

Power Electronics Notes 30A

Review of Magnetics

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Portions of these notes excerpted from the CD ROM accompanying Fitzgerald, Kingsley and Umans, *Electric Machinery*, 6th edition, McGraw Hill, 2003 and from the CD ROM accompanying Mohan, Undeland and Robbins, *Power Electronics Converters, Applications and Design*, 3d edition, John Wiley 2003. Other notes © Marc Thompson, 2005

Overview

- Review of Maxwell's equations
- Magnetic circuits
- Flux, flux linkage, inductance and energy
- Properties of magnetic materials

Review of Maxwell's Equations

James Clerk Maxwell (13 June 1831 – 5 November 1879) was a Scottish mathematician and theoretical physicist. His most significant achievement was the development of the classical electromagnetic theory, synthesizing all previous unrelated observations, experiments and equations of electricity, magnetism and even optics into a consistent theory. His set of equations—Maxwell's equations—demonstrated that electricity, magnetism and even light are all manifestations of the same phenomenon: the electromagnetic field. From that moment on, all other classical laws or equations of these disciplines became simplified cases of Maxwell's equations. Maxwell's work in electromagnetism has been called the "*second great unification in physics*", after the first one carried out by Newton.



James Clerk Maxwell
(1831-1879)

Reference: www.wikipedia.com

Review of Maxwell's Equations

- First published by James Clerk Maxwell in 1864
- Maxwell's equations couple electric fields to magnetic fields, and describe:
 - Magnetic fields
 - Electric fields
 - Wave propagation (through the wave equation)
- There are 4 Maxwell's equations, but in magnetics we generally only need 3:
 - Ampere's Law
 - Faraday's Law
 - Gauss' Magnetic Law

Ampere's Law

- Flowing current creates a magnetic field



**André-Marie Ampère
(1775-1836)**

Ampere's Law

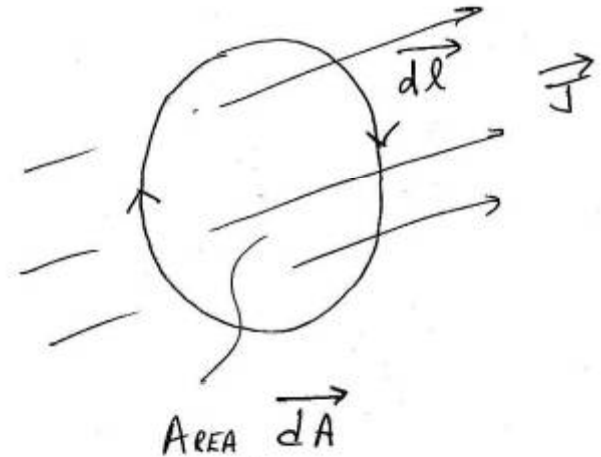
- Flowing current creates a magnetic field

$$\oint_C \vec{H} \cdot d\vec{l} = \int_S \vec{J} \cdot d\vec{A} + \frac{d}{dt} \int_S \epsilon_0 \vec{E} \cdot d\vec{A}$$

- In magnetic systems, generally there is high current and low voltage (and hence low electric field) and we can approximate for low d/dt :

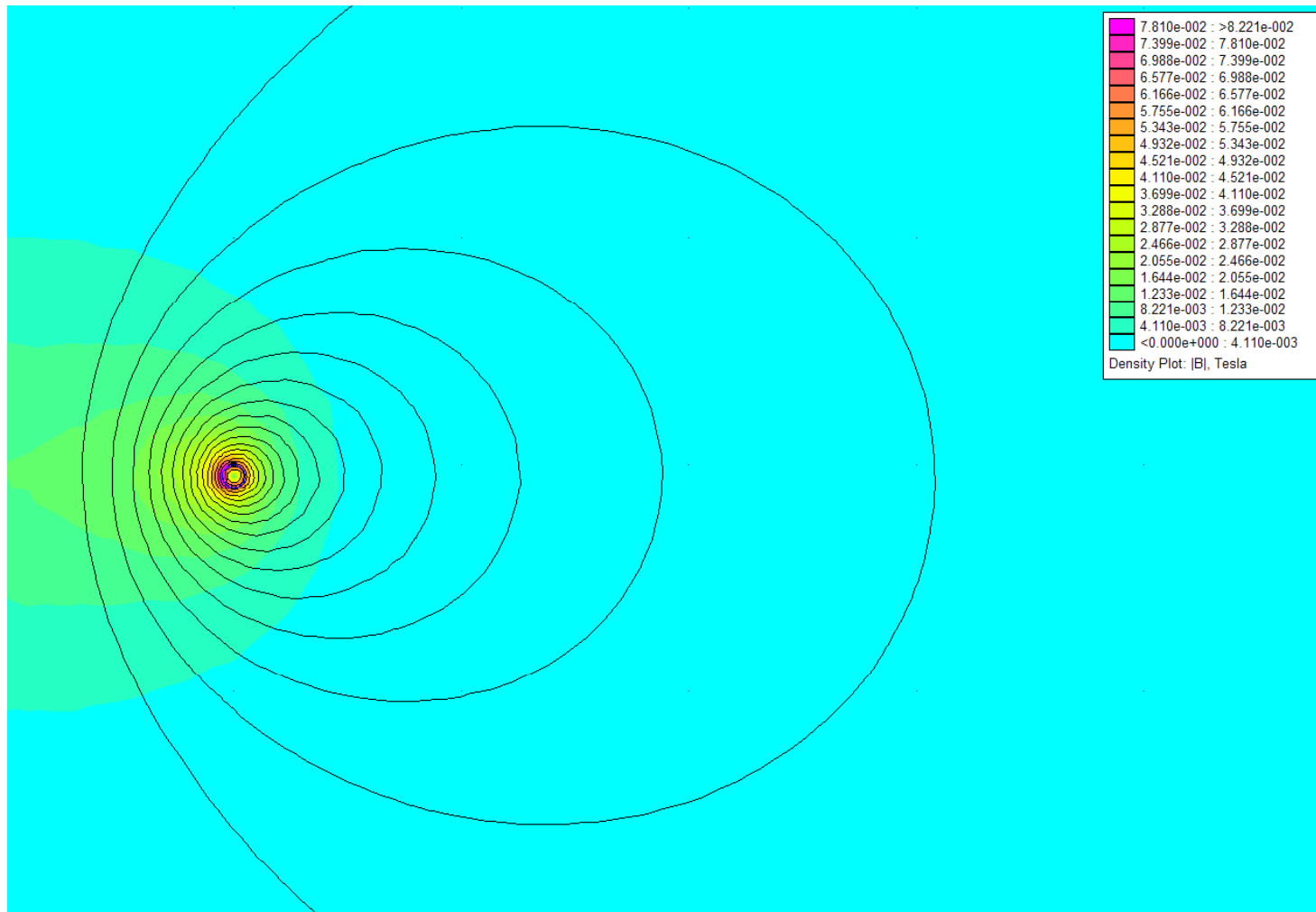
$$\oint_C \vec{H} \cdot d\vec{l} \approx \int_S \vec{J} \cdot d\vec{A}$$

- The magnetic flux density integrated around a closed contour equals the net current flowing through the surface bounded by the contour



Field From Current Loop, NI = 500 A-turns

- Coil radius $R = 1''$



Faraday's Law

- A changing magnetic flux impinging on a conductor creates an electric field and hence a current (eddy current)



Michael Faraday
(1791-1867)

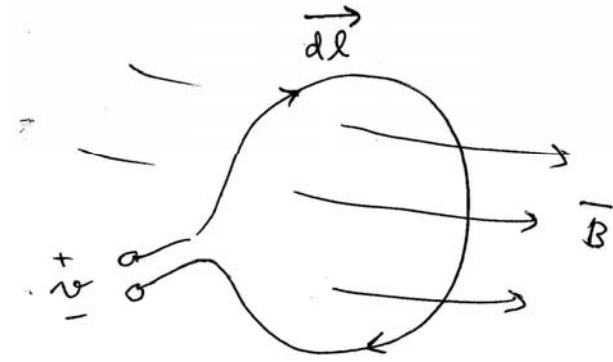
Faraday's Law

- A changing magnetic flux impinging on a conductor creates an electric field and hence a current (eddy current)

$$\oint_C \vec{E} \cdot d\vec{l} = -\frac{d}{dt} \int_S \vec{B} \cdot d\vec{A}$$

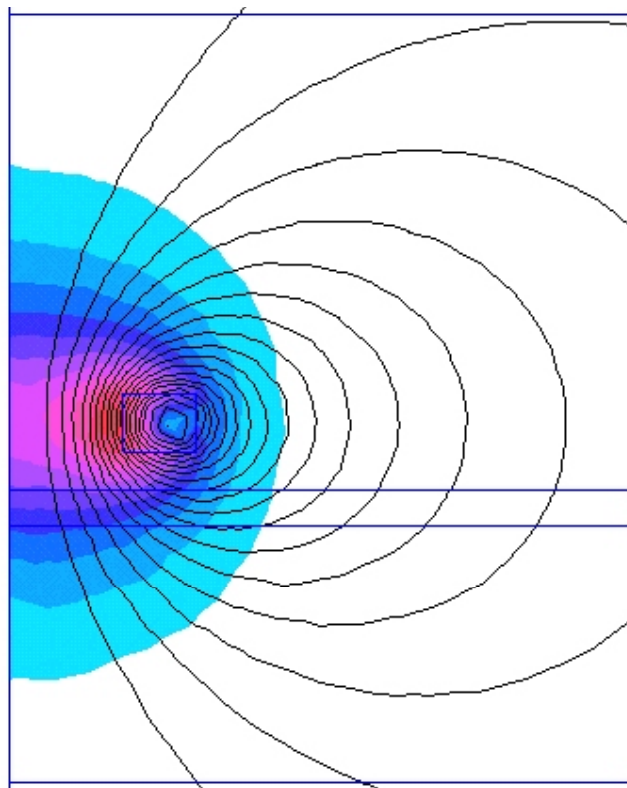
- The electric field integrated around a closed contour equals the net time-varying magnetic flux density flowing through the surface bound by the contour
- In a conductor, this electric field creates a current by: $\vec{J} = \sigma\vec{E}$

- Induction motors, brakes, etc.

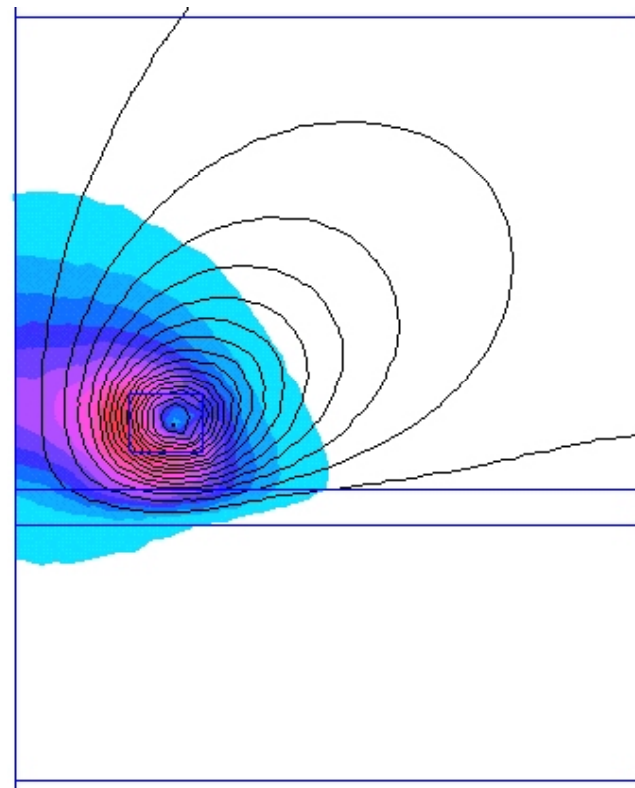


Circular Coil Above Conducting Aluminum Plate

- Flux density plots at DC and 60 Hz
- At 60 Hz, currents induced in plate via magnetic induction create lift force



DC



60 Hz

Gauss' Magnetic Law

Johann Carl Friedrich Gauss ([30 April 1777](#) – [23 February 1855](#)) was a German mathematician and scientist who contributed significantly to many fields, including number theory, statistics, analysis, differential geometry, geodesy, electrostatics, astronomy, and optics. Sometimes known as the *princeps mathematicorum* (Latin, usually translated as "the Prince of Mathematicians", although Latin *princeps* also can simply mean "the foremost") and "greatest mathematician since antiquity", Gauss had a remarkable influence in many fields of mathematics and science and is ranked as one of history's most influential mathematicians.



Carl Friedrich Gauss
(1777-1855)

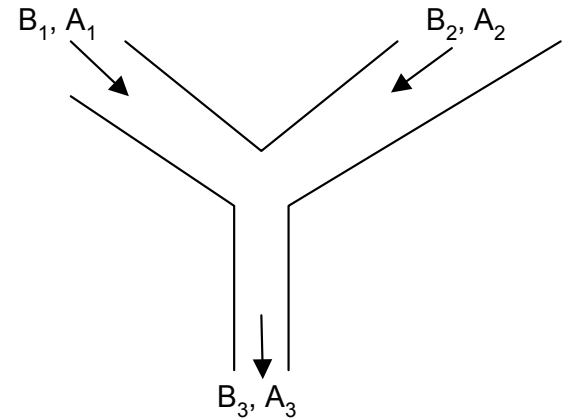
Reference: www.wikipedia.com

Gauss' Magnetic Law

- Gauss' magnetic law says that the integral of the magnetic flux density over any closed surface is zero, or:

$$\oint_S \vec{B} \cdot d\vec{A} = 0$$

- This law implies that magnetic fields are due to electric currents and that magnetic charges (“monopoles”) do not exist.
- Note: similar form to KCL in circuits !
- (We'll use this analogy later...)



$$B_3 A_3 = B_1 A_1 + B_2 A_2$$

Gauss' Law --- Continuity of Flux Lines

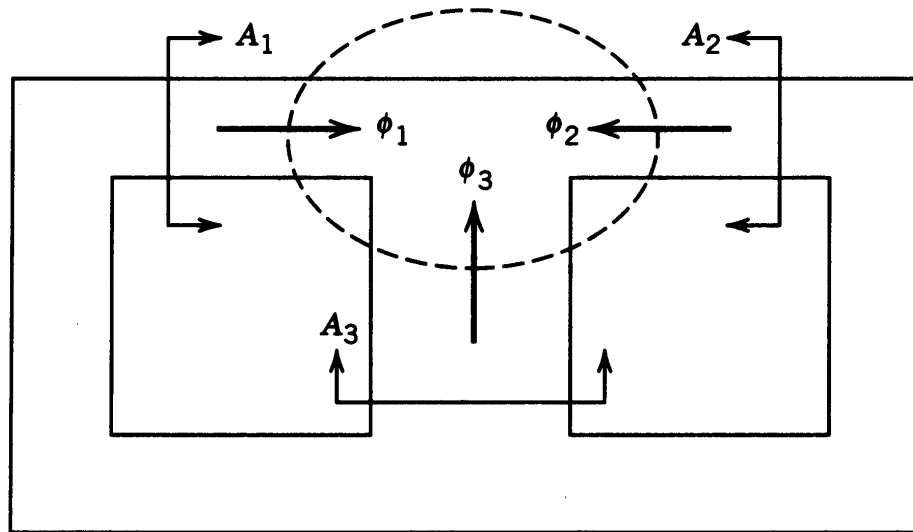


Figure 3-13 Continuity of flux.

$$\phi_1 + \phi_2 + \phi_3 = 0$$

Lorentz Force Law

- Experimentally derived rule:

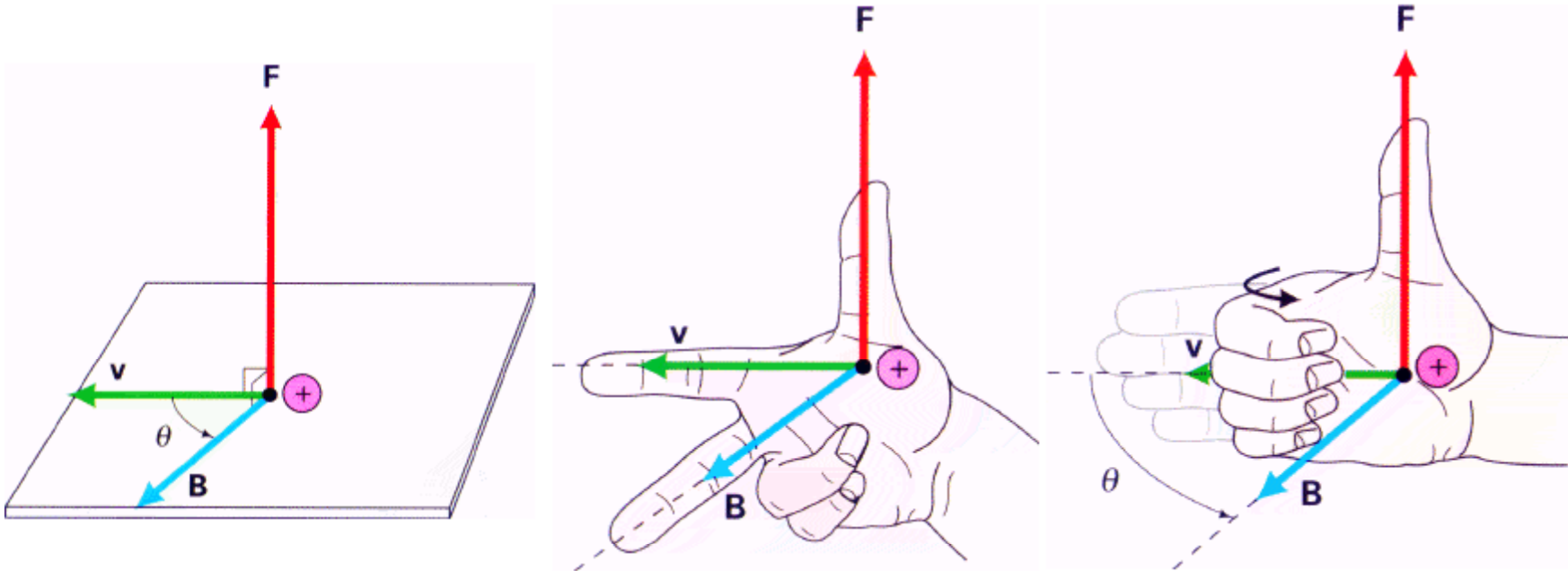
$$\vec{F} = \int \vec{J} \times \vec{B} dV$$

- For a wire of length l carrying current I perpendicular to a magnetic flux density B , this reduces to:

$$\vec{F} = IBl$$

Lorentz Force Law and the Right Hand Rule

$$\vec{F} = \int \vec{J} \times \vec{B} dV$$



Reference: <http://www.physics.brocku.ca/faculty/sternin/120/slides/rh-rule.html>

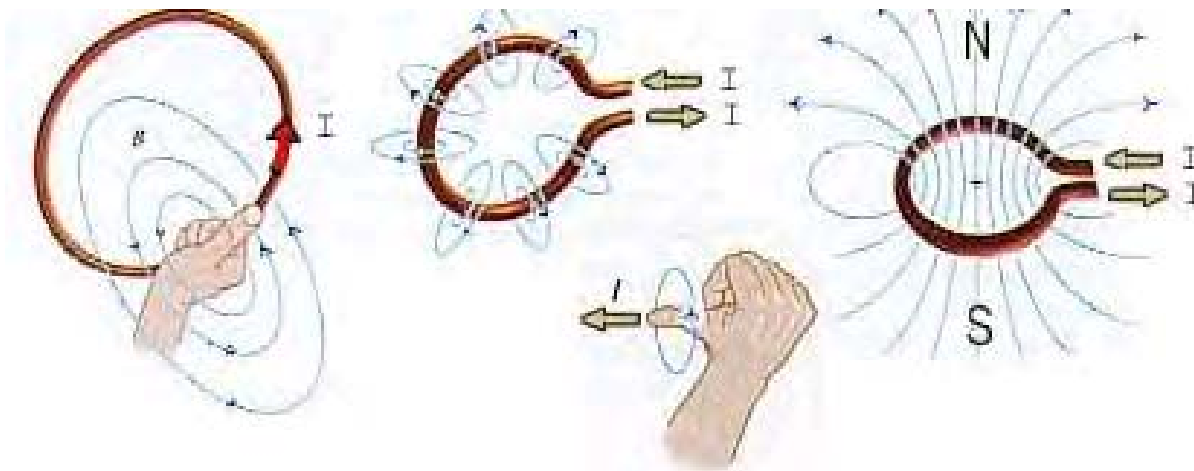
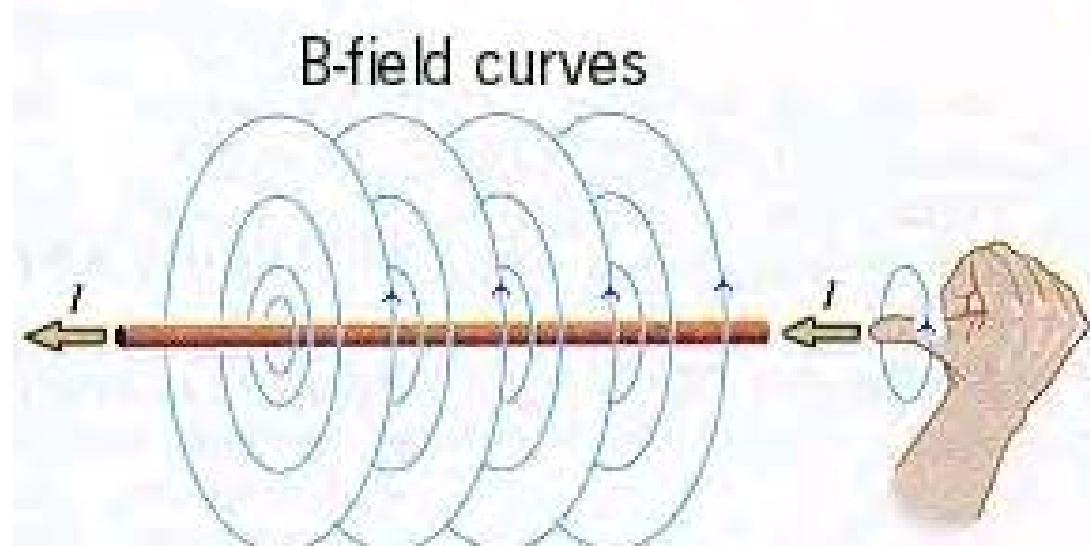
Intuitive Thinking about Magnetics

- By Ampere's Law, the current \mathbf{J} and the magnetic field \mathbf{H} are generally at right angles to one another
- By Gauss' law, magnetic field lines loop around on themselves
 - No magnetic monopole
- You can think of high- μ magnetic materials such as steel as an easy conduit for magnetic flux.... i.e. the flux easily flows thru the high- μ material

Relationship of **B** and **H**

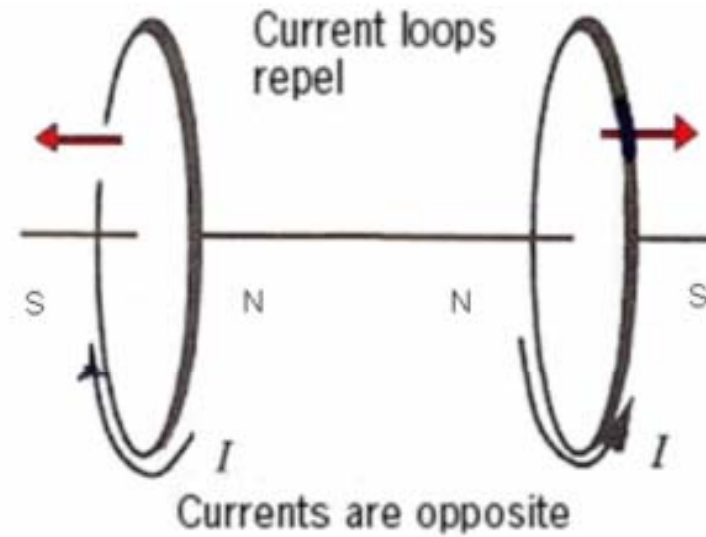
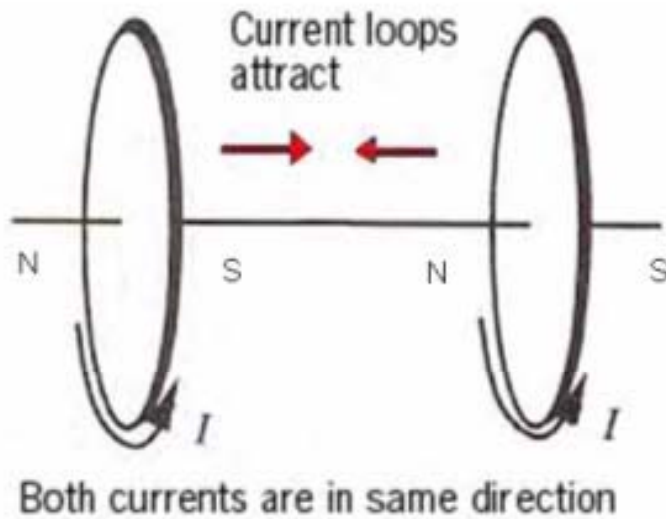
- **H** is the magnetic field (A/m in SI units) and **B** is the magnetic flux density (Weber/m², or Tesla, in SI units)
- **B** and **H** are related by the magnetic permeability μ by **$B = \mu H$**
- Magnetic permeability μ has units of Henry/meter
- In free space $\mu_0 = 4\pi \times 10^{-7}$ H/m

