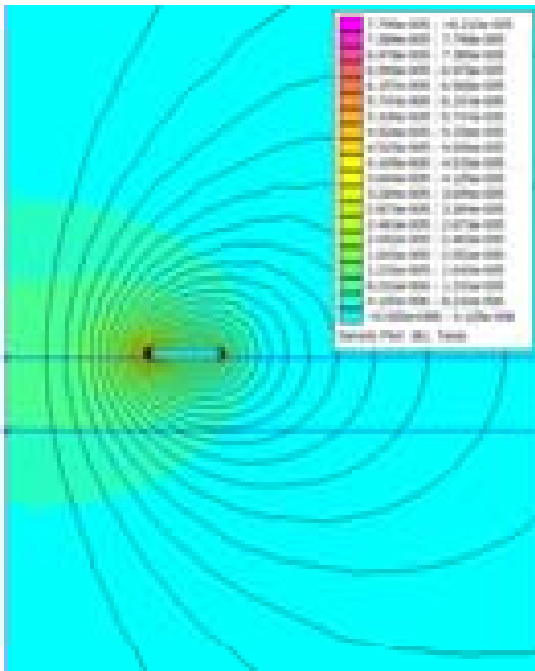


Example 4: What Happens at 100 kHz?

- Inductance is modified due to eddy currents

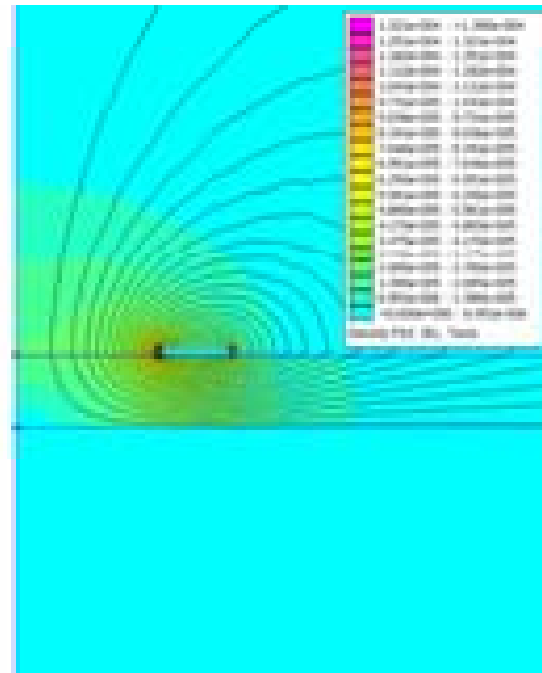
Disk coil in air

$L = 169 \text{ nH}$ at 100 kHz



Disk coil on steel

$L = 315 \text{ nH}$ at 100 kHz

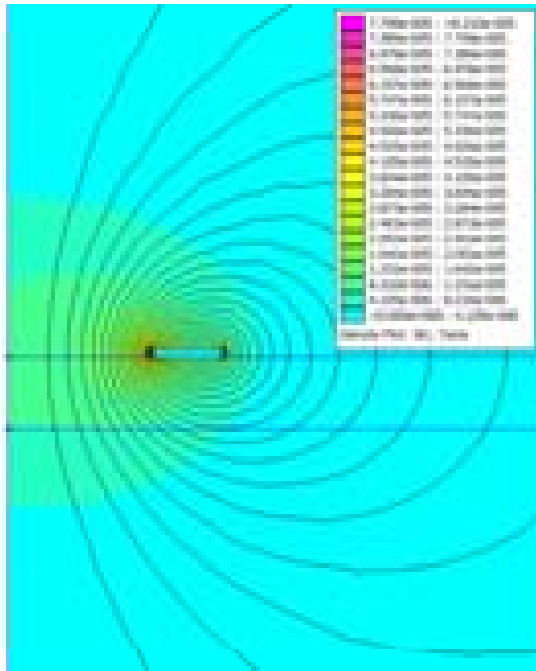


Example 5: What Happens at 100 kHz with Coil Above a Thick Copper Plate?

- Skin depth ~ 0.22 mm in copper at 100 kHz

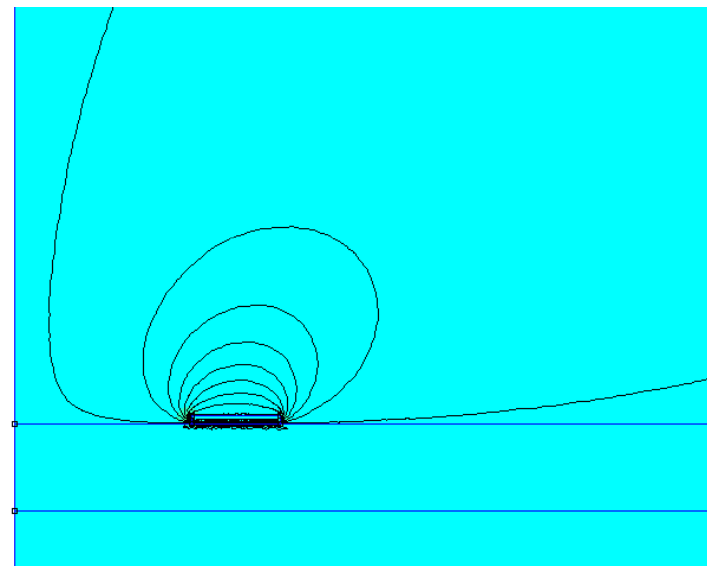
Disk coil in air

$L = 169$ nH at 100 kHz



Disk coil on copper plate

$L = 4$ nH at 100 kHz



References

1. James Clerk Maxwell, *A Treatise on Electricity and Magnetism*, reprinted by Dover Publications, New York 1954
2. H. Haus and J. Melcher, *Electromagnetic Fields and Energy*, Prentice-Hall, 1989
3. Frederick W. Grover, *Inductance Calculations: Working Formulas and Tables*, Dover Publications, New York, 1946
4. Frederick E. Terman, *Radio Engineers' Handbook*, McGraw-Hill, New York, 1943