Right Hand Rule for B and I



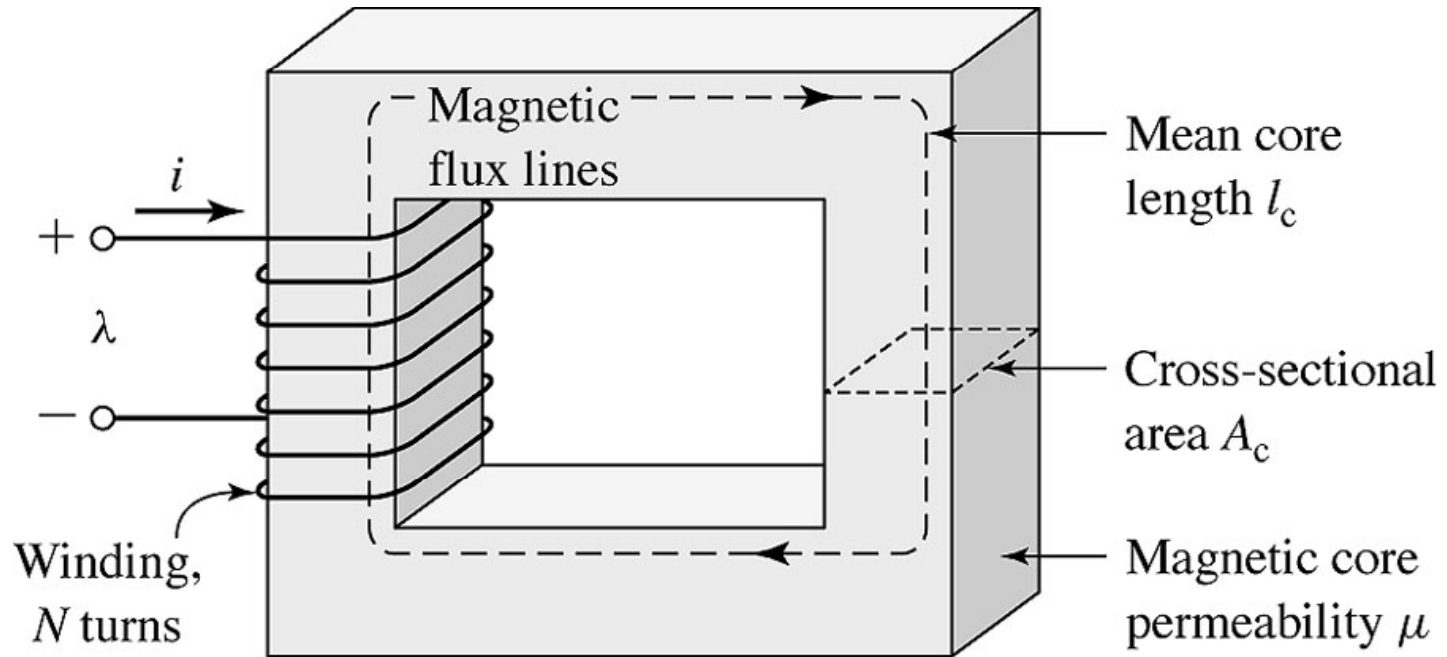
Reference: <http://sol.sci.uop.edu/~jfalward/magneticforcesfields/magneticforcesfields.html>

Forces Between Current Loops



Reference: <http://sol.sci.uop.edu/~jfalward/magneticforcesfields/magneticforcesfields.html>

Inductor Without Airgap



Constitutive relationships

In free space:

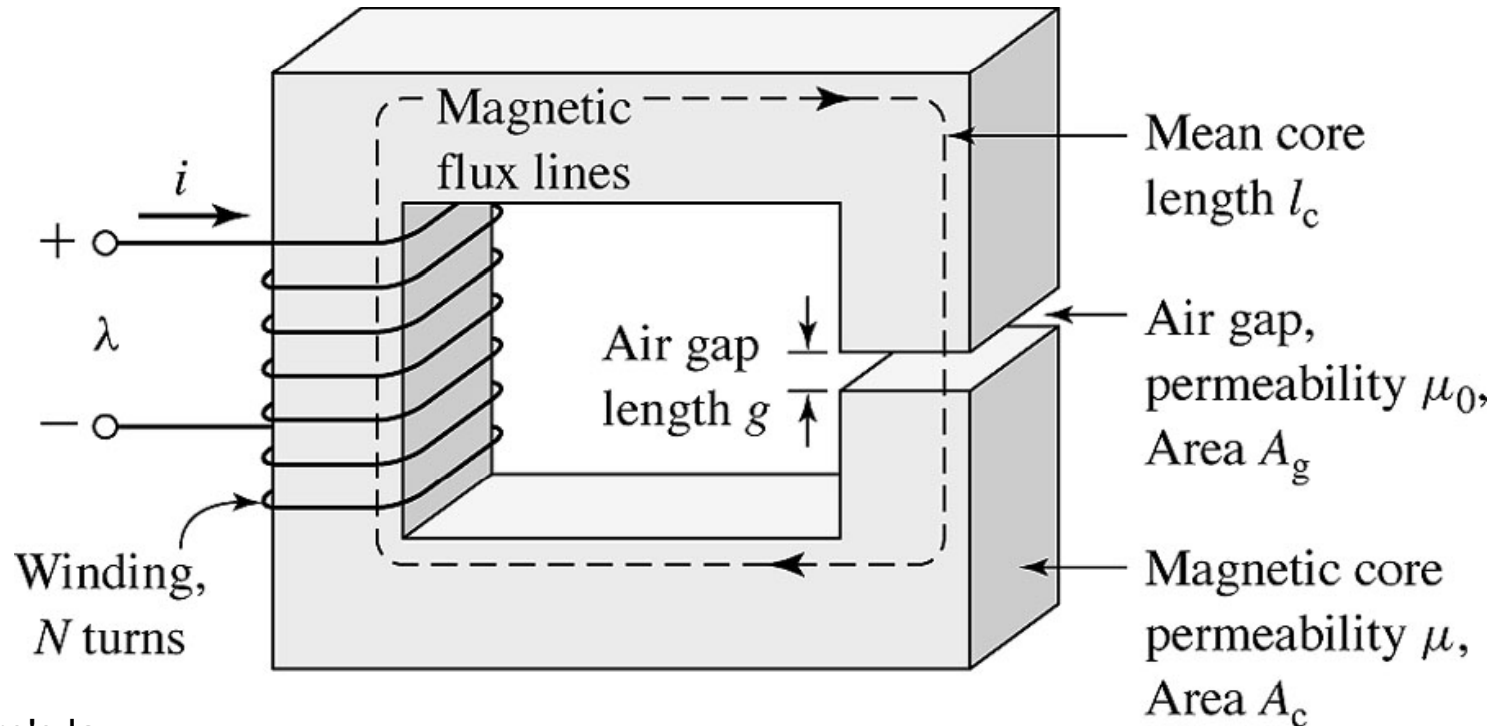
$$\vec{B} = \mu_0 \vec{H}$$

Magnetic permeability of free space $\mu_0 = 4\pi \times 10^{-7}$ Henry/meter.

In magnetic material:

$$\vec{B} = \mu \vec{H}$$

Inductor with Airgap



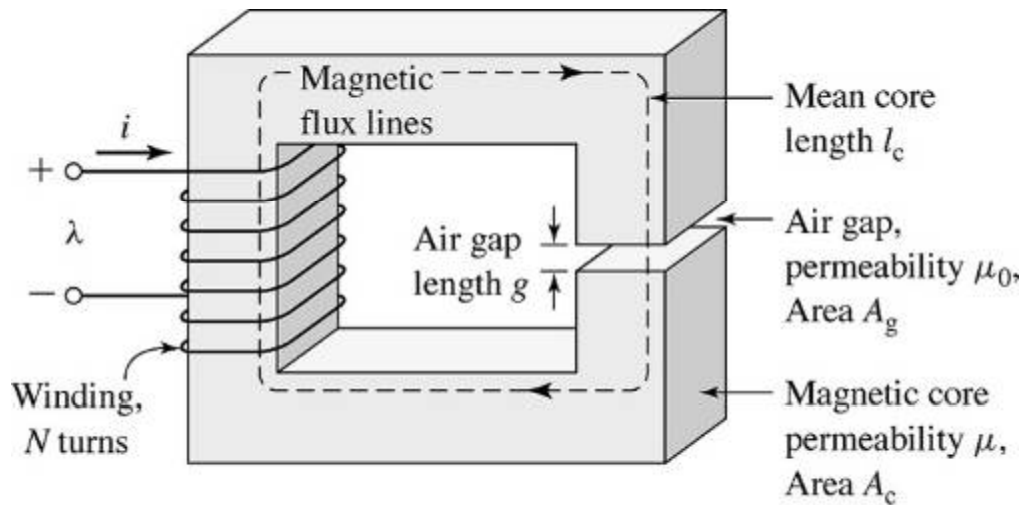
Ampere's law:

$$\oint \vec{H} \cdot d\vec{l} = \int_S \vec{J} \cdot d\vec{A} \Rightarrow H_c l_c + H_g g = NI$$

Let's use constitutive relationships:

$$\frac{B_c}{\mu_c} l_c + \frac{B_g}{\mu_o} g = NI$$

Inductor with Airgap



$$B_c = \frac{\Phi}{A_c}$$

$$B_g = \frac{\Phi}{A_g}$$

Put this into previous expression: $NI = \Phi \left(\frac{l_c}{\mu A_c} + \frac{g}{\mu_o A_g} \right)$

Solve for flux: $\Phi = \frac{NI}{\left(\frac{l_c}{\mu A_c} + \frac{g}{\mu_o A_g} \right)}$

If $g/\mu_o A_g \gg l_c/\mu A_c$

$$\Phi \approx \frac{NI}{\frac{g}{\mu_o A_g}}$$

Magnetic-Electric Circuit Analogy

- Use Ohm's law analogy to model magnetic circuits

$$V \Leftrightarrow NI$$

$$I \Leftrightarrow \Phi$$

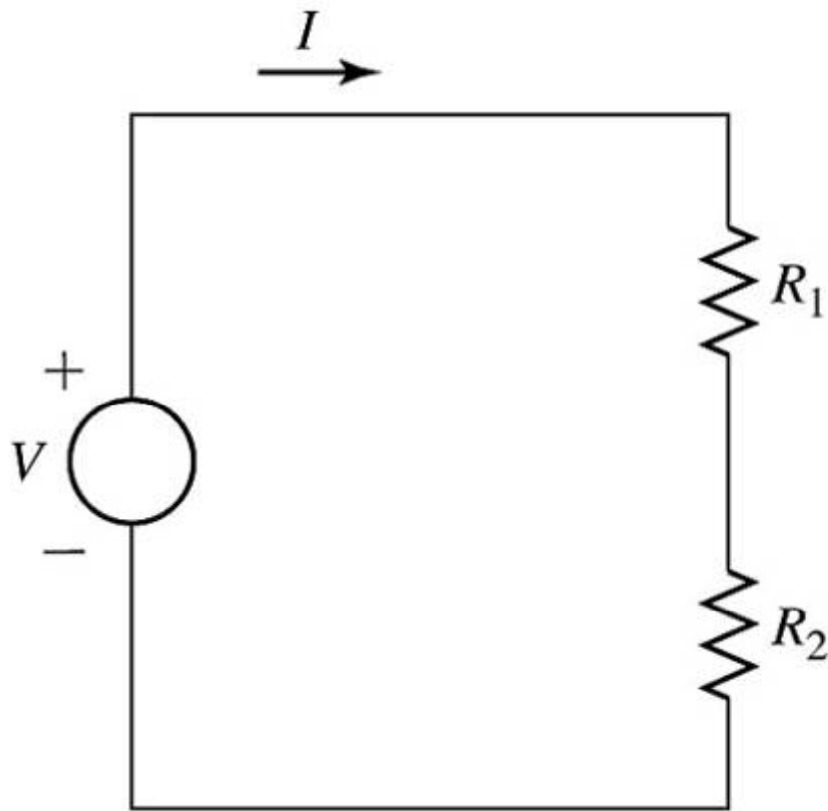
$$R \Leftrightarrow \mathfrak{R}$$

- Use magnetic “reluctance” instead of resistance

$$R = \frac{l}{\sigma A} \Leftrightarrow \mathfrak{R} = \frac{l}{\mu A}$$

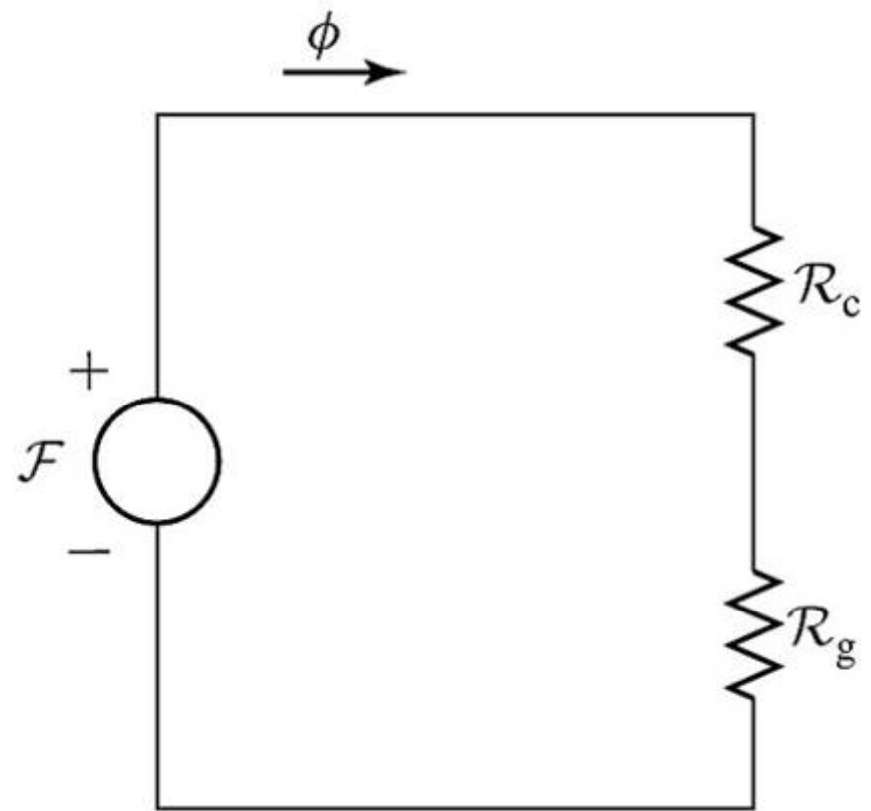
- Magnetic reluctance has units of *1/Henry* in the SI system

Magnetic-Electric Circuit Analogy



$$I = \frac{V}{(R_1 + R_2)}$$

(a)



$$\phi = \frac{\mathcal{F}}{(\mathcal{R}_c + \mathcal{R}_g)}$$

(b)

C-Core with Gap --- Using Magnetic Circuits

- Flux in the core is easily found by:

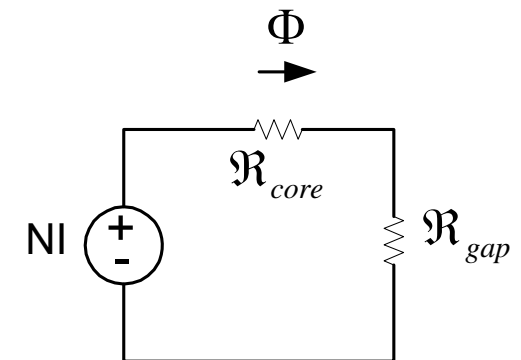
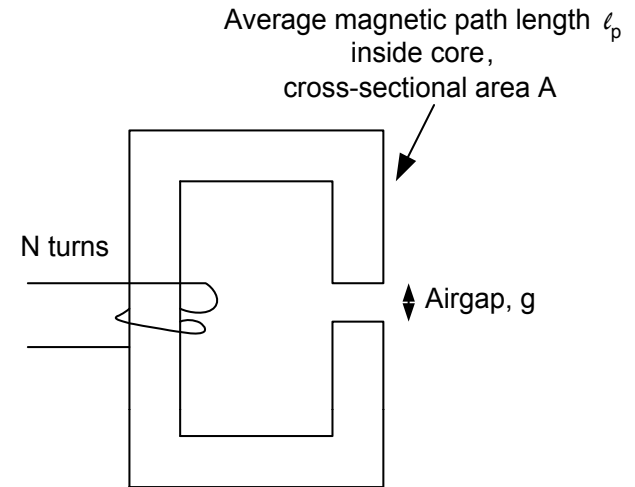
$$\Phi = \frac{NI}{\mathcal{R}_{core} + \mathcal{R}_{gap}} = \frac{NI}{\frac{l_p}{\mu_c A_c} + \frac{g}{\mu_o A_c}}$$

- Now, note what happens if $g/\mu_o \gg l_p/\mu_c$: the flux in the core is now approximately independent of the core permeability, as:

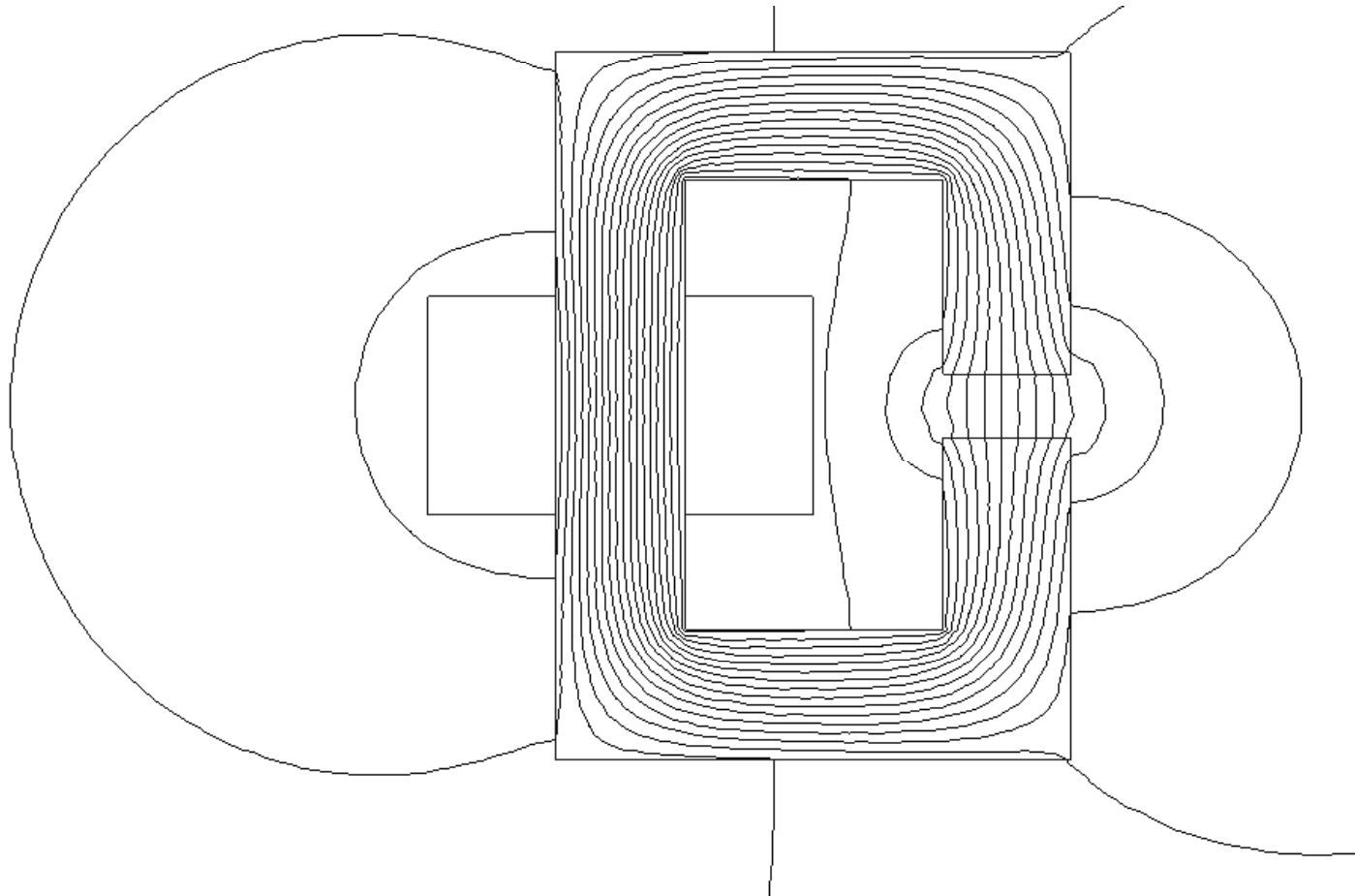
$$\Phi \approx \frac{NI}{\mathcal{R}_{gap}} \approx \frac{NI}{\frac{g}{\mu_o A_c}}$$

- Inductance:

$$L = \frac{N\Phi}{I} \approx \frac{N^2}{\mathcal{R}_{gap}} \approx \frac{N^2}{\frac{g}{\mu_o A_c}}$$

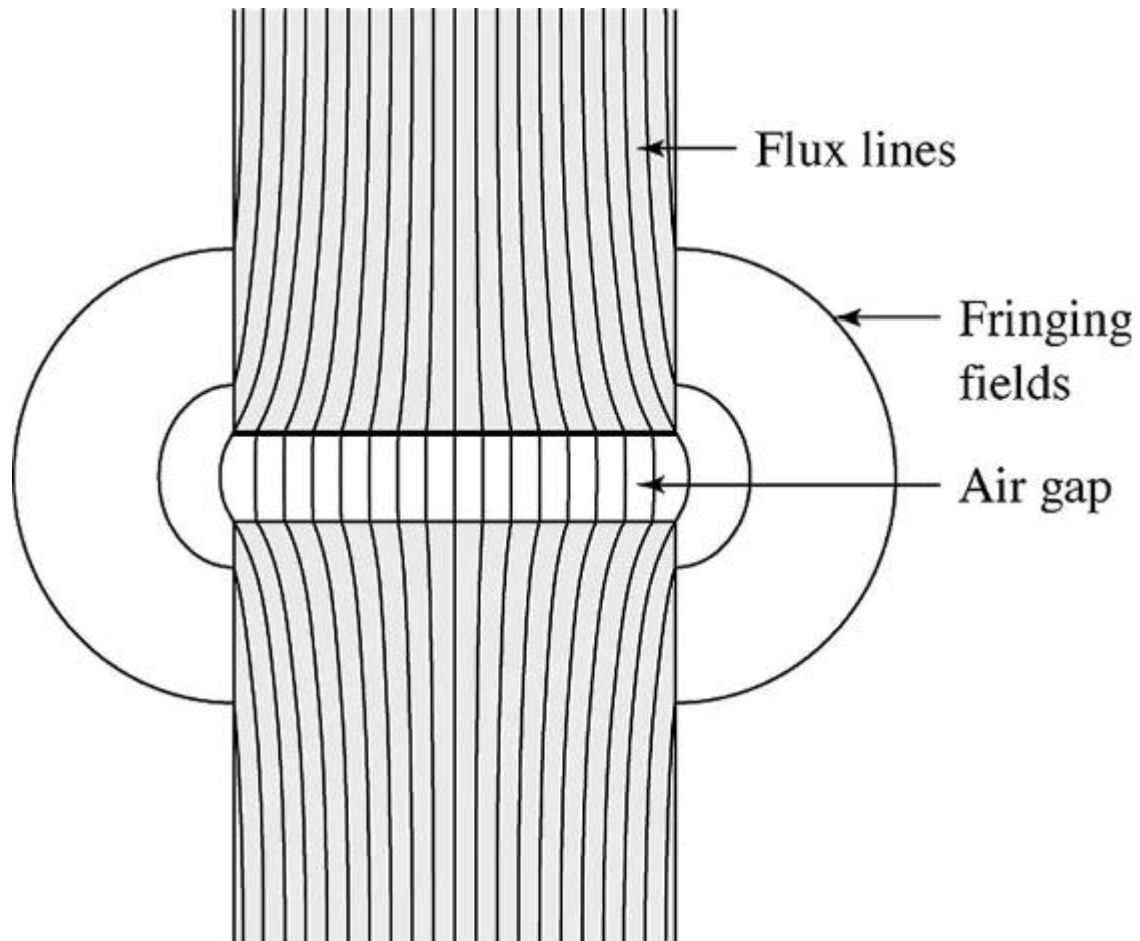


C-Core with Gap --- Finite Element Analysis (FEA)



Fringing Fields in Airgap

- If fringing is negligible, $A_c = A_g$

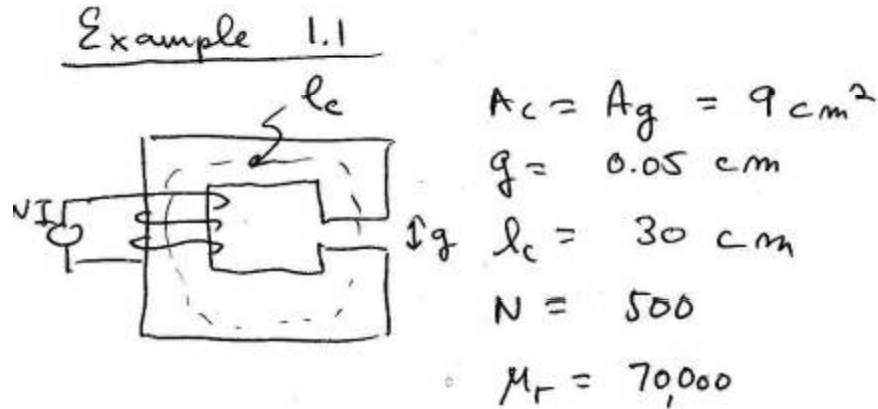


Summary of Magnetic Quantities

Item	Name	Units	Comment
H	Magnetic field	A/m	
J	Current density	A/m ²	
I	Current	A	
B	Magnetic flux density	Tesla, or <u>Wb/m²</u>	
μ_0	Magnetic permeability of free space	$4\pi \times 10^{-7}$ H/m	In magnetic steels, μ is higher
Φ	Flux	<u>Wb</u>	Integrate B (flux density) over an area to get Φ
N	Number of turns in an inductor or transformer	---	
λ	Flux linkage	<u>Wb</u> -turns	$\lambda = N\Phi$
L	Inductance	H	$L = \lambda/I$

Example 1: C-Core with Airgap

- Fitzgerald, Example 1.1: Find current I necessary to generate $B = 0.8\text{T}$ in airgap



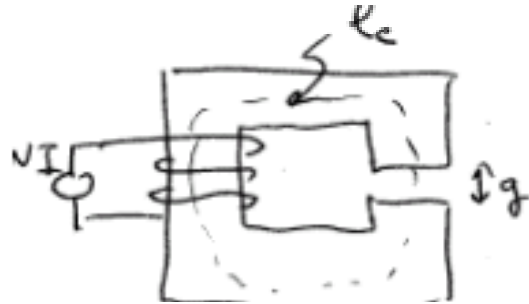
$$(a) R_c = \frac{l_c}{A_c \mu_c} = \frac{0.3}{(9 \times 10^{-4})(70,000)(4\pi \times 10^{-7})}$$

$$= 3.79 \times 10^3 \frac{\text{A-turns}}{\text{W}_b}$$

$$R_g = \frac{g}{A_g \mu_0} = \frac{5 \times 10^{-4}}{(9 \times 10^{-4})(4\pi \times 10^{-7})} = 4.42 \times 10^5 \frac{\text{A-T}}{\text{W}_b}$$

Note $R_g \gg R_c$

Example 1: C-Core with Airgap



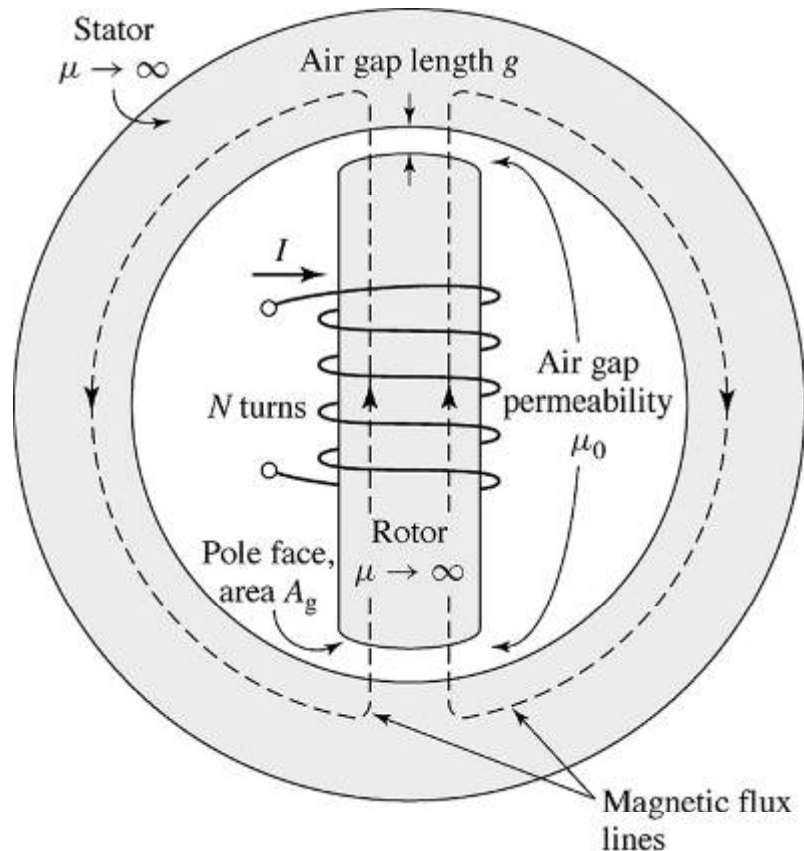
$$(b) \quad \Phi = B_c A_c = (1.0)(9 \times 10^{-4}) = \boxed{9 \times 10^{-4} \text{ Wb}}$$

$$(c) \quad NI = \Phi (R_c + R_g)$$

$$I = \frac{\Phi (R_c + R_g)}{N} = \frac{(9 \times 10^{-4})(4.46 \times 10^5)}{500}$$

$$\boxed{I = 0.8 \text{ A}}$$

Example 2: Simple Synchronous Machine



Problem:

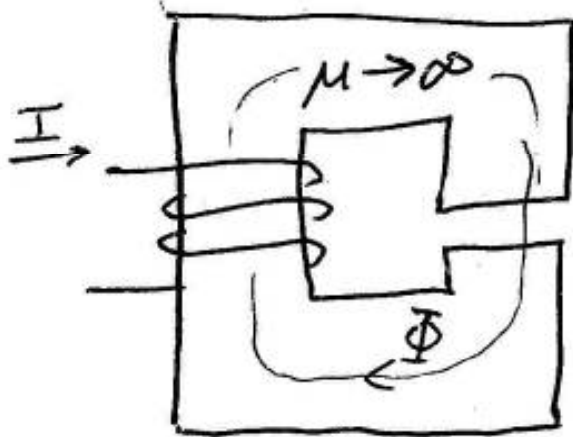
Assuming $\mu \rightarrow \infty$, find airgap flux and flux density. $I = 10\text{A}$, $N = 1000$, $g = 1\text{ cm}$ and $A_g = 2000\text{ cm}^2$.

Solution:

$$\Phi = \frac{NI}{\left(\frac{2g}{\mu_0 A_g}\right)} = \frac{(1000)(10)}{\left(\frac{2(0.01)}{(4\pi \times 10^{-7})(0.2)}\right)} = 0.13\text{ Wb}$$

$$B_g = \frac{\Phi}{A_g} = \frac{0.13}{0.2} = 0.65 \frac{\text{Wb}}{\text{m}^2} = 0.65\text{T}$$

What Does Infinite μ Imply?



In core:

$$\Phi_c = B_c A_c$$

In gap:

$$\Phi_g = B_g A_g$$

Inside core:

B_c is finite

If $\mu \rightarrow \infty$, then $H_c \rightarrow 0$ for finite B_c

Flux Linkage and Energy

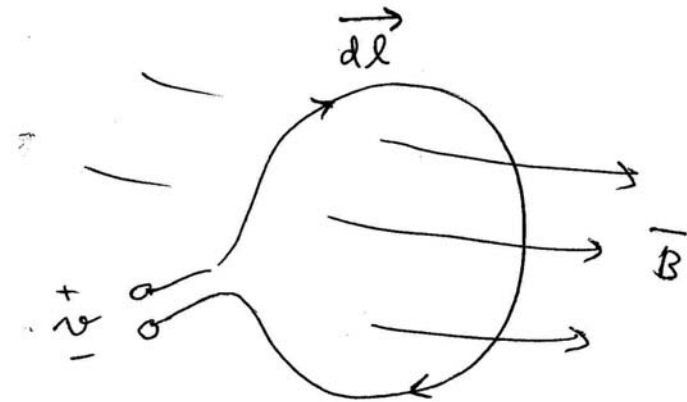
- By Faraday's law, changing magnetic flux density creates an electric field (and a voltage)

$$\oint_C \vec{E} \cdot d\vec{l} = -\frac{d}{dt} \int_S \vec{B} \cdot d\vec{A}$$

- Induced voltage:

$$v = N \frac{d\Phi}{dt} = \frac{d\lambda}{dt}$$

λ = "flux linkage" = $N\Phi$

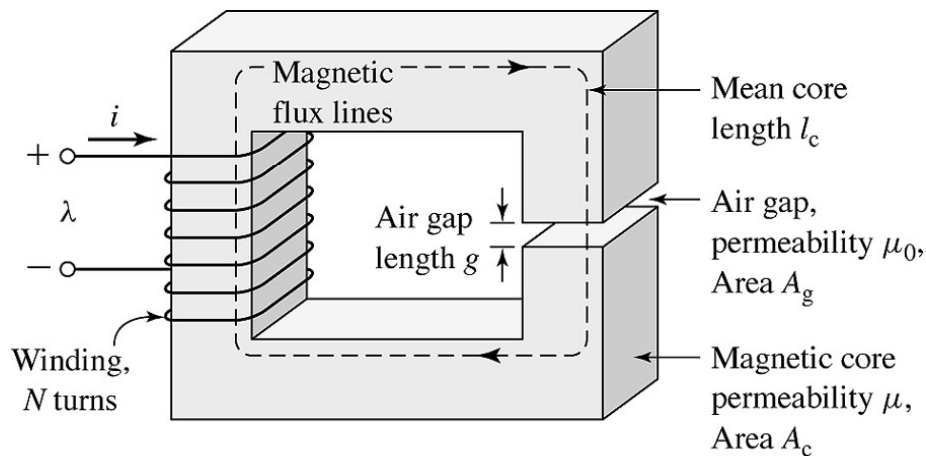


Inductance relates flux linkage to current

$$L = \frac{\lambda}{I}$$

Inductor with Airgap

- Units of inductance are Henrys, or Weber-turns per Ampere



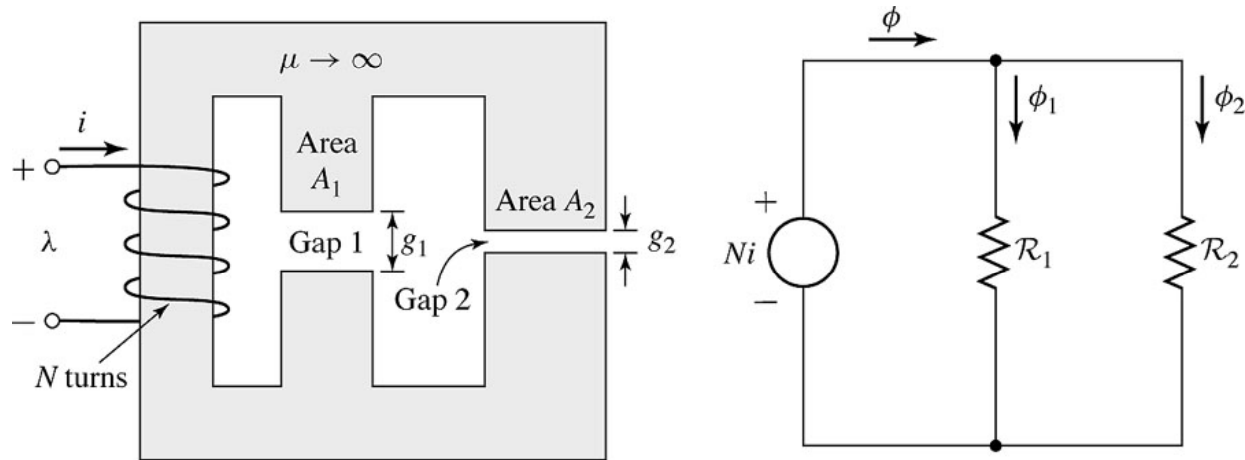
$$\Phi \approx \frac{NI}{g} \quad \text{Flux}$$

$$\frac{\mu_0 A_g}{\mu_0 A_g}$$

$$\lambda \approx N\Phi = \frac{\mu_0 A_g N^2 I}{g} \quad \text{Flux linkage}$$

$$L \approx \frac{\lambda}{I} = \frac{\mu_0 A_g N^2}{g} \quad \text{Inductance}$$

Magnetic Circuit with Two Airgaps



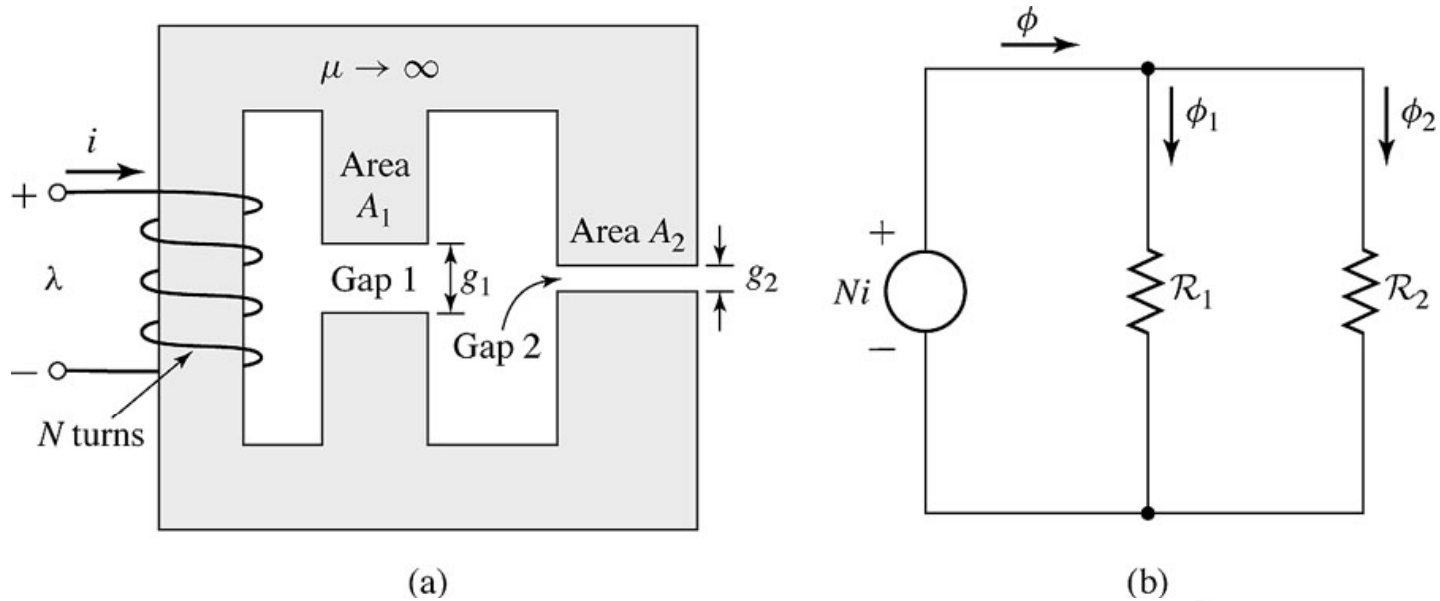
$$\text{TOTAL FLUX: } \Phi = \frac{NI}{\mathcal{R}_1 \parallel \mathcal{R}_2} = \frac{NI}{\frac{\mathcal{R}_1 \mathcal{R}_2}{\mathcal{R}_1 + \mathcal{R}_2}}$$

$$\text{RELUCTANCES: } \mathcal{R}_1 = \frac{g_1}{\mu_0 A_1} \quad \mathcal{R}_2 = \frac{g_2}{\mu_0 A_2}$$

$$\text{INDUCTANCE: } L = \frac{\lambda}{I} = \frac{N\Phi}{I} = N^2 \left(\frac{\mathcal{R}_1 + \mathcal{R}_2}{\mathcal{R}_1 \mathcal{R}_2} \right)$$

$$L = \mu_0 N^2 \left(\frac{A_1}{g_1} + \frac{A_2}{g_2} \right)$$

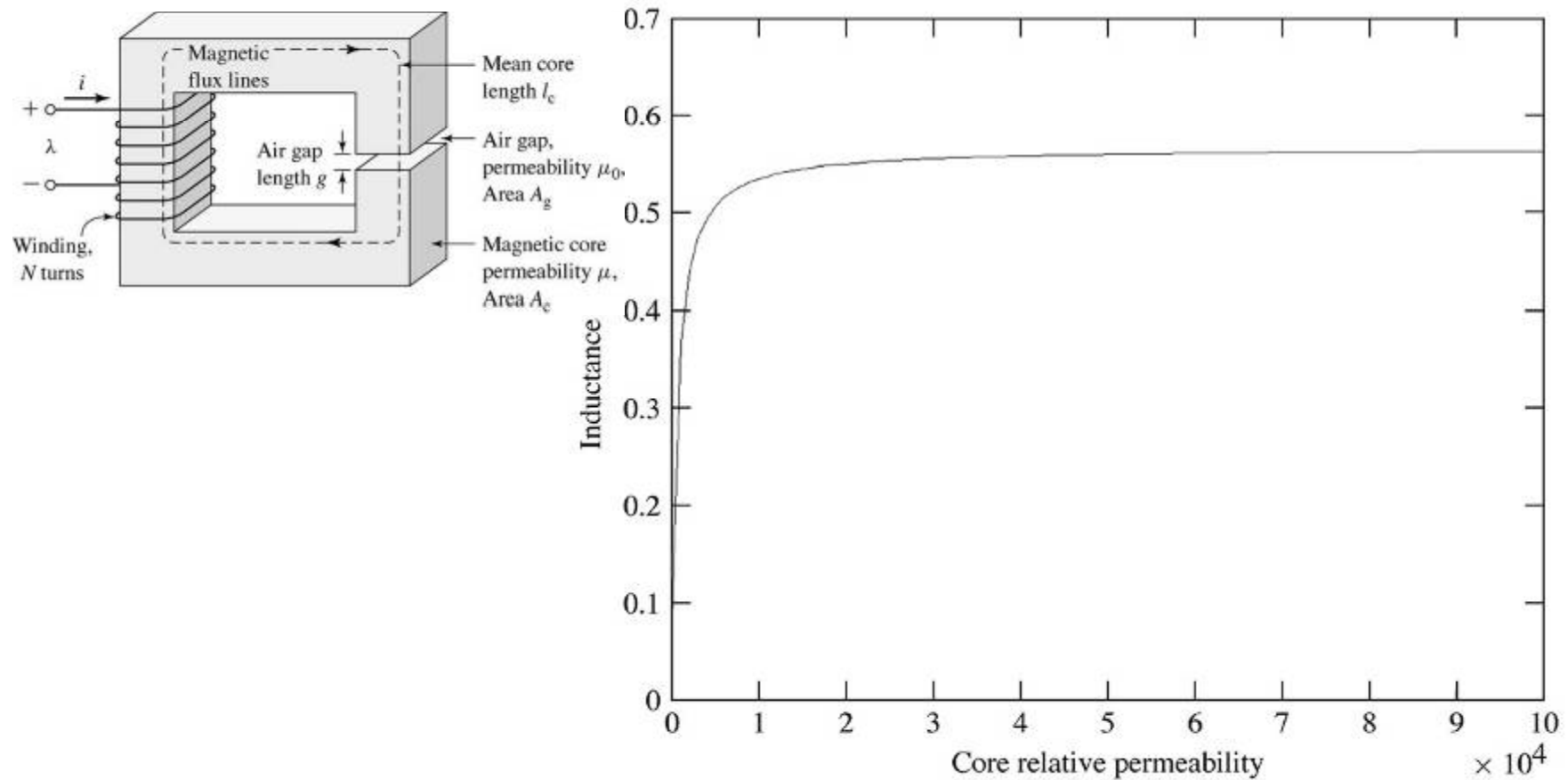
Magnetic Circuit with Two Airgaps



FLUX IN LEG #1:
$$\Phi_1 = \frac{NI}{\mathcal{R}_1} = \frac{\mu_0 A_1 NI}{g_1}$$

FLUX DENSITY IN LEG #1
$$B_1 = \frac{\Phi_1}{A_1} = \frac{\mu_0 NI}{g_1}$$

Inductance vs. Relative Permeability



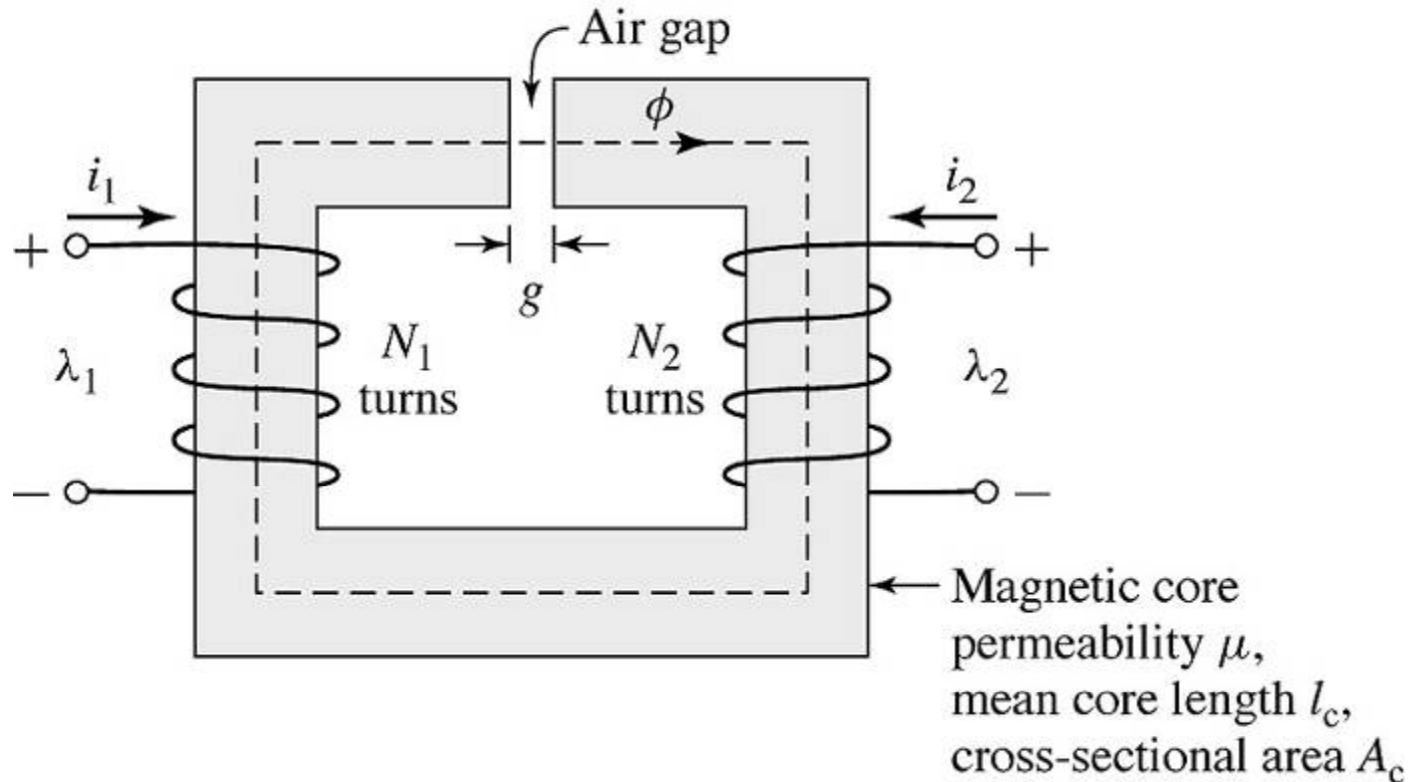
Inductance and Energy

- Magnetic stored energy (Joules) is:

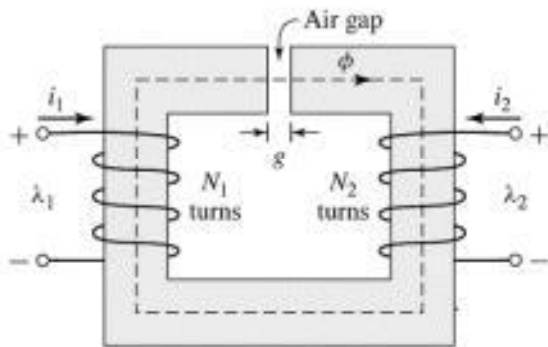
$$W = \frac{1}{2} LI^2$$

Magnetic Circuit with Two Windings

- Note that flux is the sum of flux due to i_1 and that due to i_2



Magnetic Circuit with Two Windings



FLUX:

$$\Phi = (N_1 I_1 + N_2 I_2) \frac{\mu_0 A_c}{g}$$

FLUX LINKAGE FOR COIL #1

$$\lambda_1 = N_1 \Phi = N_1^2 I_1 \frac{\mu_0 A_c}{g} + N_1 N_2 I_2 \frac{\mu_0 A_c}{g}$$

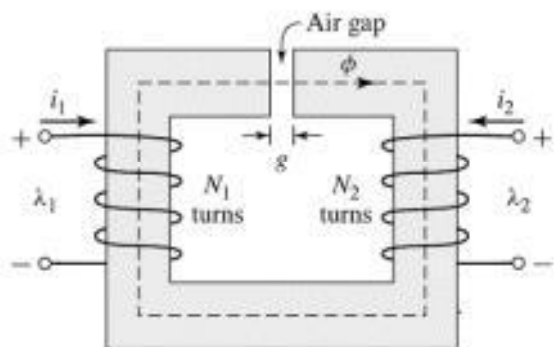
WE CAN WRITE THIS AS:

$$\lambda_1 = L_{11} I_1 + L_{12} I_2$$

L_{11} = "SELF INDUCTANCE" OF COIL #1

L_{12} = "MUTUAL INDUCTANCE" BETWEEN COILS #1 AND #2

Magnetic Circuit with Two Windings



FLUX LINKAGE FOR COIL #2

$$\lambda_2 = N_2 \Phi = N_1 N_2 I_1 \frac{\mu_0 A_c}{g} + N_2^2 I_2 \frac{\mu_0 A_c}{g}$$

$$\lambda_2 = L_{21} I_1 + L_{22} I_2$$

$$\begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix} = \begin{bmatrix} L_{11} & L_{12} \\ L_{21} & L_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

INDUCTANCE MATRIX

B-H Curve and Saturation

- Definition of magnetic permeability: slope of B-H curve

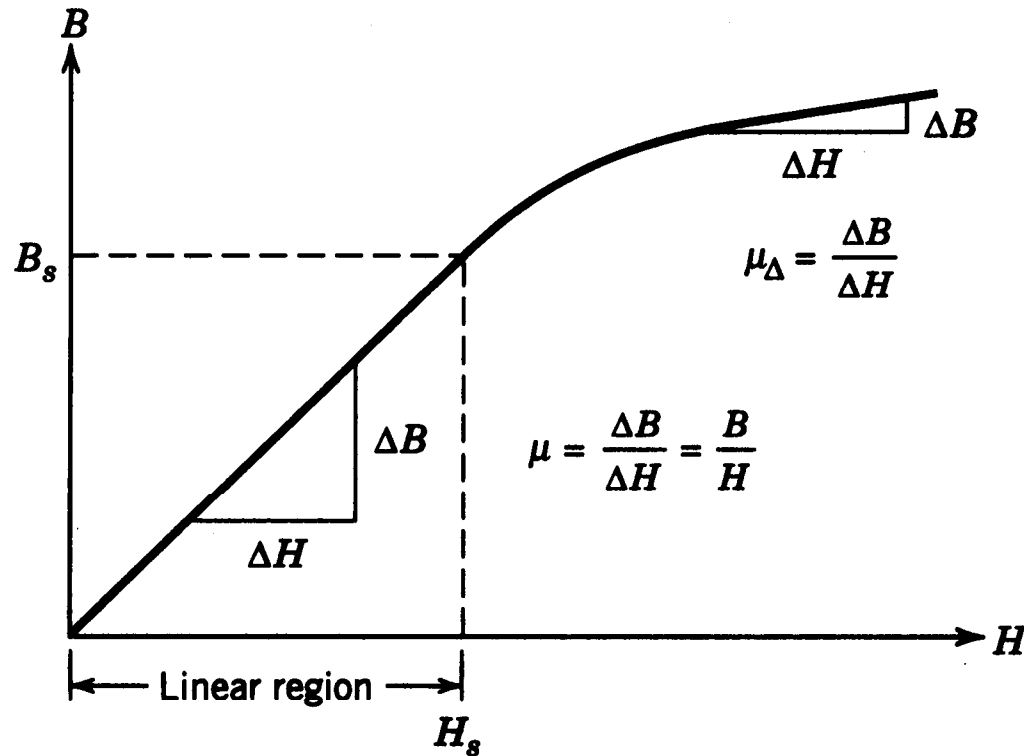
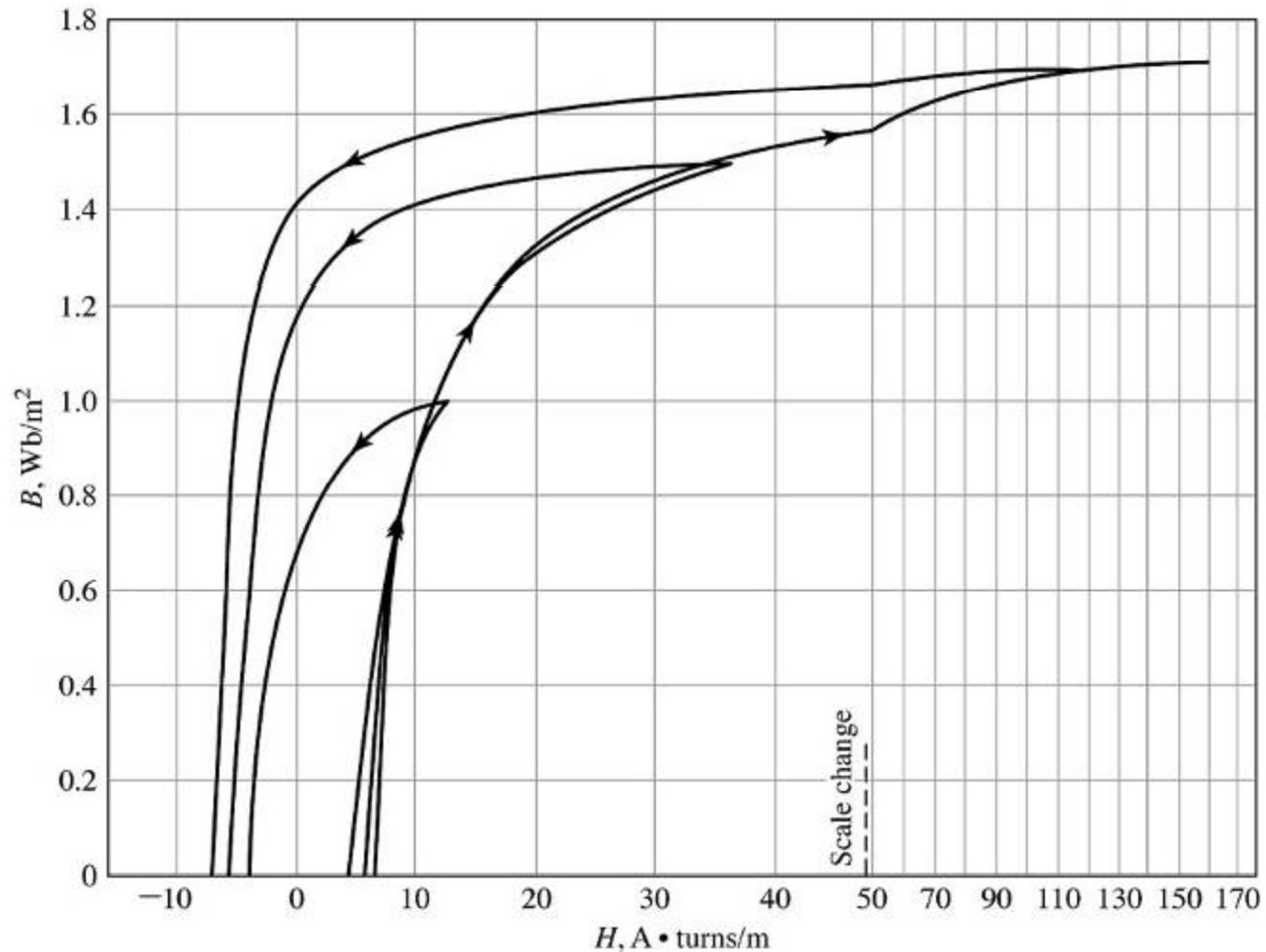


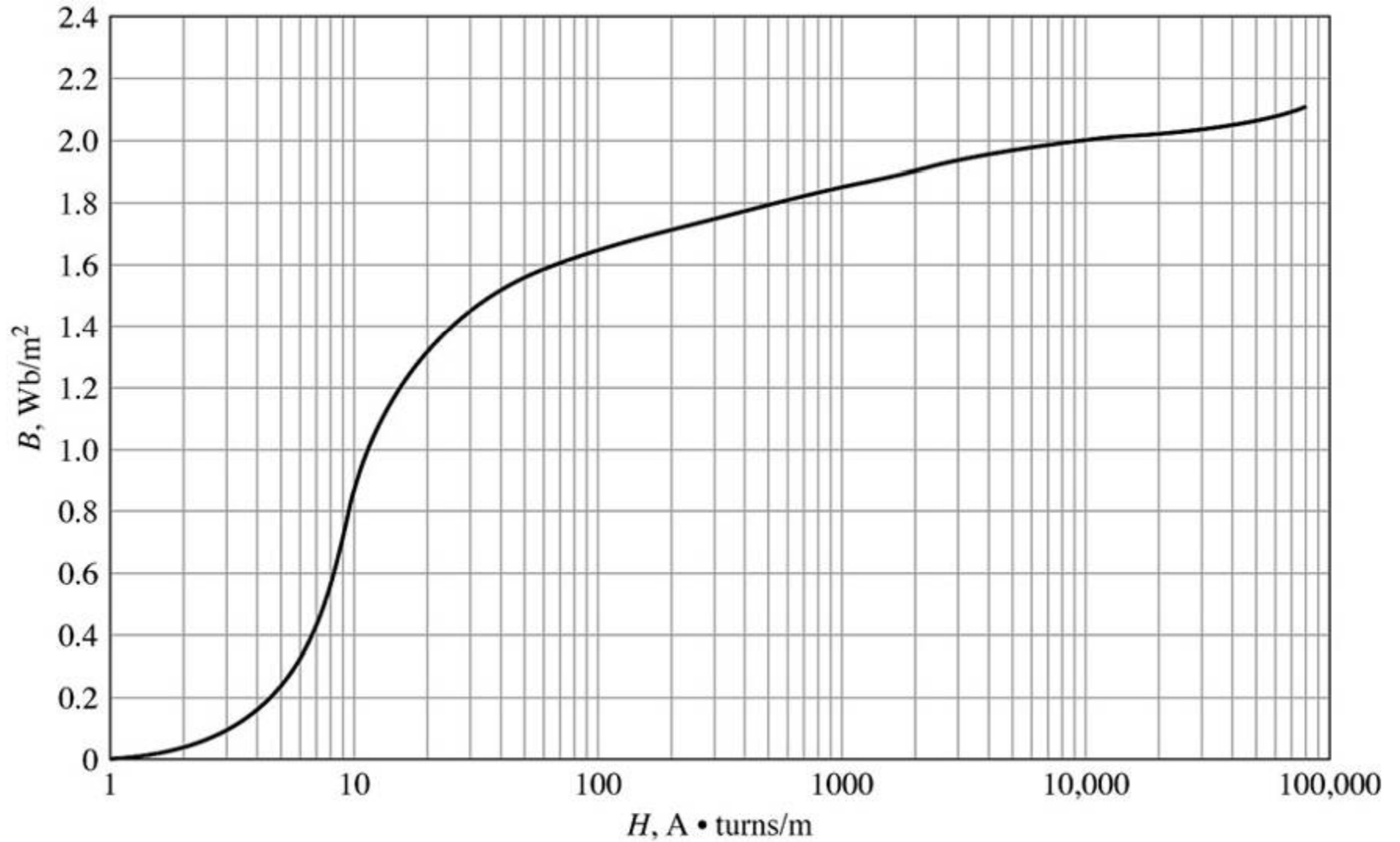
Figure 3-12 Relation between B - and H -fields.

B-H Loop for M-5 Grain-Oriented Steel

- Only the top half of the loops shown for steel 0.012" thick



DC Magnetization Curve for M-5



BH Curves for Various Soft Magnetic Materials

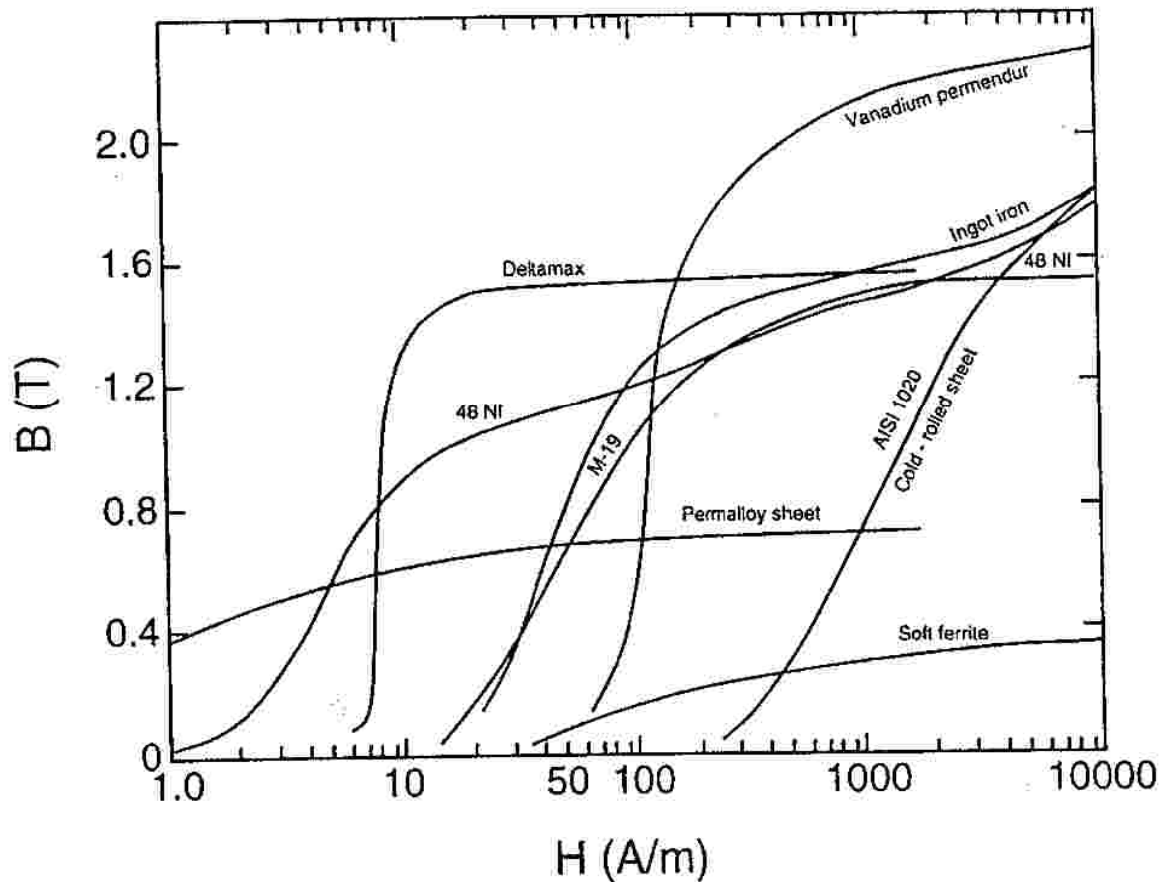
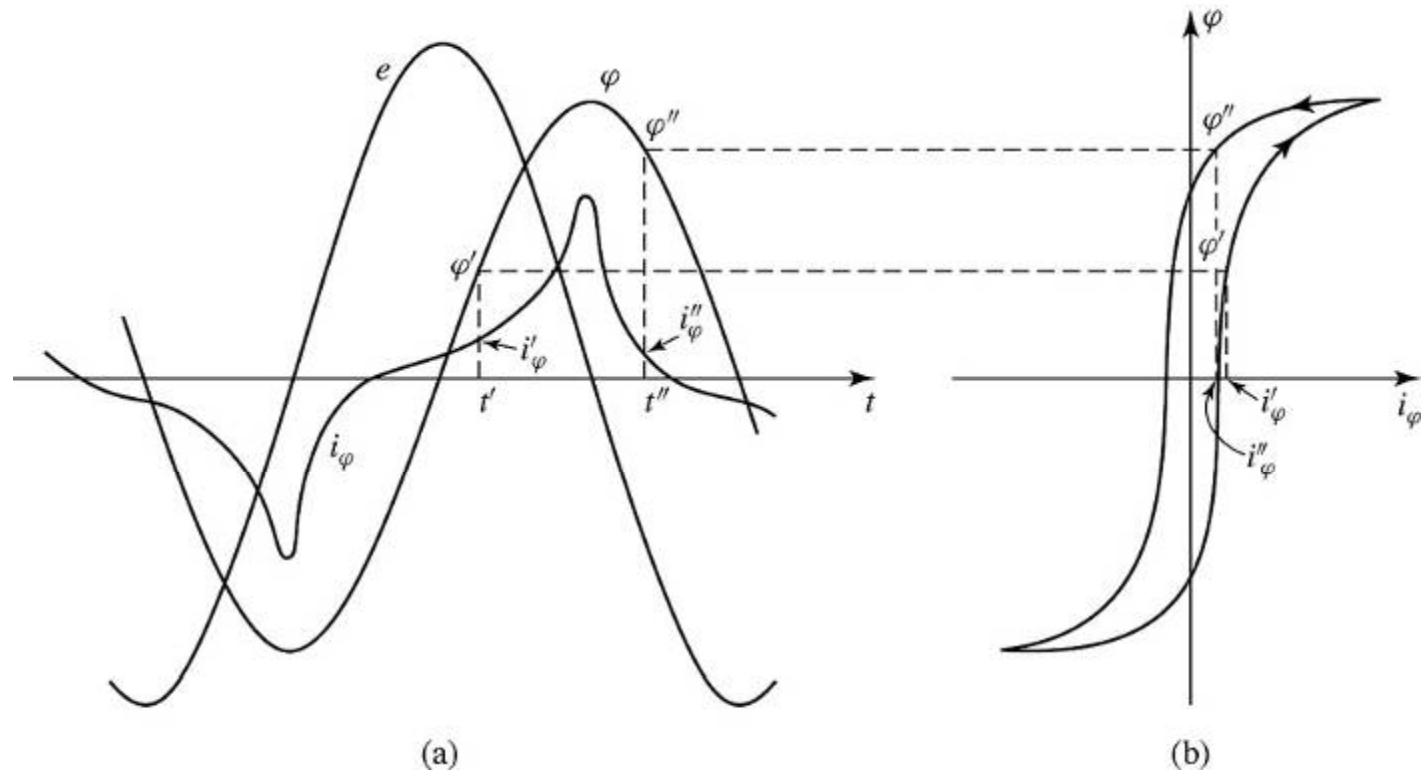
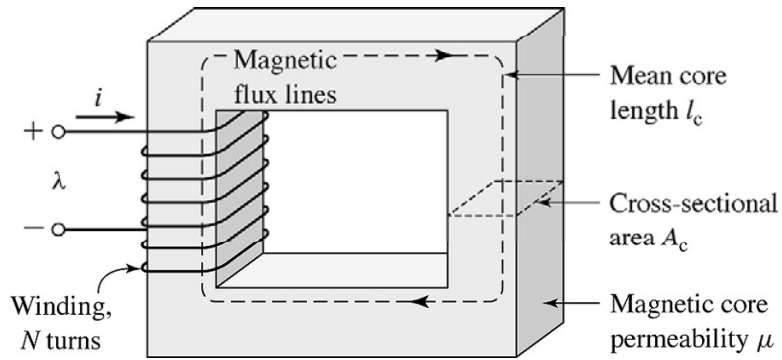


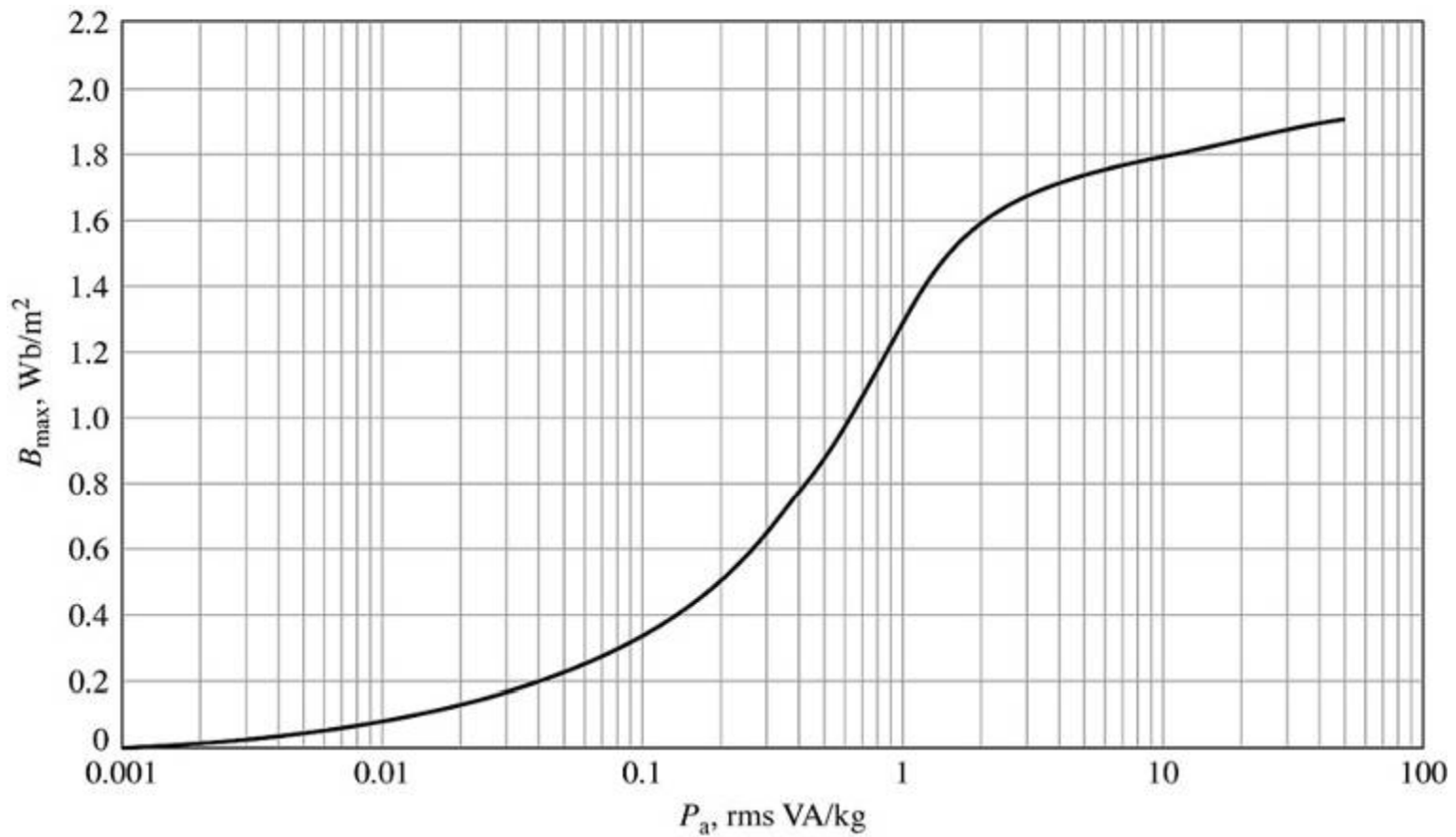
FIGURE 1.27 The B - H curves of various soft magnetic materials.

Reference: E. Furlani, *Permanent Magnet and Electromechanical Devices*, Academic Press, 2001, pp. 41

Relationship Between Voltage, Flux and Current

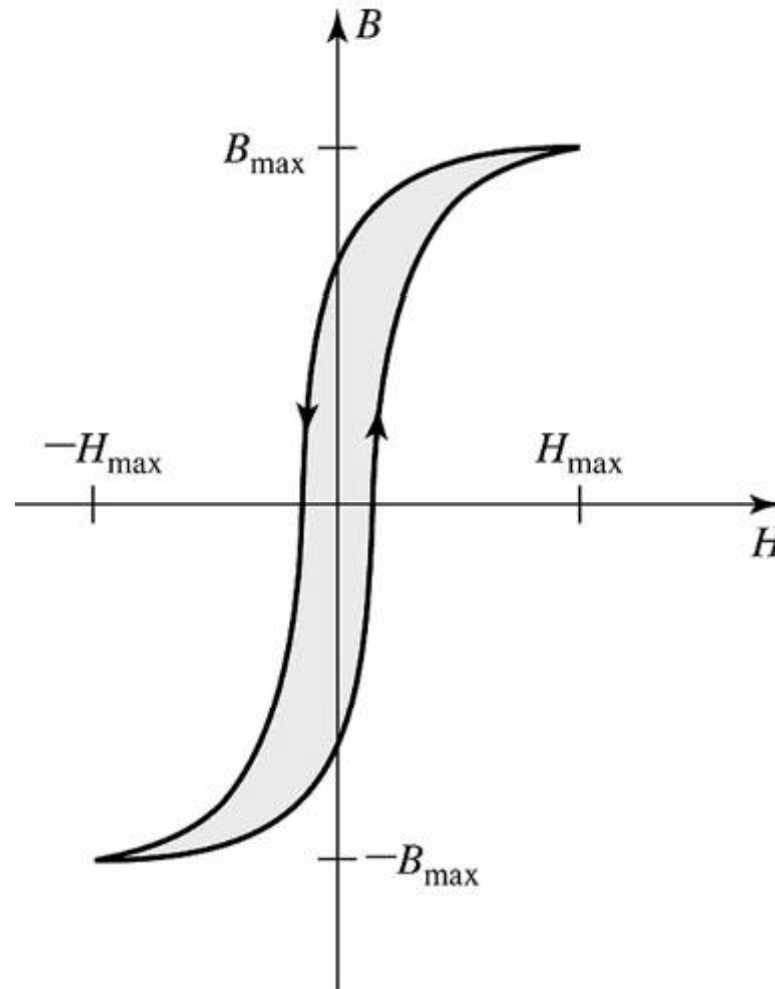


Exciting RMS VA per kg at 60 Hz



Hysteresis Loop

- Hysteresis loss (Watts) is proportional to shaded area



Hysteresis Loop Size Increases with Frequency

- Hysteresis loss increases as frequency increases

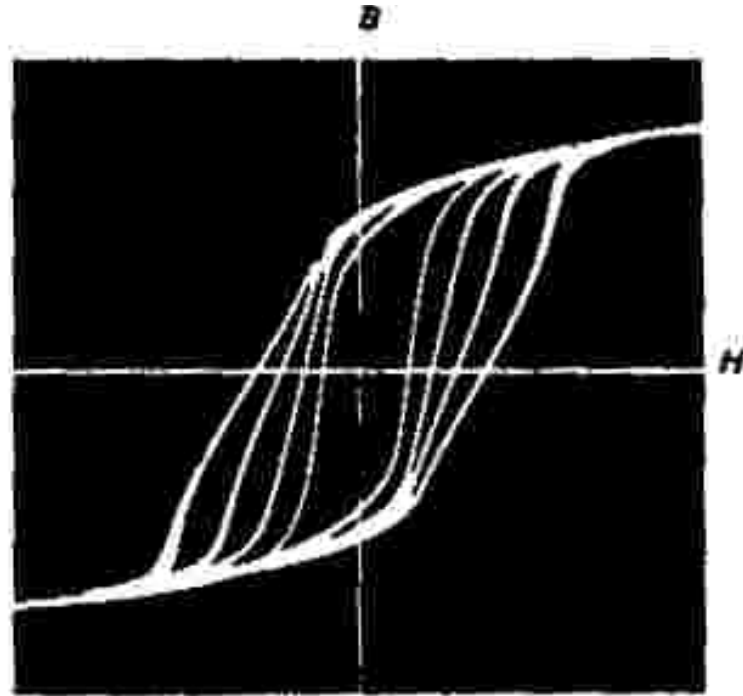
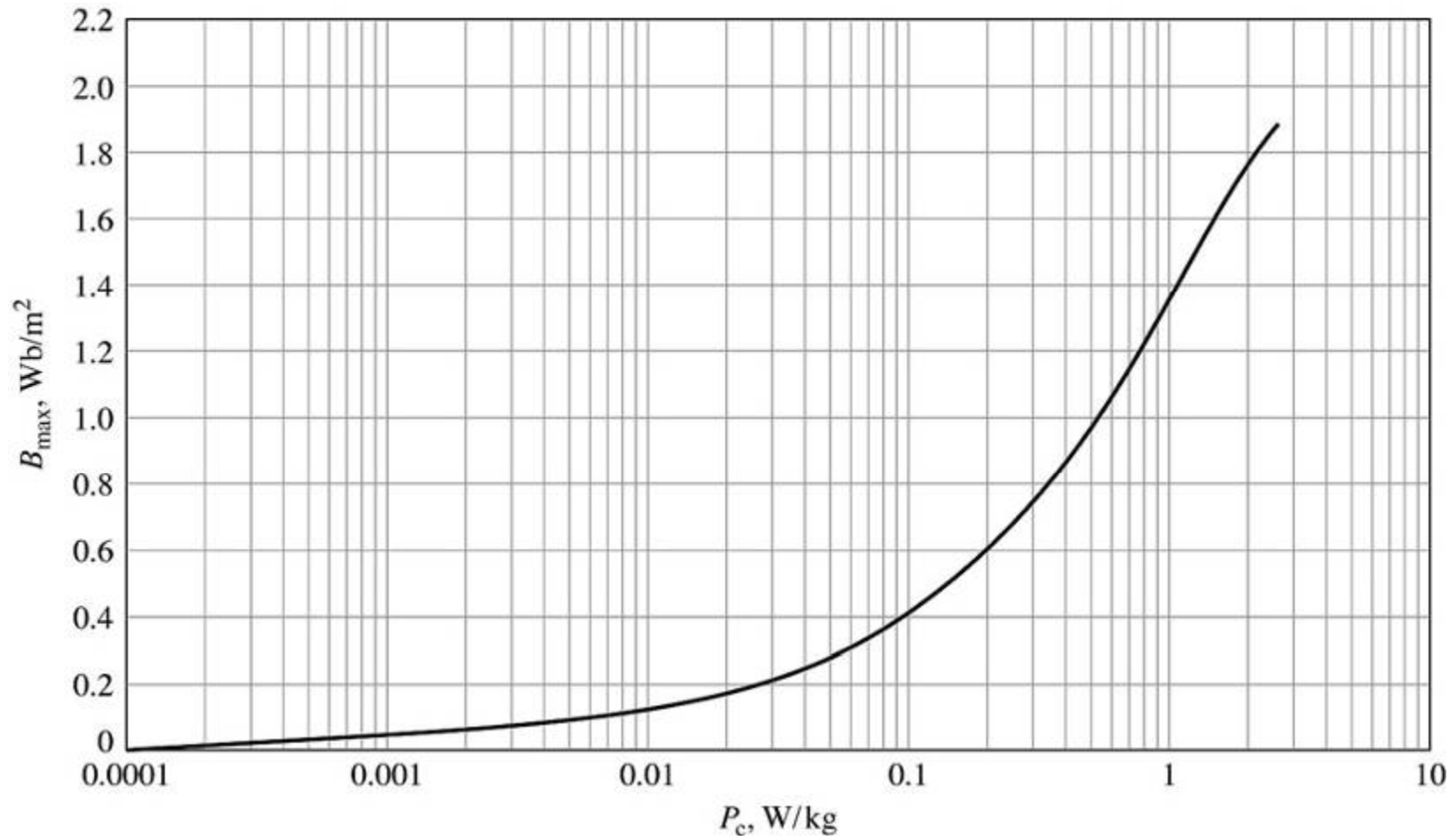


Fig. 2.3
Hysteresis loops at different frequencies

Reference: Siemens, *Soft Magnetic Materials* (Vacuumschmelze Handbook), pp. 30

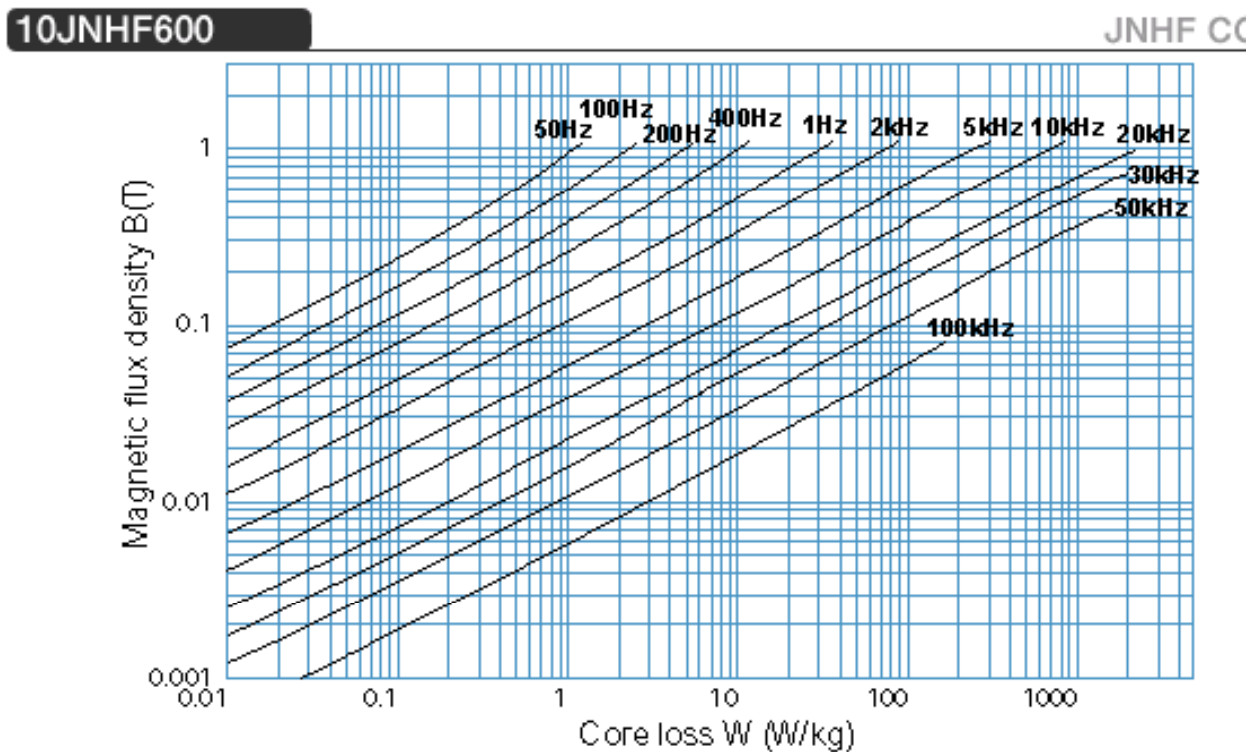
Total Core Loss

- Core loss (Watts/kg) in M-5 steel at 60 Hz vs. B_{\max}



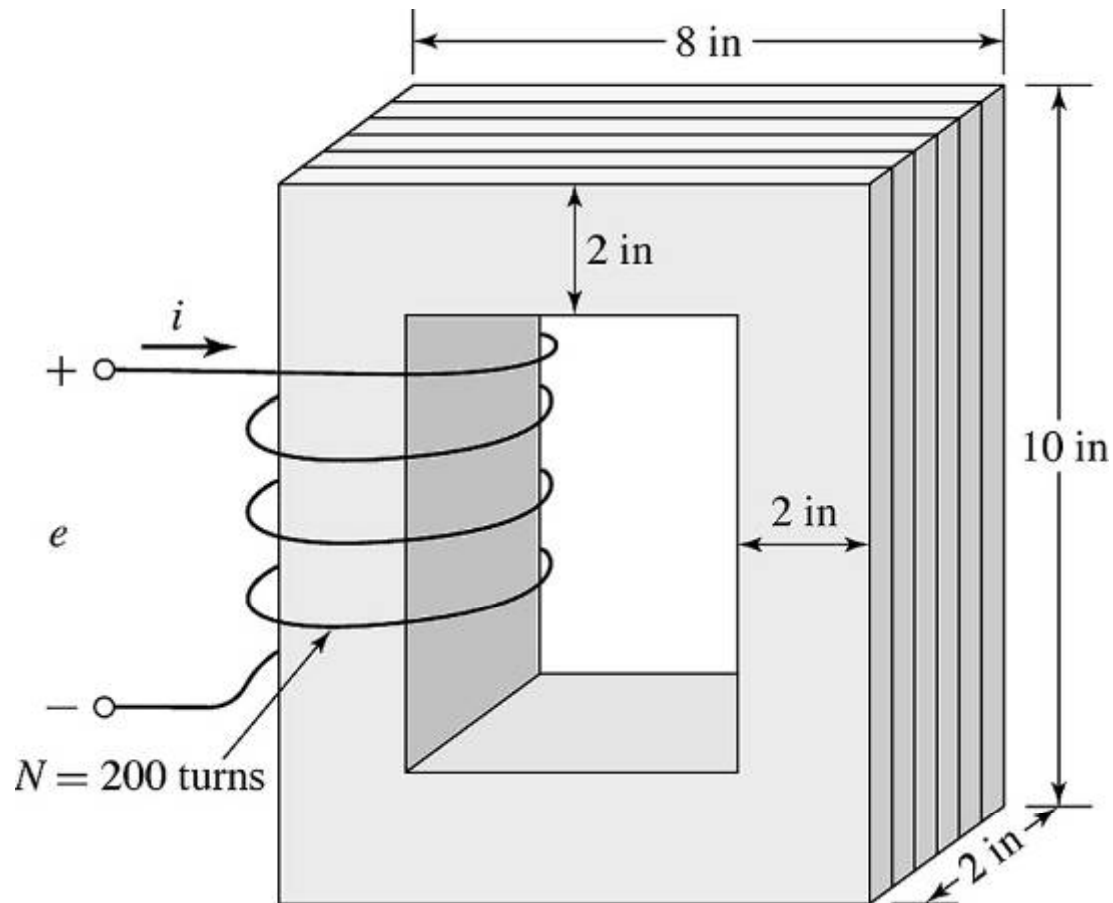
Total Core Loss vs. Frequency and Bmax

- Core loss depends on peak flux density and excitation frequency
- High frequency core material



Reference: <http://www.jfe-steel.co.jp/en/products/electrical/jnhf/02.html>

Example 3: Core Calculations



- Find maximum B
- From this B and switching frequency, find core loss per kg
- Total loss is power density \times mass of core

Example 3: Find V, I and Core Loss

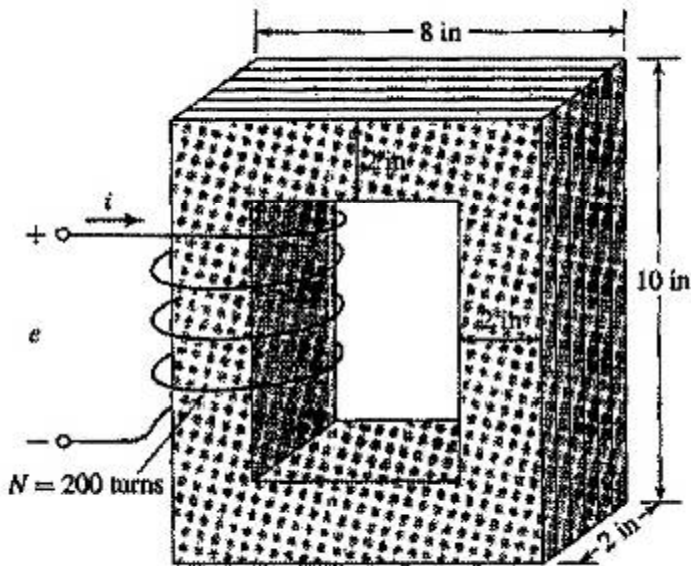


Figure 1.15 Laminated steel core with winding for Example 1.8.

MATERIAL M-5

$$f = 60 \text{ Hz}$$

$$B_c = 1.5 \sin \omega t \text{ Tesla}$$

Steel is 94% of cross section

$$\text{Density of steel} = 7.65 \text{ g/cm}^3$$

- FIND
- (a) Applied voltage
 - (b) Peak current
 - (c) RMS current
 - (d) Core loss

Example 3: Excitation Voltage

(a) From Faraday's law:

$$e = \frac{d\lambda}{dt} = N \frac{d\phi}{dt} = NA_c \frac{dB_c}{dt}$$

$$A_c = 2 \text{ in} \times 2 \text{ in} \times 0.94 = 2.42 \times 10^{-3} \text{ m}^2$$

$$\begin{aligned} \frac{dB_c}{dt} &= 1.5 \omega \cos \omega t = (1.5)(2\pi \times 60) \cos \omega t \\ &= (1.5)(377) \cos \omega t \\ &= 565 \cos \omega t \end{aligned}$$

$$e = (200)(2.42 \times 10^{-3} \text{ m}^2)(565 \cos \omega t)$$

$$\therefore e = 274 \cos \omega t$$

Example 3: Peak Current in Winding

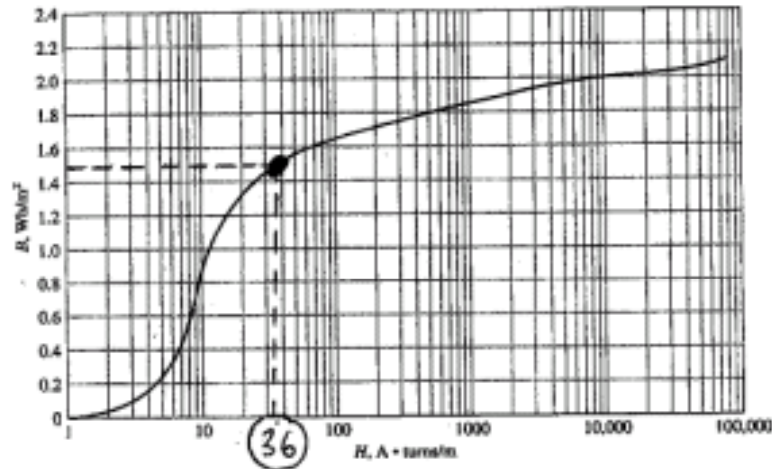
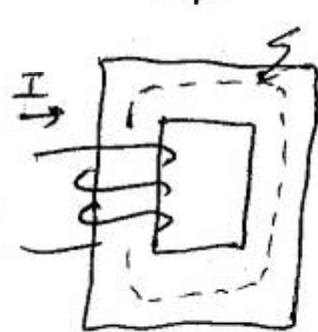


Figure 1.10 DC magnetization curve for M-5 grain-oriented electrical steel 0.012 in thick. (Armco Inc.)

(b) Peak current

- From Fig 1.10, $B = 1.5 \text{ T}$ requires $H = 36 \text{ AT/m}$

- From Ampere's law $Hl_c = NI$



$l_c = \text{average path length}$

$$= 2 \times (8'' + 6'') = 28'' = 0.71 \text{ m}$$

$$I = \frac{Hl_c}{N} = \frac{(36)(0.71)}{200} = 0.13 \text{ A}$$

Example 3: RMS Winding Current

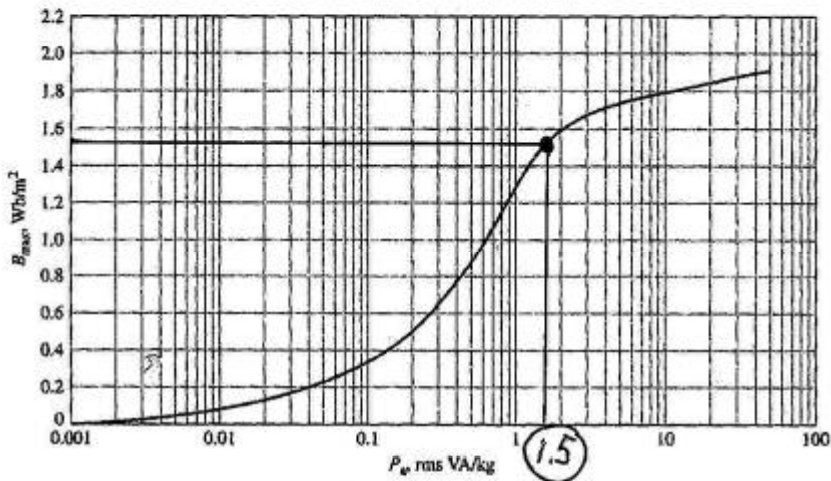


Figure 1.12 Exciting rms voltamperes per kilogram at 60 Hz for M-5 grain-oriented electrical steel 0.012 in thick. (Armco Inc.)

(c) RMS winding current

From Fig 1.12, at $B_{max} = 1.5 \text{ T}$, $P_a = 1.5 \text{ VA/kg}$

Core volume $V_c = (4 \text{ in}^2)(28 \text{ in})(0.94) = 105 \text{ in}^3 \left(\frac{2.54}{100}\right)^3$

$$V_c = 1.72 \times 10^{-3} \text{ m}^3$$

Core mass $M_c = V_c \rho_c = (1.72 \times 10^{-3} \text{ m}^3) \left(\frac{7650 \text{ kg}}{\text{m}^3}\right)$

$$M_c = 13.2 \text{ kg}$$

$$P_c = 1.5 \frac{\text{VA}}{\text{kg}} \times 13.2 \text{ kg} = 19.7 \text{ V-A}$$

$$I_{RMS} = \frac{P_c}{E_{RMS}} = \frac{19.7}{274/\sqrt{2}} = \boxed{0.10 \text{ A}}$$

Example 3: Core Loss

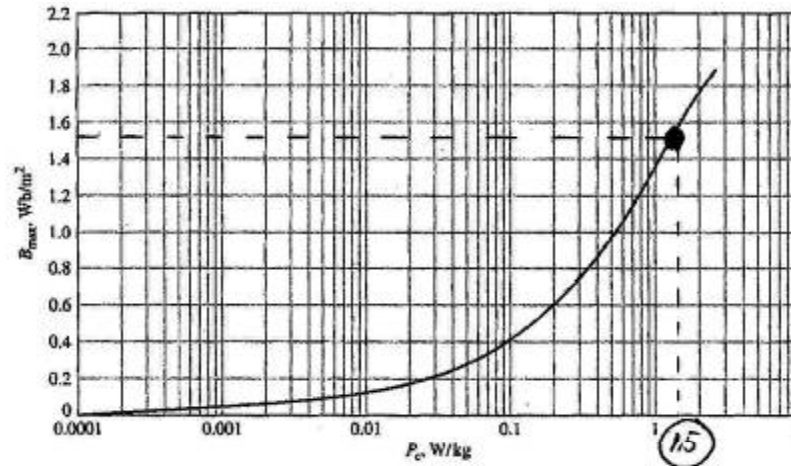


Figure 1.14 Core loss at 60 Hz in watts per kilogram for M-5 grain-oriented electrical steel 0.012 in thick. (Armco Inc.)

(d) Core loss

From Fig 1.14, core loss density ≈ 1.5 W/kg
at $B_{max} = 1.5$ Tesla. Total core loss is

$$P_c = M_c \times 1.5 \text{ W/kg} = (13.2)(1.5) = \boxed{20 \text{ Watts}}$$

Concept of Magnetic Reluctance

- Flux is related to ampere-turns by reluctance

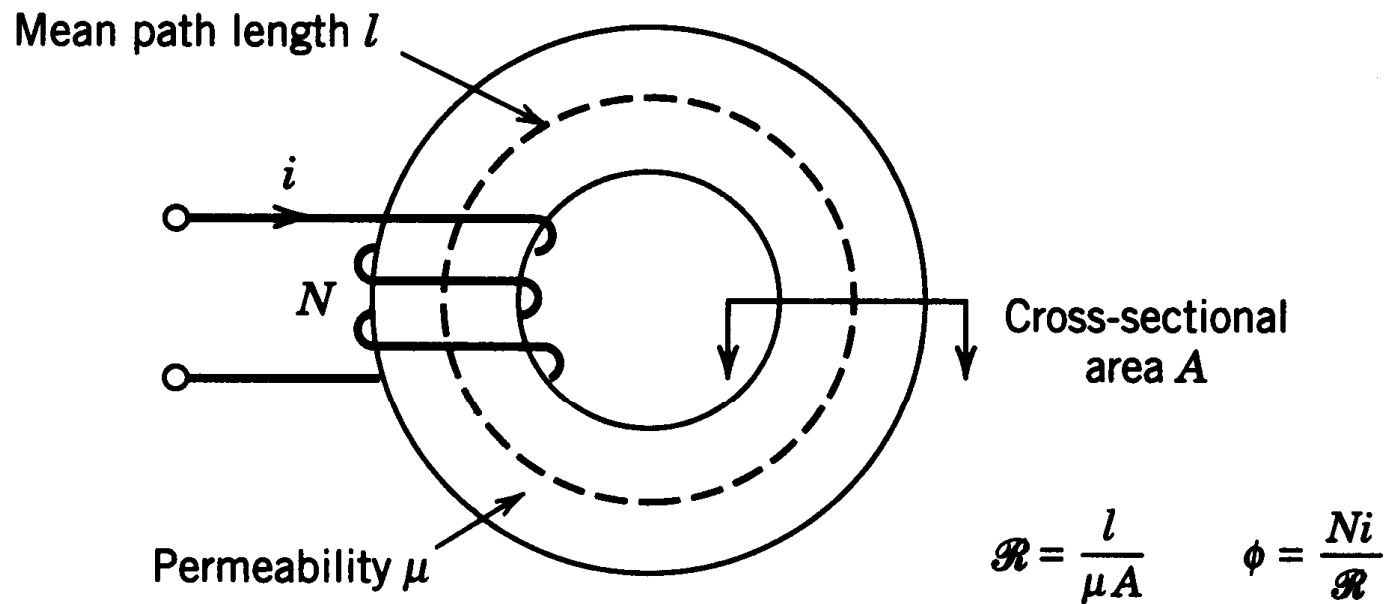


Figure 3-14 Magnetic reluctance.

Analogy between Electrical and Magnetic Worlds

Table 3-2 Electrical–Magnetic Analogy

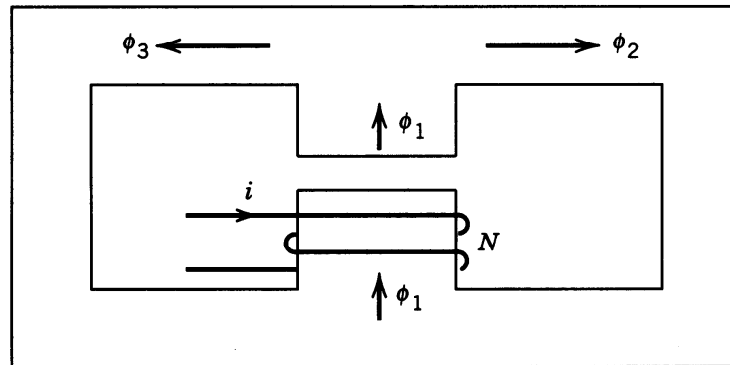
<i>Magnetic Circuit</i>	<i>Electric Circuit</i>
mmf Ni	v
Flux ϕ	i
reluctance \mathcal{R}	R
permeability μ	$1/\rho$, where ρ = resistivity

Analogy between Equations in Electrical and Magnetic Circuits

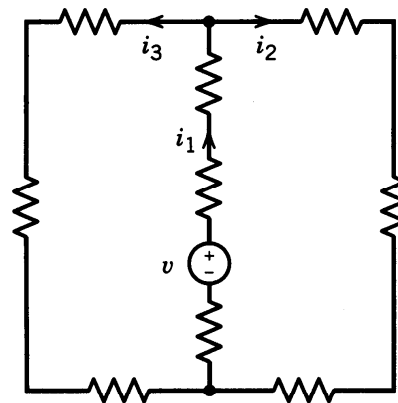
Table 3-3 Magnetic–Electrical Circuit Equation Analogy

<i>Magnetic</i>	<i>Electrical (dc)</i>
$\frac{Ni}{\phi} = \mathcal{R} = \frac{l}{\mu A}$	Ohm's law: $\frac{v}{i} = R = \frac{l}{A/\rho}$
$\phi \sum_k \mathcal{R}_k = \sum_m N_m i_m$	Kirchhoff's voltage law: $i \sum_k R_k = \sum_m v_m$
$\sum \phi_k = 0$	Kirchhoff's current law: $\sum_k i_k = 0$

Magnetic Circuit and its Electrical Analog



(a)



(b)

Figure 3-15 (a) Magnetic circuit. (b) An electrical analog.

Faraday's Law and Lenz's Law

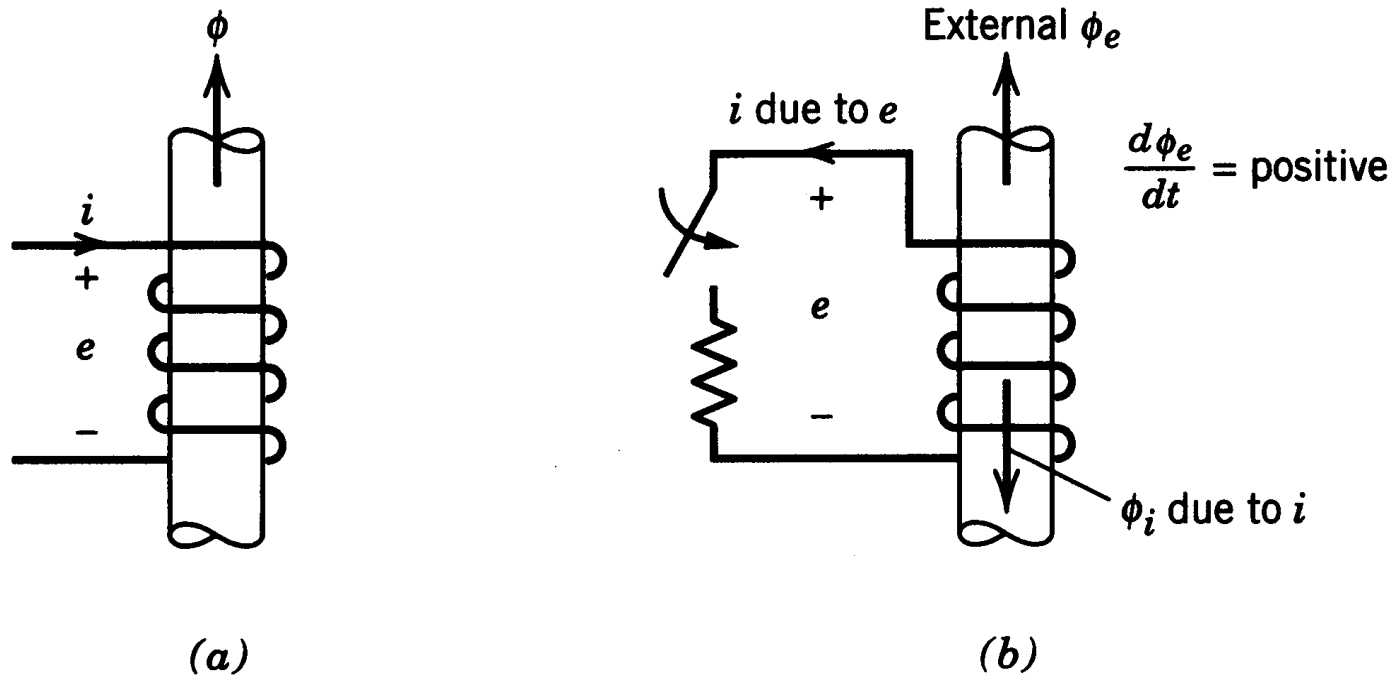
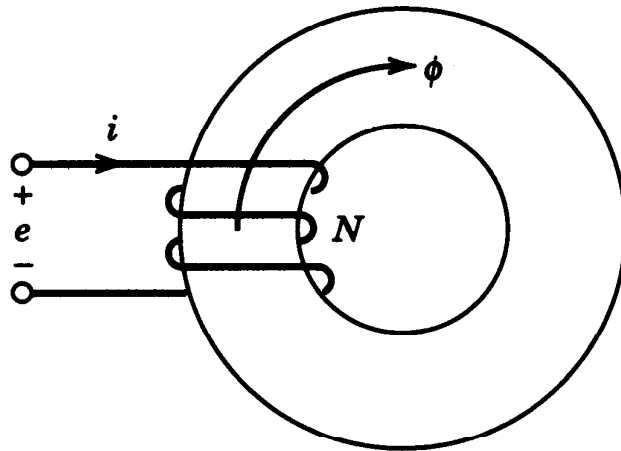


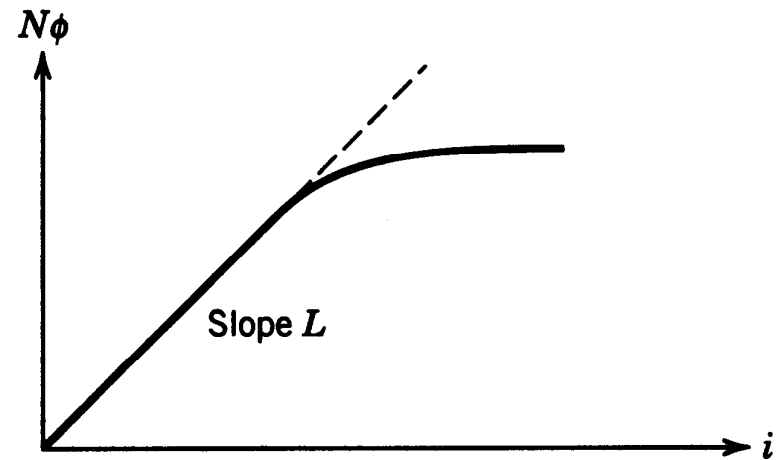
Figure 3-16 (a) Flux direction and voltage polarity.
(b) Lenz's law.

Inductance L

- Inductance relates flux-linkage λ to current I



(a)



(b)

Figure 3-17 Self-inductance L .

Analysis of a Transformer

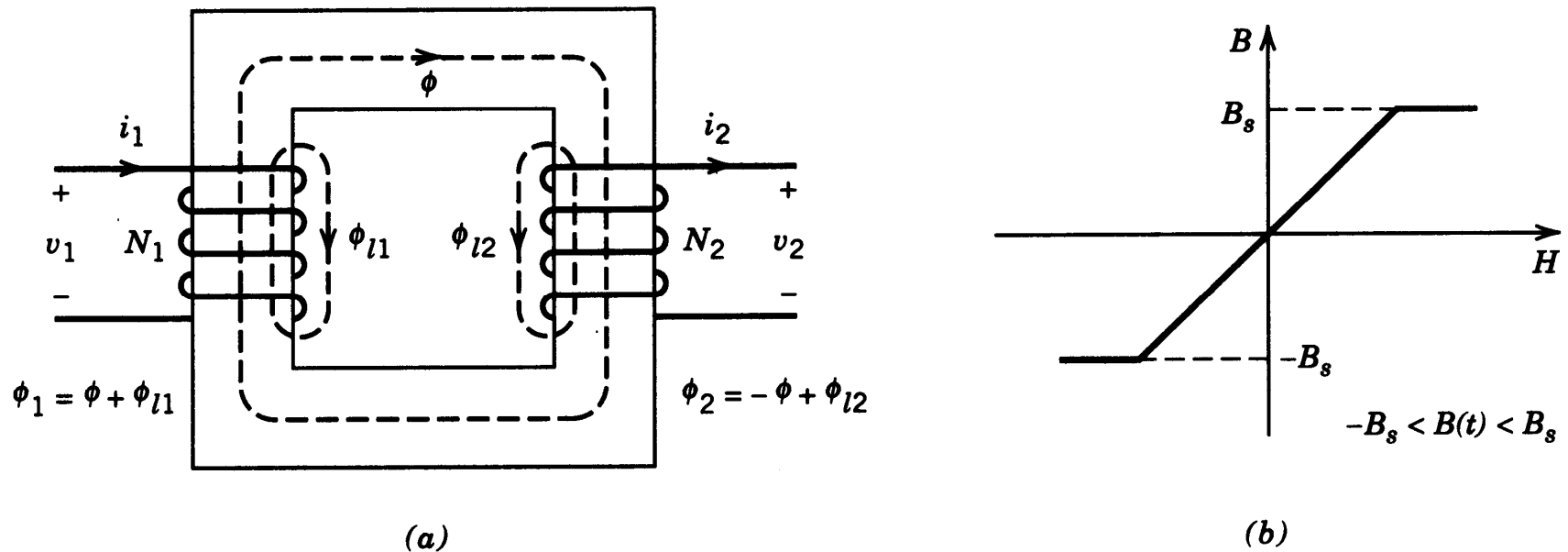
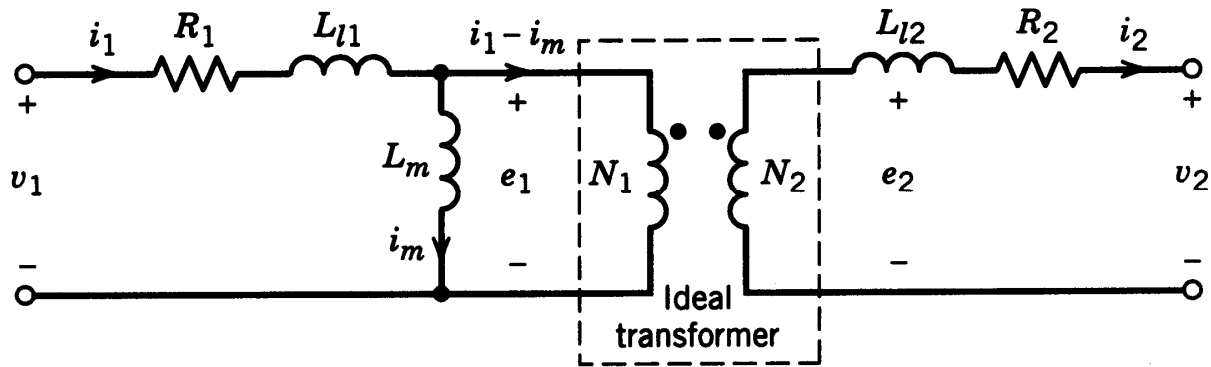
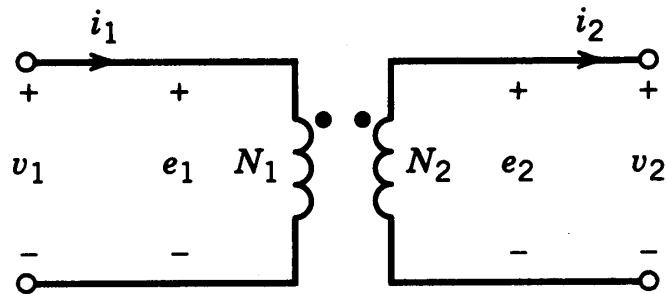


Figure 3-18 (a) Cross section of a transformer. (b) The $B-H$ characteristics of the core.

Transformer Equivalent Circuit



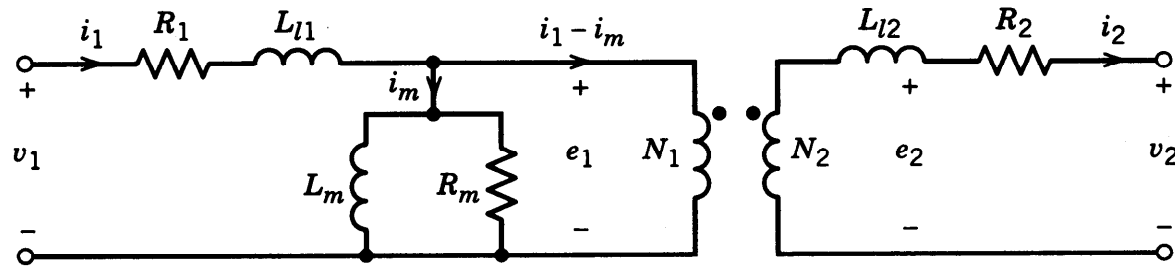
(a)



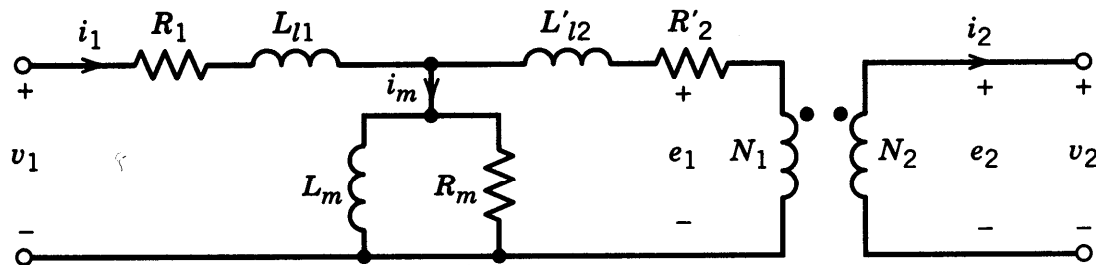
(b)

Figure 3-19 Equivalent circuit for (a) a physically realizable transformer wound on a lossless core and (b) an ideal transformer.

Including the Core Losses



(a)



(b)

Figure 3-21 Equivalent circuit of a transformer including the effects of hysteresis loss. (a) Circuit components are on both sides (coil 1 and coil 2 sides) of the ideal transformer. (b) Components from the secondary (coil 2) side are reflected across the ideal transformer to the primary (coil 1) side.

Transformer Core Characteristic

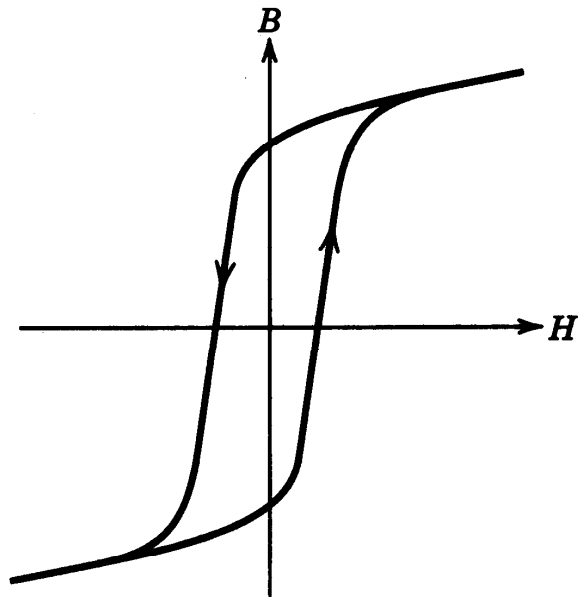
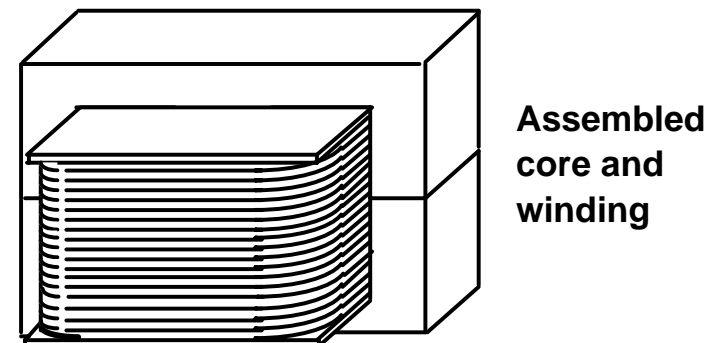
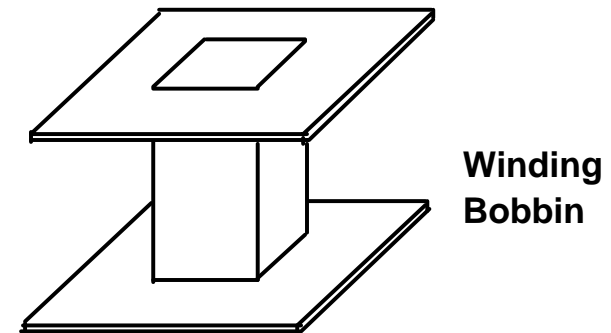
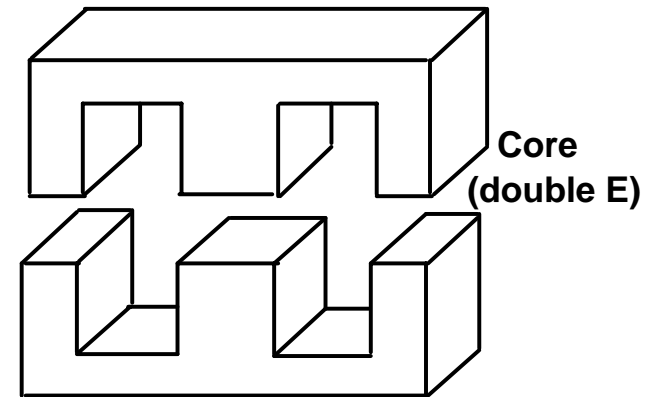


Figure 3-20 B - H characteristic of a transformer core having hysteresis and hence magnetic losses.

Magnetic Design Issues

- Ratings for inductors and transformers in power electronic circuits vary too much for commercial vendors to stock full range of standard parts.
- Instead only magnetic cores are available in a wide range of sizes, geometries, and materials as standard parts.
- Circuit designer must design the inductor/transformer for the particular application.
- Design consists of:
 1. Selecting appropriate core material, geometry, and size
 2. Selecting appropriate copper winding parameters: wire type, size, and number of turns.



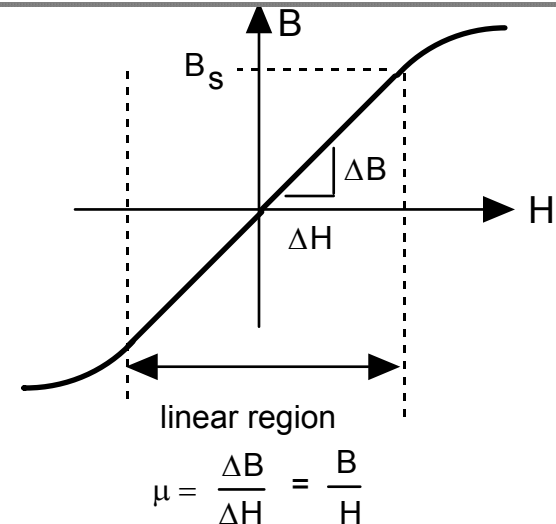
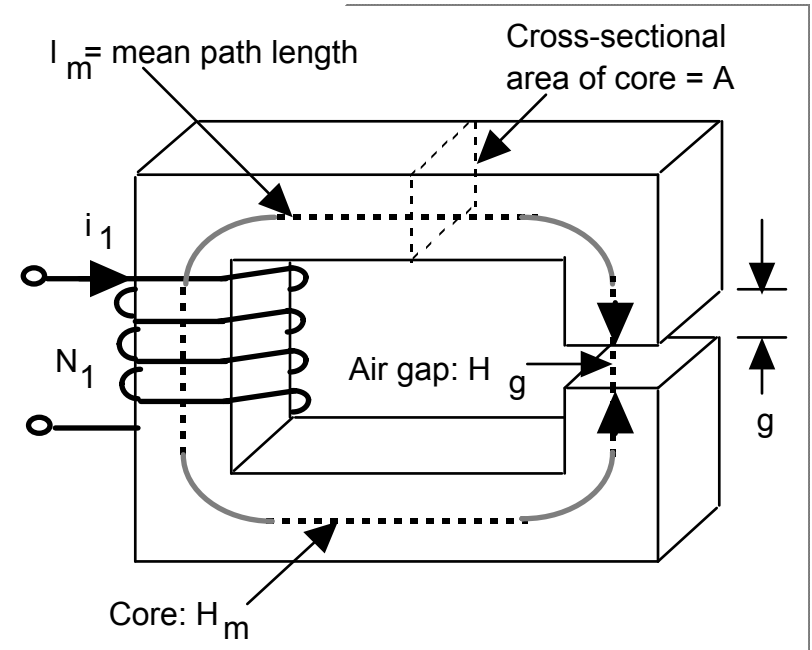
Inductor Fundamentals

- Assumptions
 - No core losses or copper winding losses
 - Linearized B-H curve for core with $\mu_m \gg \mu_o$
 - $l_m \gg g$ and $A \gg g^2$
 - Magnetic circuit approximations (flux uniform over core cross-section, no fringing flux)

- Starting equations
 - $H_m l_m + H_g g = N I$ (Ampere's Law)
 - $B_m A = B_g A = \phi$ (Continuity of flux assuming no leakage flux)
 - $\mu_m H_m = B_m$ (linearized B-H curve) ;
 $\mu_o H_g = B_g$

- Results

- $B_s > B_m = B_g = \frac{N I}{l_m / \mu_m + g / \mu_o} = \phi / A$
- $LI = N\phi$; $L = \frac{A N^2}{l_m / \mu_m + g / \mu_o}$



Transformer Fundamentals

- Assumptions same as for inductor

- Starting equations

- $H_1 L_m = N_1 I_1$; $H_2 L_m = N_2 I_2$

(Ampere's Law)

- $H_m L_m = (H_1 - H_2) L_m = N_1 I_1 - N_2 I_2$

- $\mu_m H_m = B_m$ (linearized B-H curve)

- $v_1 = N_1 \frac{d\phi_1}{dt}$; $v_2 = N_2 \frac{d\phi_2}{dt}$

(Faraday's Law)

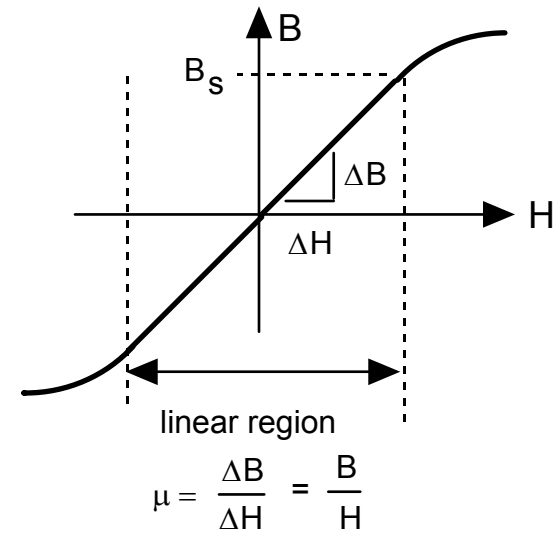
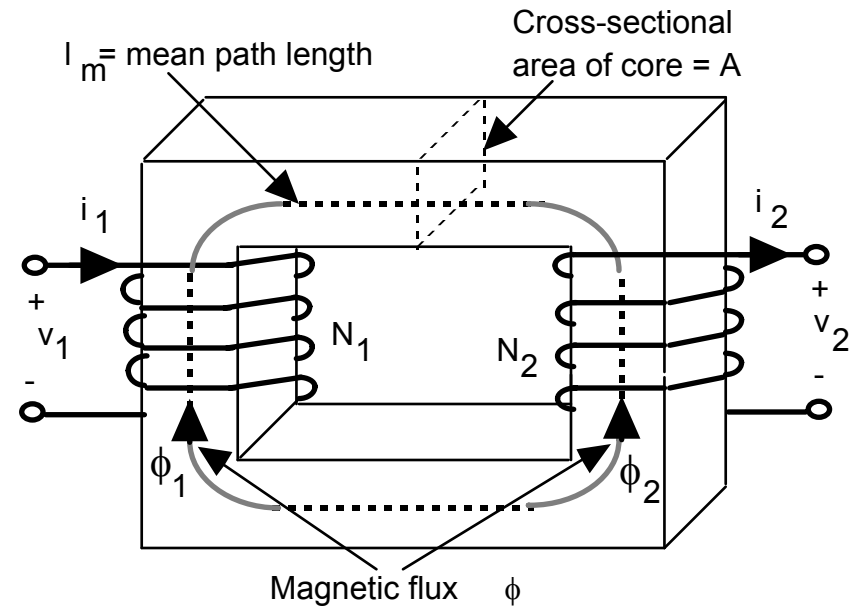
- Net flux $\phi = \phi_1 - \phi_2 = \mu_m H_m A$

$$= \frac{\mu_m A (N_1 I_1 - N_2 I_2)}{L_m}$$

- Results assuming $\mu_m \Rightarrow \infty$, i.e. ideal core or ideal transformer approximation.

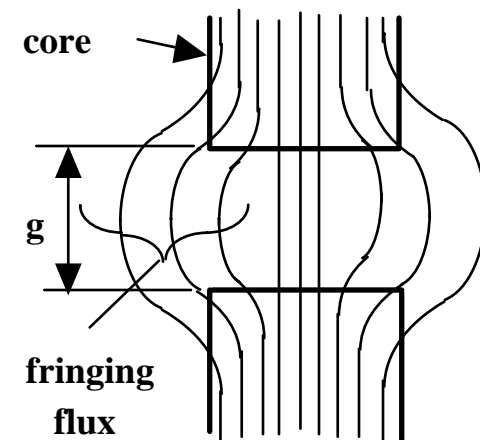
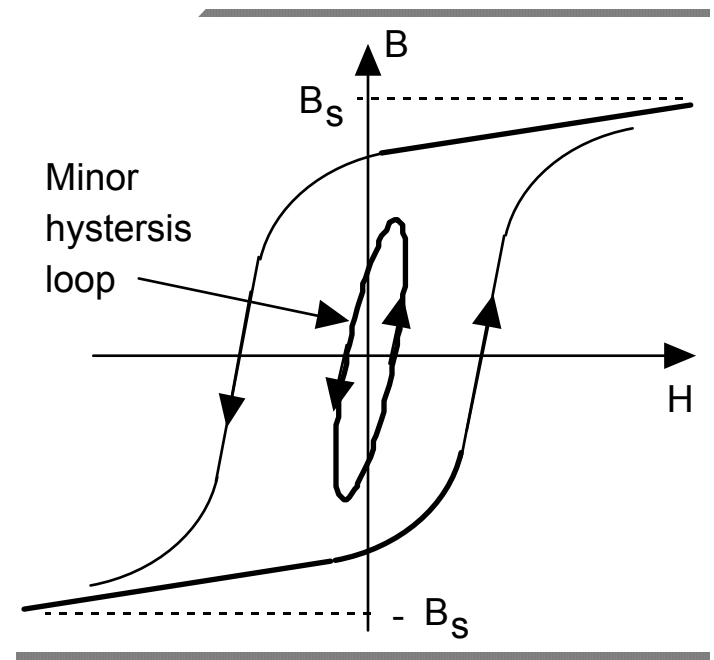
- $\frac{\phi}{\mu_m} = 0$ and thus $N_1 I_1 = N_2 I_2$

- $\frac{d(\phi_1 - \phi_2)}{dt} = 0 = \frac{v_1}{N_1} - \frac{v_2}{N_2}$; $\frac{v_1}{N_1} = \frac{v_2}{N_2}$



Scaling Issues

- Larger electrical ratings require larger current I and larger flux density B .
 - Core loss (hysteresis, eddy currents) increase as B^2 (or greater)
 - Winding (ohmic) loss increases as I^2 and increases at high frequencies (skin effect, proximity effect)
 - To control component temperature, surface area of component and thus size of component must be increased to reject increased heat to ambient.
 - At constant winding current density J and core flux density B , heat generation increases with volume V but surface area only increases as $V^{2/3}$.
 - Maximum J and B must be reduced as electrical ratings increase.
- Flux density B must be $< B_s$
 - Higher electrical ratings \square larger total flux
 - \square larger component size
 - Flux leakage, nonuniform flux distribution complicate design

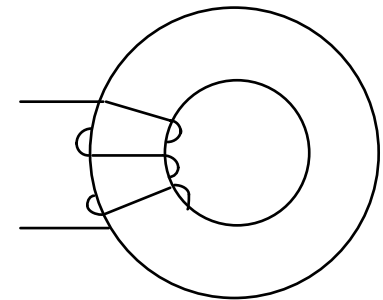
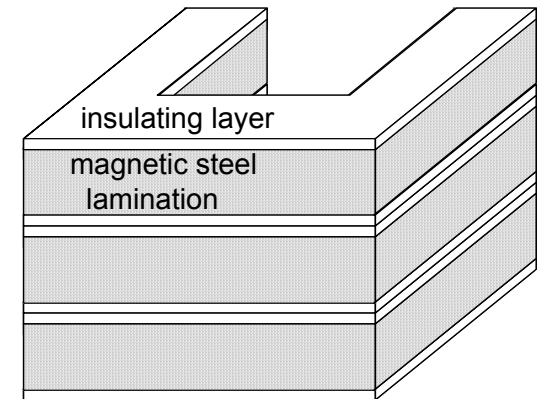
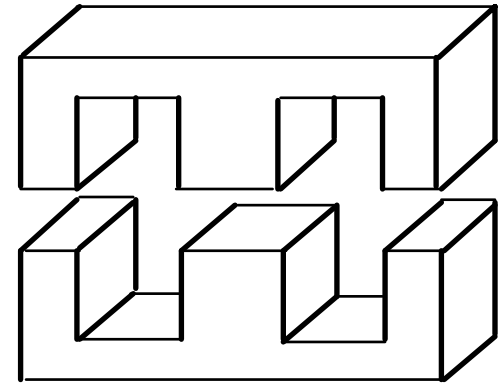


Overview of the Magnetic Design Problem

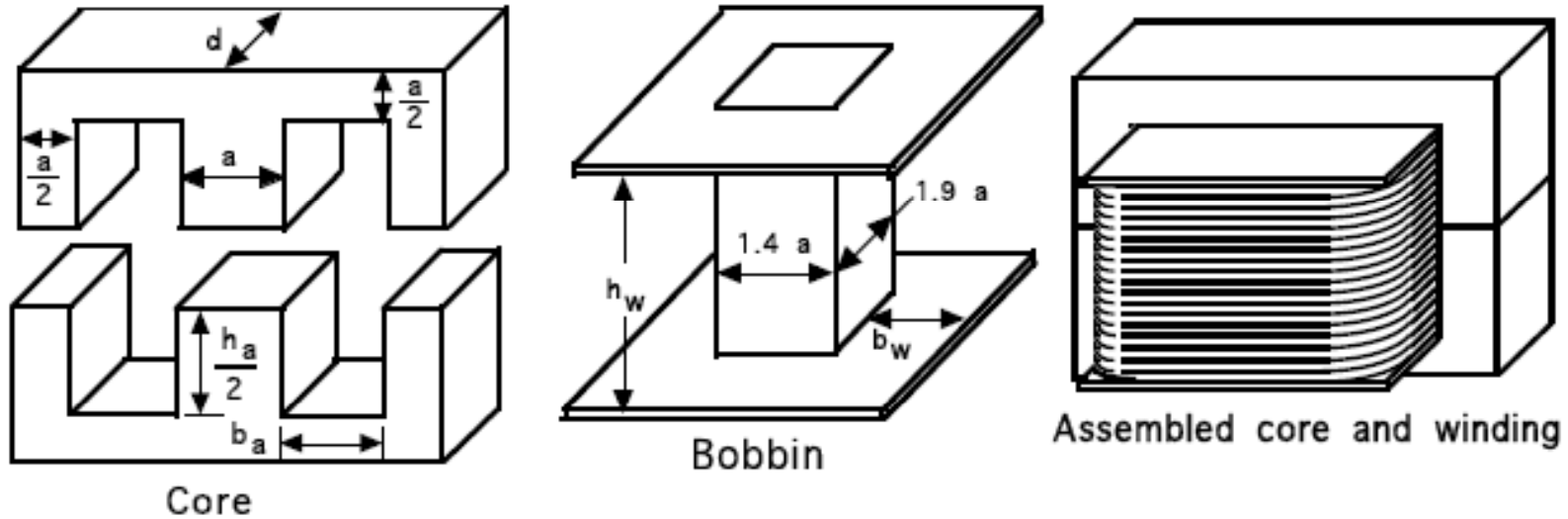
- Challenge - conversion of component operating specs in converter circuit into component design parameters.
- Goal - simple, easy-to-use procedure that produces component design specs that result in an acceptable design having a minimum size, weight, and cost.
- Inductor electrical (e.g. converter circuit) specifications.
 - Inductance value L
 - Inductor currents rated peak current I , rated rms current I_{rms} , and rated dc current (if any) I_{dc}
 - Operating frequency f .
 - Allowable power dissipation in inductor or equivalently maximum surface temperature of the inductor T_s and maximum ambient temperature T_a .
- Transformer electrical (converter circuit) specifications.
 - Rated rms primary voltage V_{pri}
 - Rated rms primary current I_{pri}
 - Turns ratio $N_{\text{pri}}/N_{\text{sec}}$
 - Operating frequency f
 - Allowable power dissipation in transformer or equivalently maximum temperatures T_s and T_a
- Design procedure outputs.
 - Core geometry and material.
 - Core size (A_{core} , A_w)
 - Number of turns in windings.
 - Conductor type and area A_{cu} .
 - Air gap size (if needed).
- Three impediments to a simple design procedure.
 1. Dependence of J_{rms} and B on core size.
 2. How to choose a core from a wide range of materials and geometries.
 3. How to design low loss windings at high operating frequencies.
- Detailed consideration of core losses, winding losses, high frequency effects (skin and proximity effects), heat transfer mechanisms required for good design procedures.

Magnetic Core Shapes and Sizes

- Magnetic cores available in a wide variety of sizes and shapes.
 - Ferrite cores available as U, E, and I shapes as well as pot cores and toroids.
 - Laminated (conducting) materials available in E, U, and I shapes as well as tape wound toroids and C-shapes.
 - Open geometries such as E-core make for easier fabrication but more stray flux and hence potentially more severe EMI problems.
 - Closed geometries such as pot cores make for more difficult fabrication but much less stray flux and hence EMI problems.
- Bobbin or coil former provided with most cores.
- Dimensions of core are optimized by the manufacturer so that for a given rating (i.e. stored magnetic energy for an inductor or V-I rating for a transformer), the volume or weight of the core plus winding is minimized or the total cost is minimized.
 - Larger ratings require larger cores and windings.
 - Optimization requires experience and computerized optimization algorithm.
- Vendors usually are in much better position to do the optimization than the core user.



Double-E Core Example



Characteristic	Relative Size	Absolute Size for $a = 1 \text{ cm}$
Core area A_{core}	$1.5 a^2$	1.5 cm^2
Winding area A_w	$1.4 a^2$	1.4 cm^2
Area product $AP = A_w A_c$	$2.1 a^4$	2.1 cm^4
Core volume V_{core}	$13.5 a^3$	13.5 cm^3
Winding volume V_w	$12.3 a^3$	12.3 cm^3
Total surface area of assembled core and winding	$59.6 a^2$	59.6 cm^2

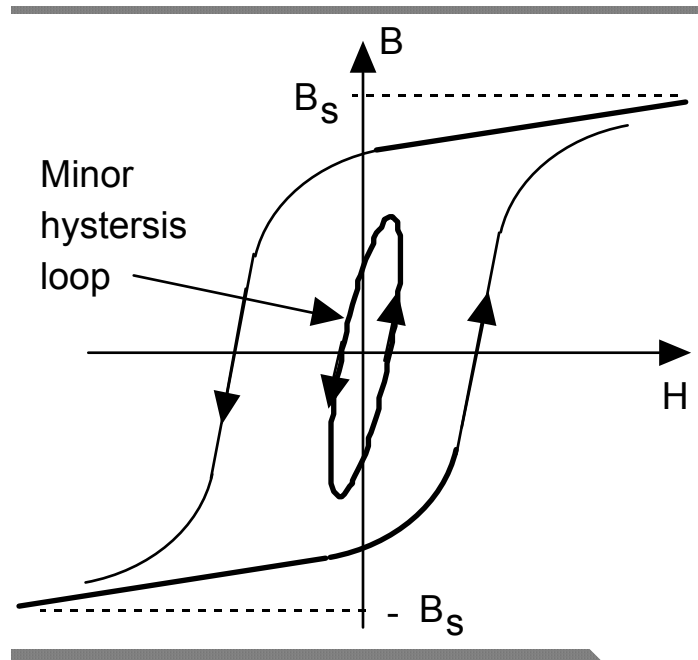
High Frequency Effects

- At DC, current flow in a wire spreads out uniformly across cross section of wire
- For high frequencies, skin effect and proximity effect affect current flow patterns
 - For an isolated wire at high frequency, current is concentrated on the surface of the wire in a thin layer approximately one “skin depth” thick
 - Proximity effect may be important if you have adjacent windings

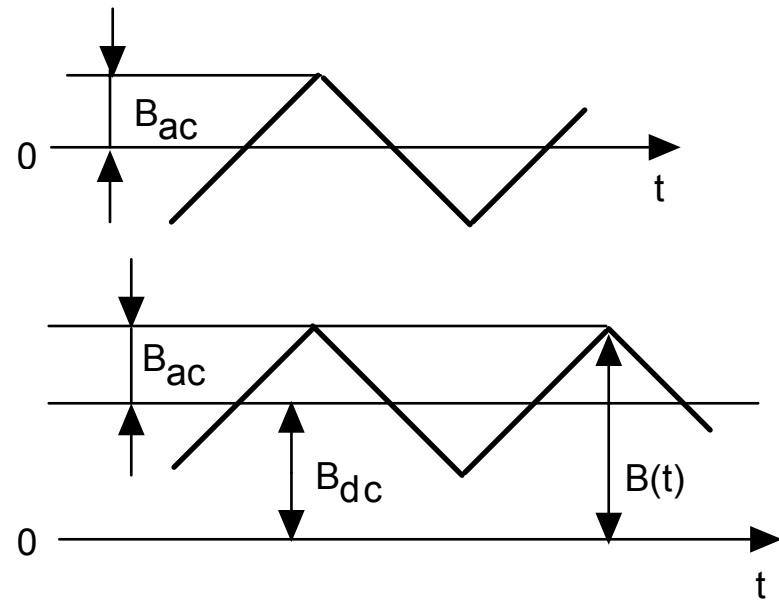
Types of Core Materials

- Iron-based alloys
 - Various compositions
 - Fe-Si (few percent Si)
 - Fe-Cr-Mn
 - METGLASS (Fe-B, Fe-B-Si, plus many other compositions)
 - Important properties
 - Resistivity = (10 - 100) ρ_{Cu}
 - $B_S = 1 - 1.8 \text{ T}$ (T = tesla = 10^4 Gauss)
 - METGLASS materials available only as tapes of various widths and thickness.
 - Other iron alloys available as laminations of various shapes.
 - Powdered iron can be sintered into various core shapes. Powdered iron cores have larger effective resistivities.
- Ferrite cores
 - Various compositions - iron oxides, Fe-Ni-Mn oxides
 - Important properties
 - Resistivity ρ very large (insulator) - no ohmic losses and hence skin effect problems at high frequencies.
 - $B_S = 0.3 \text{ T}$

Hysteresis Loss



- Area encompassed by hysteresis loop equals work done on material during one cycle of applied ac magnetic field. Area times frequency equals power dissipated per unit volume

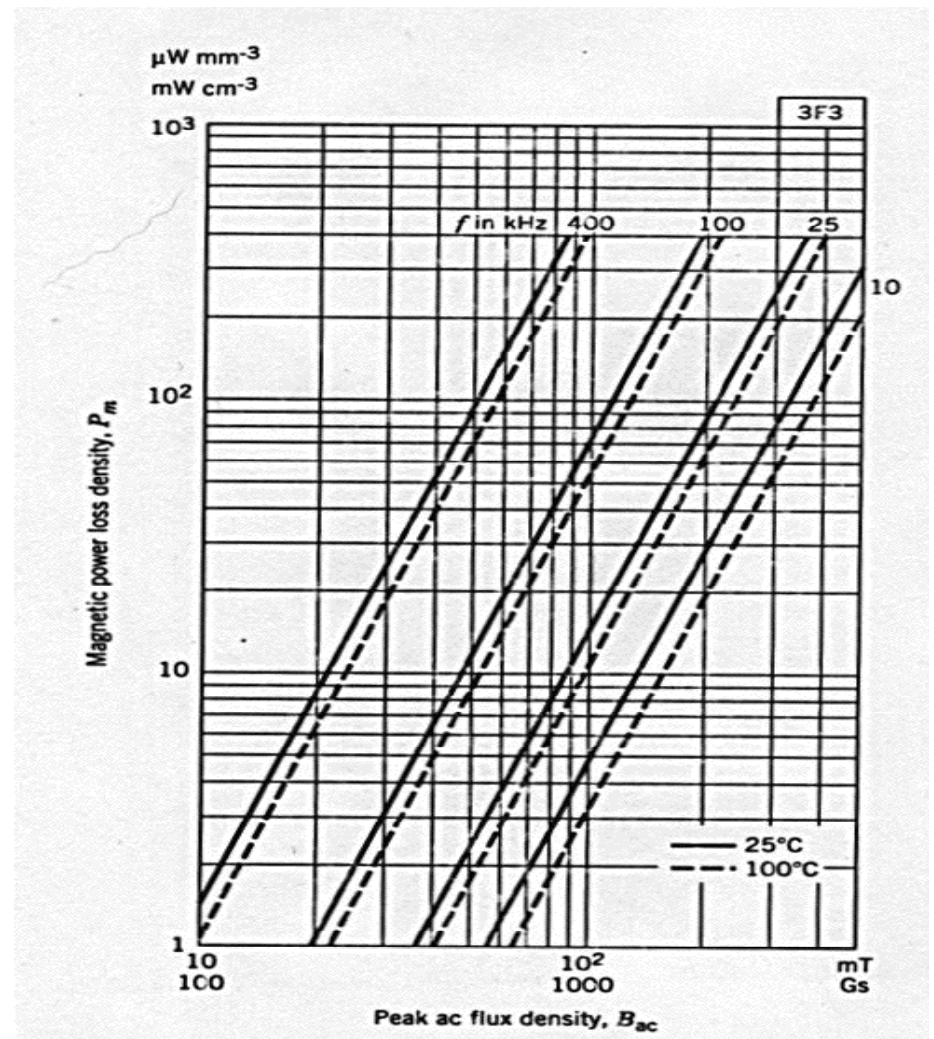


- Typical waveforms of flux density, $B(t)$ versus time, in an inductor
- Only B_{ac} contributes to hysteresis loss

Total Core Loss

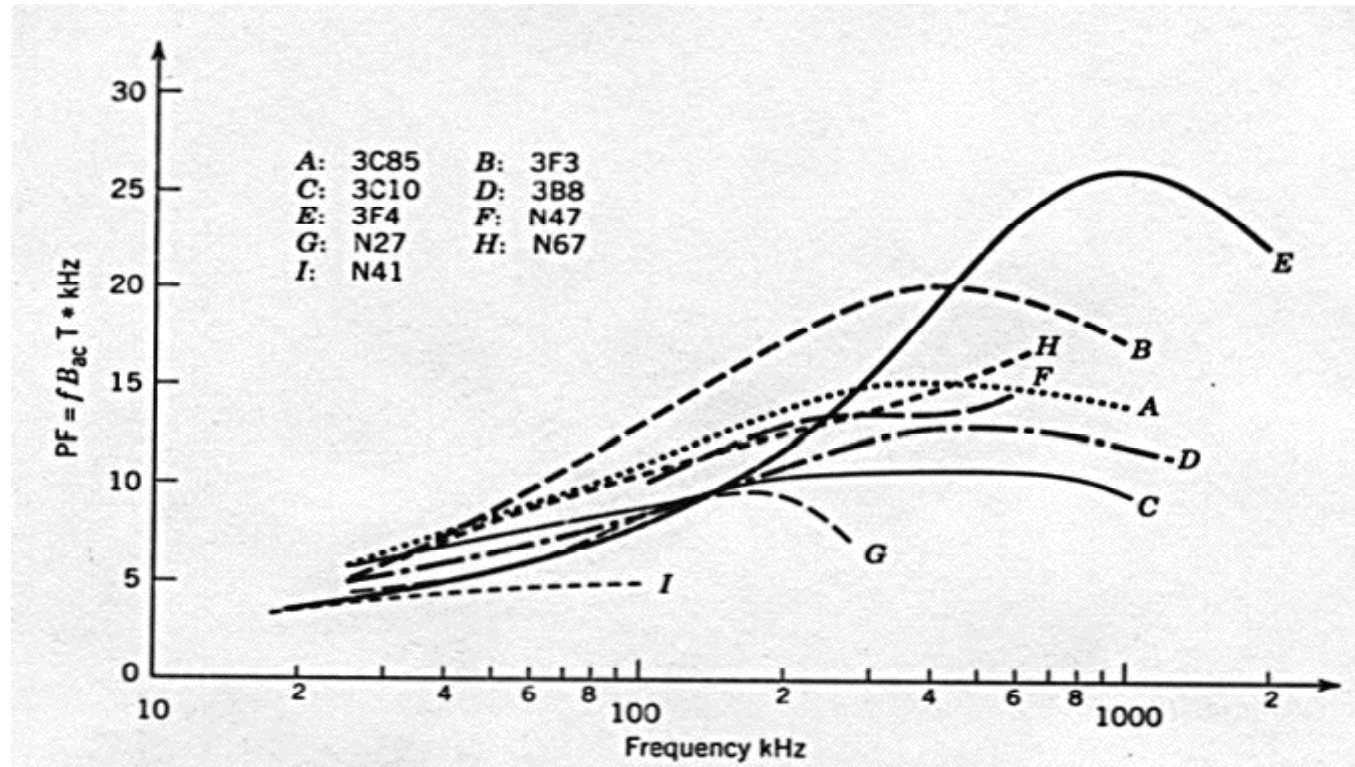
- Eddy current loss plus hysteresis loss = core loss.
- Empirical equation - $P_{m,sp} = k f^a [B_{ac}]^d$.
 f = frequency of applied field. B_{ac} = base-to-peak value of applied ac field. k , a , and d are constants which vary from material to material
- $P_{m,sp} = 1.5 \times 10^{-6} f^{1.3} [B_{ac}]^{2.5}$ mW/cm³ for 3F3 ferrite. (f in kHz and B in mT)
- $P_{m,sp} = 3.2 \times 10^{-6} f^{1.8} [B_{ac}]^2$ mW/cm³ METGLAS 2705M (f in kHz and B in mT)
- Example: 3F3 ferrite with $f = 100$ kHz and $B_{ac} = 100$ mT, $P_{m,sp} = 60$ mW/cm³

- 3F3 core losses in graphical form.

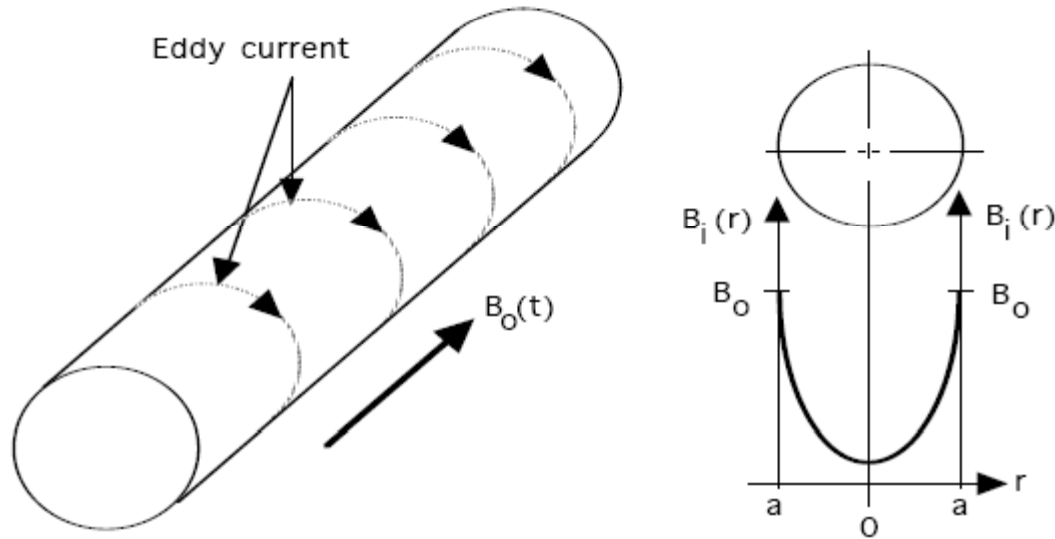


Core Material Performance Factor

- Volt-amp (V-A) rating of transformers is proportional to $f B_{ac}$
- Core materials have different allowable values of B_{ac} at a specific frequency. B_{ac} limited by allowable $P_{m,sp}$.
- Most desirable material is one with largest B_{ac} .
- Choosing best material aided by defining an empirical performance factor $PF = f B_{ac}$. Plots of PF versus frequency for a specified value of $P_{m,sp}$ permit rapid selection of best material for an application.
- Plot of PF versus frequency at $P_{m,sp} = 100 \text{ mW/cm}^3$ for several different ferrites shown below.
- For instance, 3F3 is best material in 40 kHz to 420 kHz range



Eddy Current Losses in Magnetic Cores



- AC magnetic fields generate eddy currents in conducting magnetic materials.
 - Eddy currents dissipate power.
 - Shield interior of material from magnetic field.

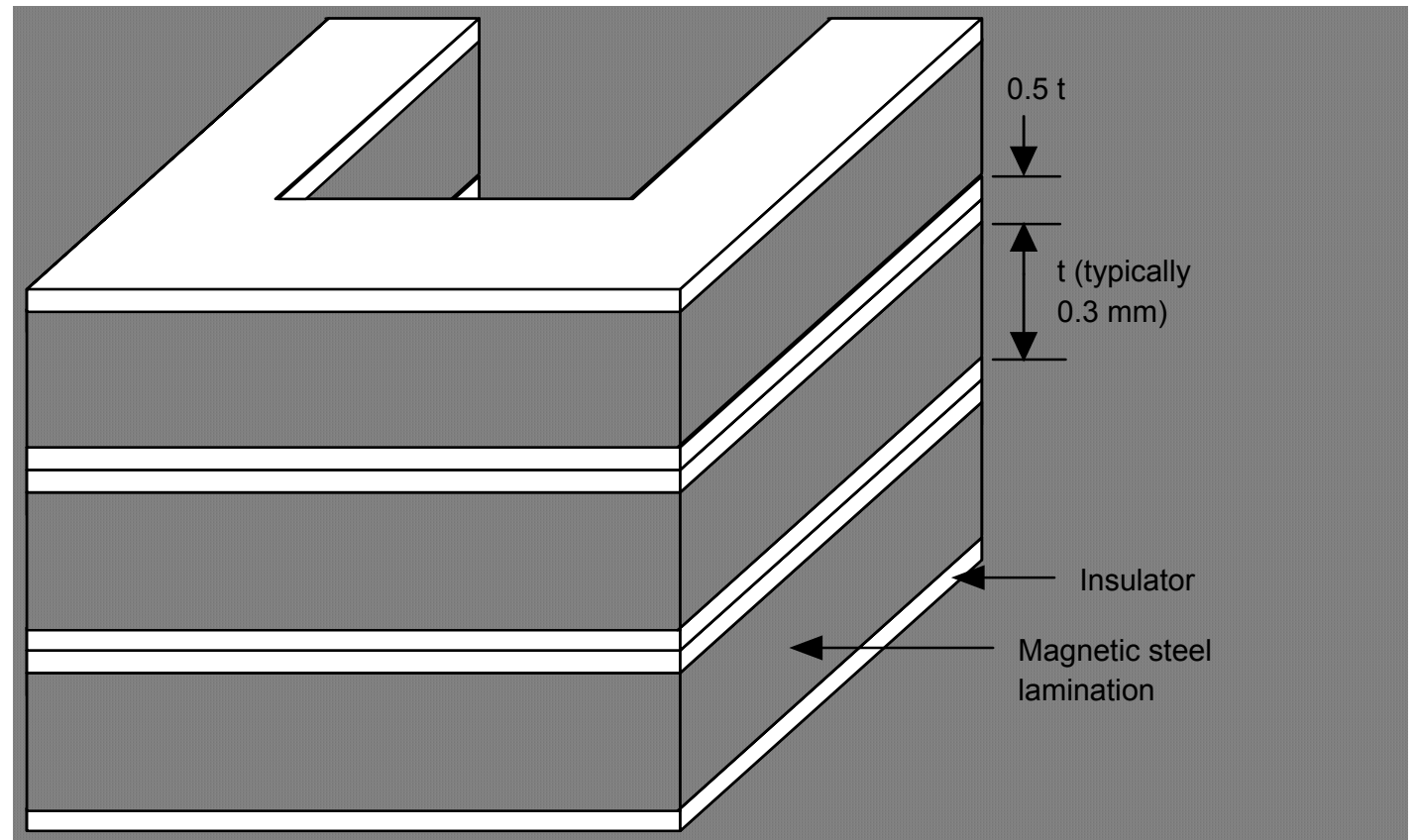
- $\frac{B_i(r)}{B_o} = \exp(\{r - a\}/\delta)$
- $\delta = \text{skin depth} = \sqrt{\frac{2}{\omega\mu\sigma}}$
 - $\omega = 2\pi f$, $f = \text{frequency}$
 - $\mu = \text{magnetic permeability}$; $\mu >$ for magnetic materials.
 - $\sigma = \text{conductivity of material}$.

- Numerical example

- $\sigma = 0.05 \sigma_{\text{Cu}}$; $\mu = 10^3 \mu_0$
- $f = 100 \text{ Hz}$
- $\delta = 1 \text{ mm}$

Laminated Core

- Cores made from conductive magnetic materials must be made of many thin laminations. Lamination thickness $<$ skin depth.



Eddy Current Loss in Lamination

- Flux $\phi(t)$ intercepted by current loop of area $2xw$ given by $\phi(t) = 2xw B(t)$
- Voltage in current loop $v(t) = 2xw \frac{dB(t)}{dt}$
 $= 2wx\omega B \cos(\omega t)$
- Current loop resistance $r = \frac{2w \rho_{\text{core}}}{L dx}$; $w \gg d$
- Instantaneous power dissipated in thin loop
 $\delta p(t) = \frac{[v(t)]^2}{r}$
- Average power P_{ec} dissipated in lamination

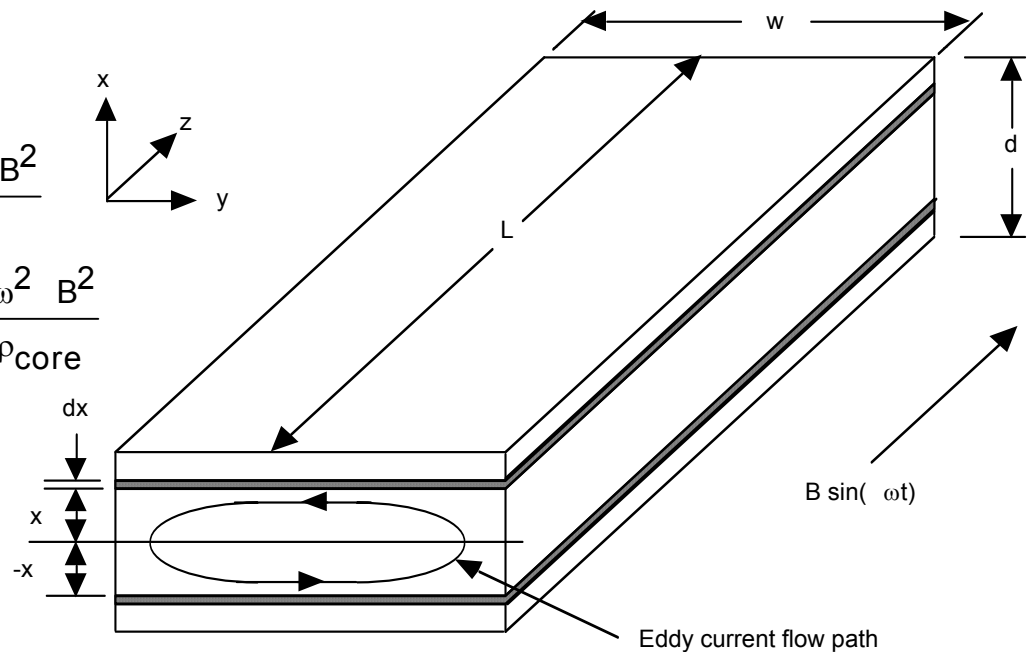
$$\text{given by } P_{\text{ec}} = \langle \int \delta p(t) dV \rangle = \frac{w L d^3 \omega^2 B^2}{24 \rho_{\text{core}}}$$

$$P_{\text{ec,sp}} = \frac{P_{\text{ec}}}{V} = \frac{w L d^3 \omega^2 B^2}{24 \rho_{\text{core}}} \frac{1}{dwL} = \frac{d^2 \omega^2 B^2}{24 \rho_{\text{core}}}$$

- Average power P_{ec} dissipated in lamination

$$\text{given by } P_{\text{ec}} = \langle \int \delta p(t) dV \rangle = \frac{w L d^3 \omega^2 B^2}{24 \rho_{\text{core}}}$$

$$P_{\text{ec,sp}} = \frac{P_{\text{ec}}}{V} = \frac{w L d^3 \omega^2 B^2}{24 \rho_{\text{core}}} \frac{1}{dwL} = \frac{d^2 \omega^2 B^2}{24 \rho_{\text{core}}}$$



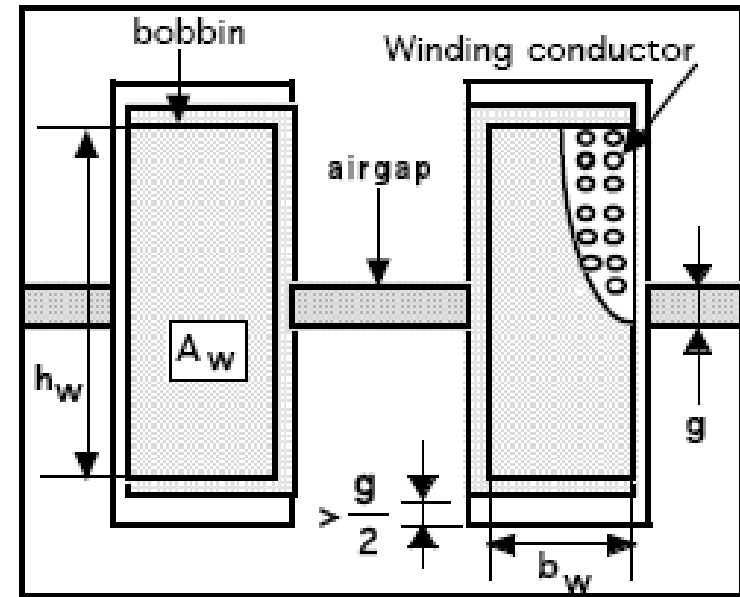
Power Dissipation in Windings

- Average power per unit volume of copper dissipated in copper winding = $P_{cu,sp} = \rho_{cu} (J_{rms})^2$ where $J_{rms} = I_{rms}/A_{cu}$ and ρ_{cu} = copper resistivity.

- Average power dissipated per unit volume of winding = $P_{w,sp} = k_{cu} \rho_{cu} (J_{rms})^2$; $V_{cu} = k_{cu} V_w$ where V_{cu} = total volume of copper in the winding and V_w = total volume of the winding.

- Copper fill factor $k_{cu} = \frac{N A_{cu}}{A_w} < 1$

- N = number of turns; A_{cu} = cross-sectional area of copper conductor from which winding is made;
 $A_w = b_w l_w$ = area of winding window.
- $k_{cu} = 0.3$ for Litz wire; $k_{cu} = 0.6$ for round conductors; $k_{cu} \approx 0.7-0.8$ for rectangular conductors.

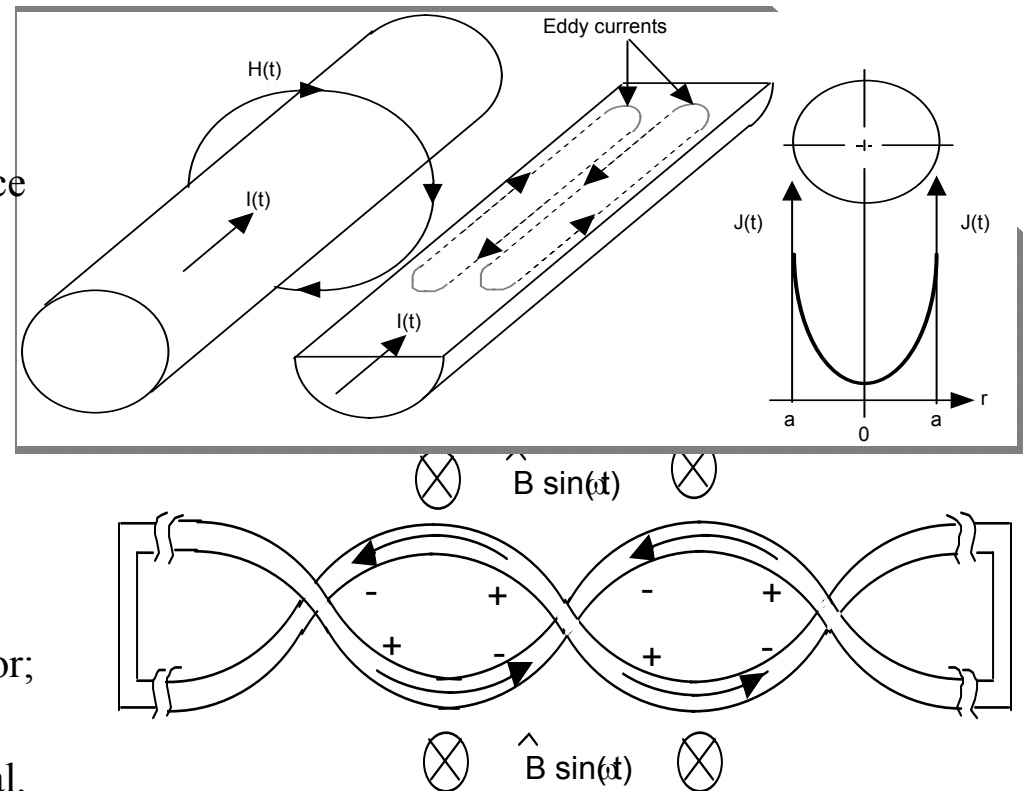


- $k_{cu} < 1$ because:
 - Insulation on wire to avoid shorting out adjacent turns in winding.
 - Geometric restrictions. (e.g. tight-packed circles cannot cover 100% of a square area.)

Eddy Currents Increase Winding Loss

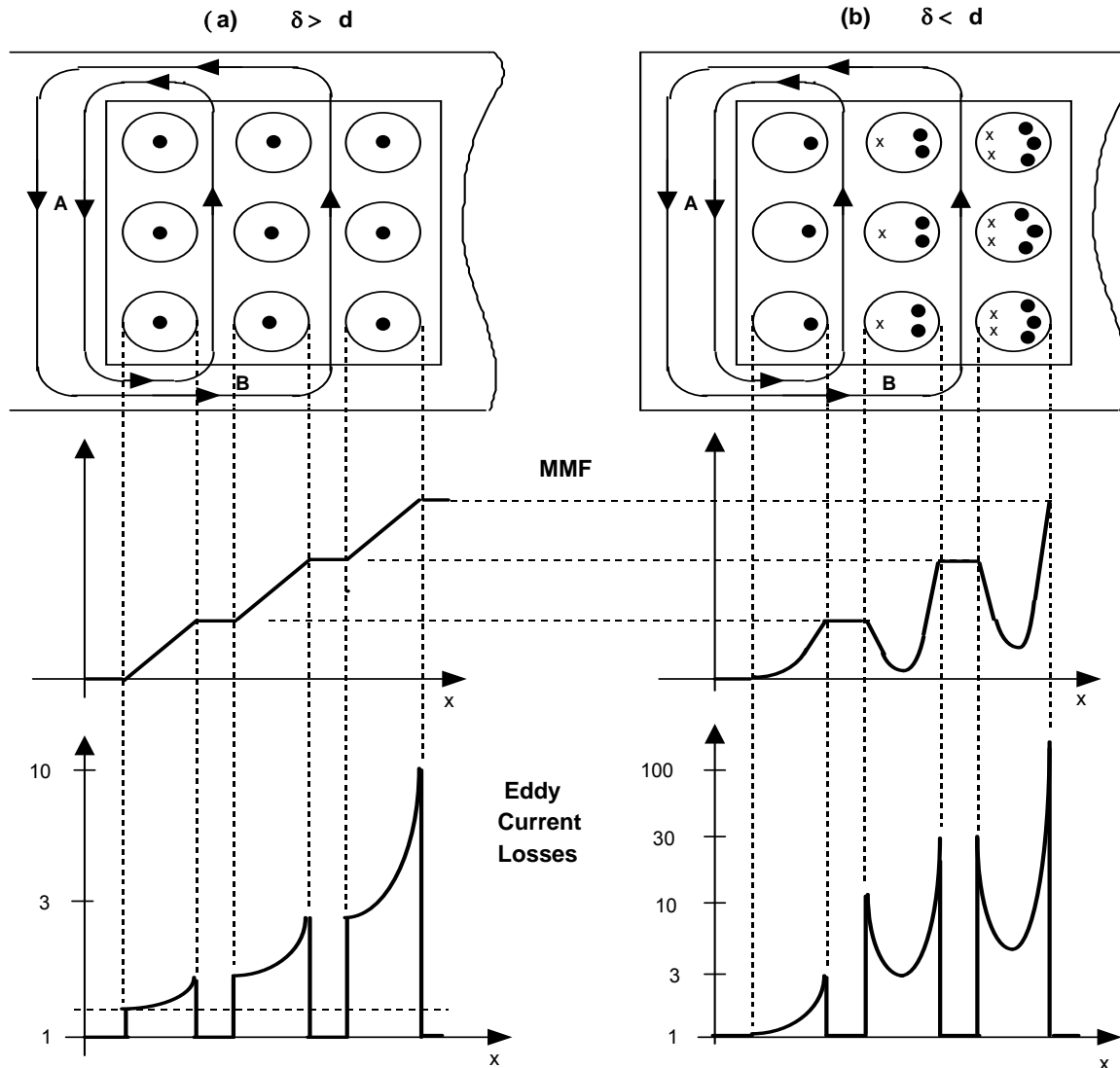
- AC currents in conductors generate ac magnetic fields which in turn generate eddy currents that cause a nonuniform current density in the conductor. Effective resistance of conductor increased over dc value.
- $P_{w,sp} > k_{cu} \rho_{cu} (J_{rms})^2$ if conductor dimensions greater than a skin depth.
- $\frac{J(r)}{J_0} = \exp(\{-r - a\}/\delta)$
- $\delta = \text{skin depth} = \sqrt{\frac{2}{\omega\mu\sigma}}$
 - $\omega = 2\pi f$, f = frequency of ac current
 - μ = magnetic permeability of conductor; $\mu = \mu_0$ for nonmagnetic conductors.
 - σ = conductivity of conductor material.
- Numerical example using copper at 100 °C

Frequency	50 Hz	5 kHz	20 kHz	500 kHz
Skin Depth	10.6 mm	1.06 mm	0.53 mm	0.106 mm



- Minimize eddy currents using Litz wire bundle. Each conductor in bundle has a diameter less than a skin depth.
- Twisting of paralleled wires causes effects of intercepted flux to be canceled out between adjacent twists of the conductors. Hence little if any eddy currents.

Proximity Effect Can Increase Winding Loss



- Proximity effect - losses due to eddy current generated by the magnetic field experienced by a particular conductor section but generated by the current flowing in the rest of the winding.

- Design methods for minimizing proximity effect losses discussed later.

Transformer Winding Loss --- AC Effects

$$\varphi = \sqrt{\eta} \sqrt{\frac{\pi}{4} \frac{d}{\delta}} \approx \frac{d}{\delta}$$

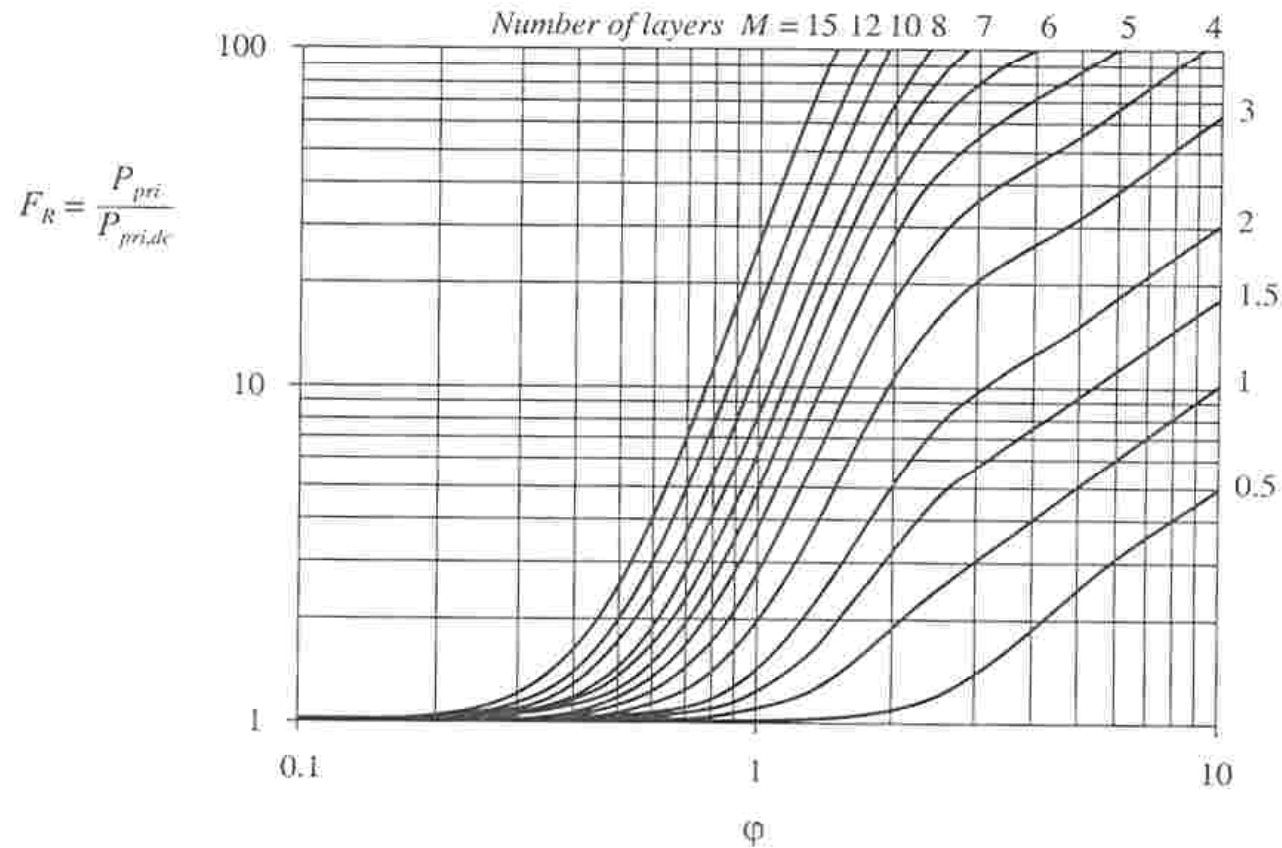


Fig. 13.34 Increased total winding copper loss in the two-winding transformer example, as a function of φ and number of layers M , for sinusoidal excitation.

Minimum Winding Loss

- $P_w = P_{dc} + P_{ec}$; P_{ec} = eddy current loss.

Optimum conductor size

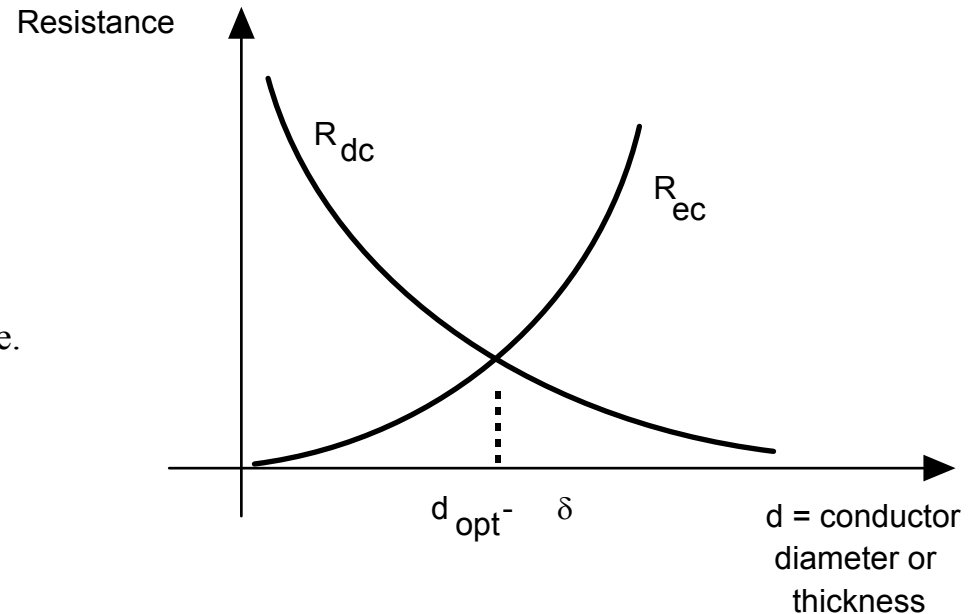
- $P_w = \{ R_{dc} + R_{ec} \} [I_{rms}]^2 = R_{ac} [I_{rms}]^2$

- $R_{ac} = F_R R_{dc} = [1 + R_{ec}/R_{dc}] R_{dc}$

- Minimum winding loss at optimum conductor size.

- $P_w \approx 1.5 P_{dc}$

- $P_{ec} = 0.5 P_{dc}$



- High frequencies require small conductor sizes minimize loss.
- P_{dc} kept small by putting many small-size conductors in parallel using Litz wire or thin but wide foil conductors.

Thermal Considerations

- Losses (winding and core) raise core temperature. Common design practice to limit maximum interior temperature to 100-125 °C.
 - Core losses (at constant flux density) increase with temperature above 100 °C
 - Saturation flux density B_s decreases with temp.
 - Nearby components such as power semi-conductor devices, integrated circuits, capacitors have similar limits.
 - Temperature limitations in copper windings
 - Copper resistivity increases with temperature increases. Losses, at constant current density increase with temperature.
 - Reliability of insulating materials degrades with temperature increase.
 - Surface temperature of component nearly equal to interior temperature. Minimal temperature gradient between interior and exterior surface.
 - Power dissipated uniformly in component volume.
 - Large cross-sectional area and short path lengths to surface of components.
 - Core and winding materials have large thermal conductivity.
 - Thermal resistance (surface to ambient) of magnetic component determines its temperature.
- $$P_{sp} = \frac{T_s - T_a}{R_{\theta sa}(V_w + V_c)} ; R_{\theta sa} = \frac{h}{A_s}$$
 - h = convective heat transfer coefficient = 10 °C-m²/W
 - A_s = surface area of inductor (core + winding). Estimate using core dimensions and simple geometric considerations.
 - Uncertain accuracy in h and other heat transfer parameters do not justify more accurate thermal modeling of inductor.

Core and Winding Scaling

- Power per unit volume, P_{sp} , dissipated in magnetic component is $P_{sp} = k_1/a$; $k_1 = \text{constant}$ and $a = \text{core scaling dimension}$.

- $P_{w,sp} V_w + P_{m,sp} V_m = \frac{T_s - T_a}{R_{\theta sa}}$:
 $T_a = \text{ambient temperature}$ and $R_{\theta sa} = \text{surface-to-ambient thermal resistance of component}$.

- For optimal design $P_{w,sp} = P_{c,sp} = P_{sp}$:
Hence $P_{sp} = \frac{T_s - T_a}{R_{\theta sa}(V_w + V_c)}$

- $R_{\theta sa}$ proportional to a^2 and $(V_w + V_c)$ proportional to a^3

- $J_{rms} = \sqrt{\frac{P_{sp}}{k_{cu} r_{cu}}} = k_2 \frac{1}{\sqrt{k_{cu} a}}$; $k_2 = \text{constant}$

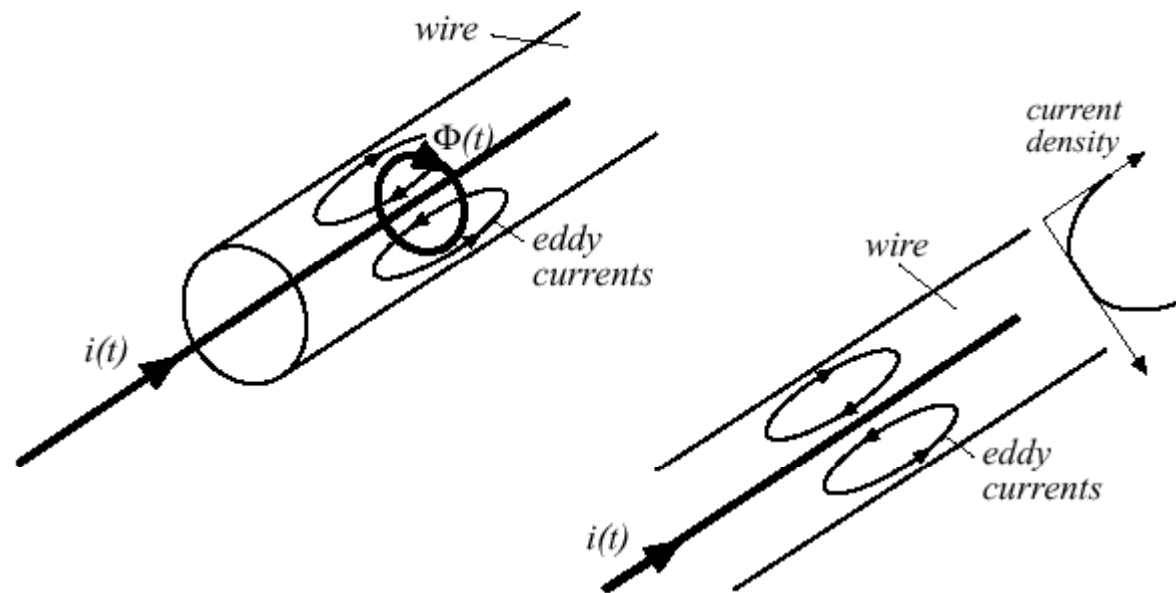
- $P_{m,sp} = P_{sp} = k f^b [B_{ac}]^d$; Hence

$$B_{ac} = \sqrt[d]{\frac{P_{sp}}{k f^b}} = \frac{k_3}{\sqrt[d]{f^b a}} \text{ where } k_3 = \text{constant}$$

- Plots of J_{rms} , B_{ac} , and P_{sp} versus core size (scale factor a) for a specific core material, geometry, frequency, and $T_s - T_a$ value very useful for picking appropriate core size and winding conductor size.

Causes of Skin Effect

- Self-field of wire causes current to crowd on the surface of the wire
- Method:
 - Current $i(t) \Rightarrow$ changing magnetic flux density $B(t) \Rightarrow$ reaction currents



Reference: <http://ece-www.colorado.edu/~pwrelect/book/slides/Ch12slide.pdf>

Effects of Skin Effect

- For high frequencies, current is concentrated in a layer approximately one skin depth δ thick
- Skin depth varies with frequency

$$\delta = \sqrt{\frac{2}{\omega\mu\sigma}} = \sqrt{\frac{1}{\pi f\mu\sigma}}$$

- σ = electrical conductivity = $5.9 \times 10^7 \Omega^{-1} \text{ m}^{-1}$ for copper at 300K
- μ = magnetic permeability of material = $4\pi \times 10^{-7} \text{ H/m}$ in free space

<i>Skin depth in copper at 300K</i>		
cond	5.90E+07	
mu	1.26E-06	
<i>f</i>	<i>skin depth (meter)</i>	
1	6.55E-02	
10	2.07E-02	
100	6.55E-03	
1000	2.07E-03	
1.00E+04	6.55E-04	
1.00E+05	2.07E-04	
1.00E+06	6.55E-05	

Skin Effect --- Increase in Wire Resistance

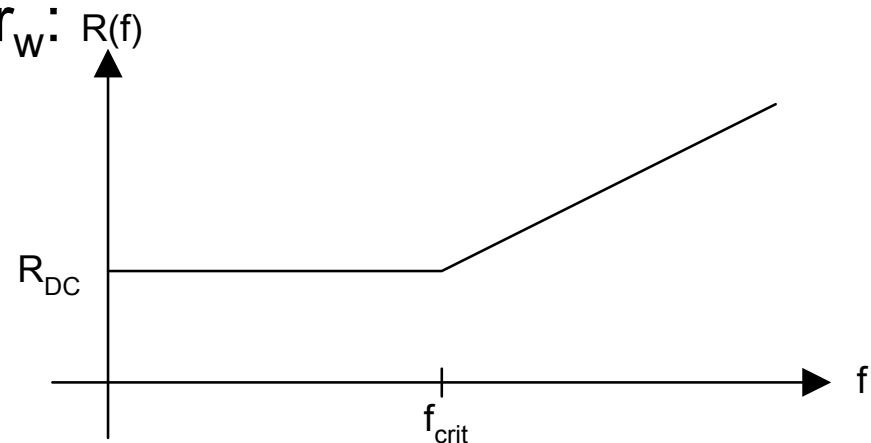
- For high frequencies, resistance of wire increases

- DC resistance of wire: $R_{DC} = \frac{l}{\sigma(\pi r_w^2)}$

- For frequencies above critical frequency where skin depth equals wire radius r_w :

$$R_{AC} = \frac{l}{\sigma(2\pi r_w \delta)} = R_{DC} \left(\frac{2\delta}{r_w} \right)$$

$$f_{crit} = \frac{1}{\pi r_w^2 \mu \sigma}$$

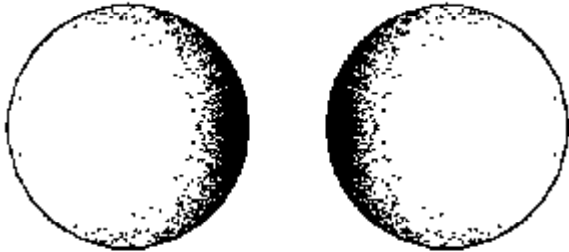


- Result: for high frequency operation, don't bother making wire radius $> \delta$
 - Skin depth in copper at 60 Hz is approximately 8 mm

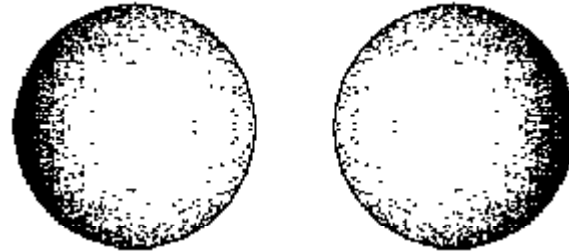
Proximity Effect

- In multiple-layer windings in inductors and transformers, the proximity effect can also greatly increase the winding resistance
- Field from one wire affects the current profile in another

CURRENTS IN
OPPOSITE DIRECTION

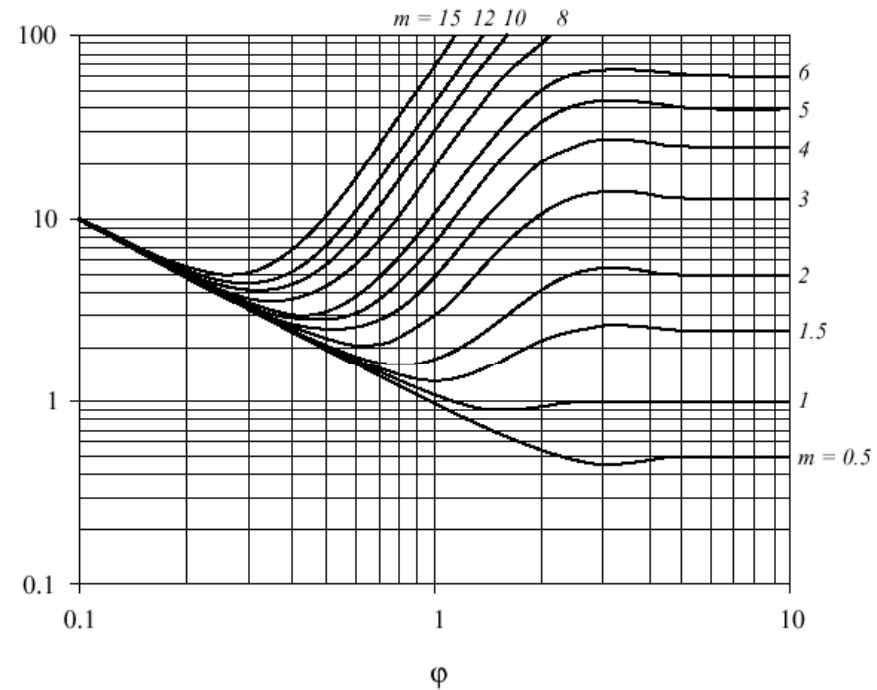


CURRENTS IN
SAME DIRECTION



Losses vs. Winding Layer Thickness

- There is an optimal wire diameter to minimize losses in your magnetics
- Optimal wire thickness is generally somewhat smaller than a skin depth
- If wire is thick, DC losses are low but skin and proximity losses are high
- If wire is skinny, DC losses are high but skin and proximity effects are minimized



Reference: <http://ece-www.colorado.edu/~pwrelect/book/slides/Ch12slide.pdf>

References

- Vacuumschmelze, *Soft Magnetic Materials*, 1979
- Magnetic Materials Producers Association, “Soft Ferrites Guide,” Standard SFG-98