Electromagnetic and Electromechanical Engineering Principles
Notes 02
Magnetics and Energy Conversion

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Portions of these notes reprinted from *Fitzgerald*, Electric Motors, 6th edition

Course Overview --- Day 2

2	<u>**Magnetics and energy conversion**</u>		
	- Review of Maxwell's equations; demo; concepts of magnetic fields; Ampere's, Faraday's and Gauss' magnetic laws; Lorentz force law; magnetic circuits……	7:30-9:30	1-54
	- Morning break……	9:30-9:45	
	- Soft magnetic materials (steel); hysteresis and core losses……	9:45-10:45	55-76
	- Hard magnetic (permanent magnets) and PM circuits……	10:45-12:00	77-139
	- Lunch ……	12:00-1:00	
	- Comments on superconductors; basic transformers: analysis, equivalent circuits……	1:00-2:45	140-208
	- Afternoon break……	2:45-3:00	
	- Per-unit system……	3:00-3:15	209-216
	- Begin electromechanical conversion; begin forces and torques……	3:15-4:00	217-240
	- Summarize……	4:00	241

Overview of Magnetics

- Review of Maxwell’s equations
- Ampere’s law, Gauss’ law, Faraday’s law
- Magnetic circuits
- Flux, flux linkage, inductance and energy

Review of Maxwell’s Equations

- First published by James Clerk Maxwell in 1864
- Maxwell’s equations couple electric fields to magnetic fields, and describe:
  - Magnetic fields
  - Electric fields
  - Wave propagation (through the wave equation)
- There are 4 Maxwell’s equations, but in magnetics we generally only need 3:
  - Ampere’s Law
  - Faraday’s Law
  - Gauss’ Magnetic Law

James Clerk Maxwell

Review of Maxwell’s Equations

- We’ll review Maxwell’s equations in words, followed by a little bit of mathematics and some computer simulations showing the magnetic fields

Ampere's Law

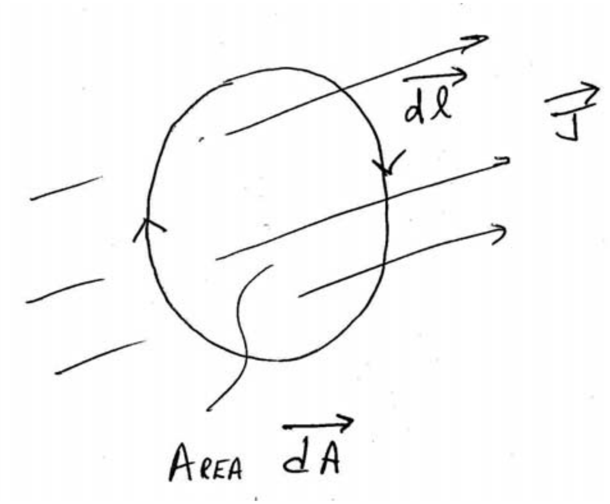
- Flowing current creates a magnetic field

$$\oint_C \vec{H} \cdot d\vec{l} = \int_S \vec{J} \cdot d\vec{A} + \frac{d}{dt}\int_S \varepsilon_o \vec{E} \cdot d\vec{A}$$

- In magnetic systems, generally there is high current and low voltage (and hence low electric field) and we can approximate for low *d/dt*:

$$\oint_C \vec{H} \cdot d\vec{l} \approx \int_S \vec{J} \cdot d\vec{A}$$

- In words: the magnetic flux density integrated around <u>any</u> closed contour equals the net current flowing through the surface bounded by the contour



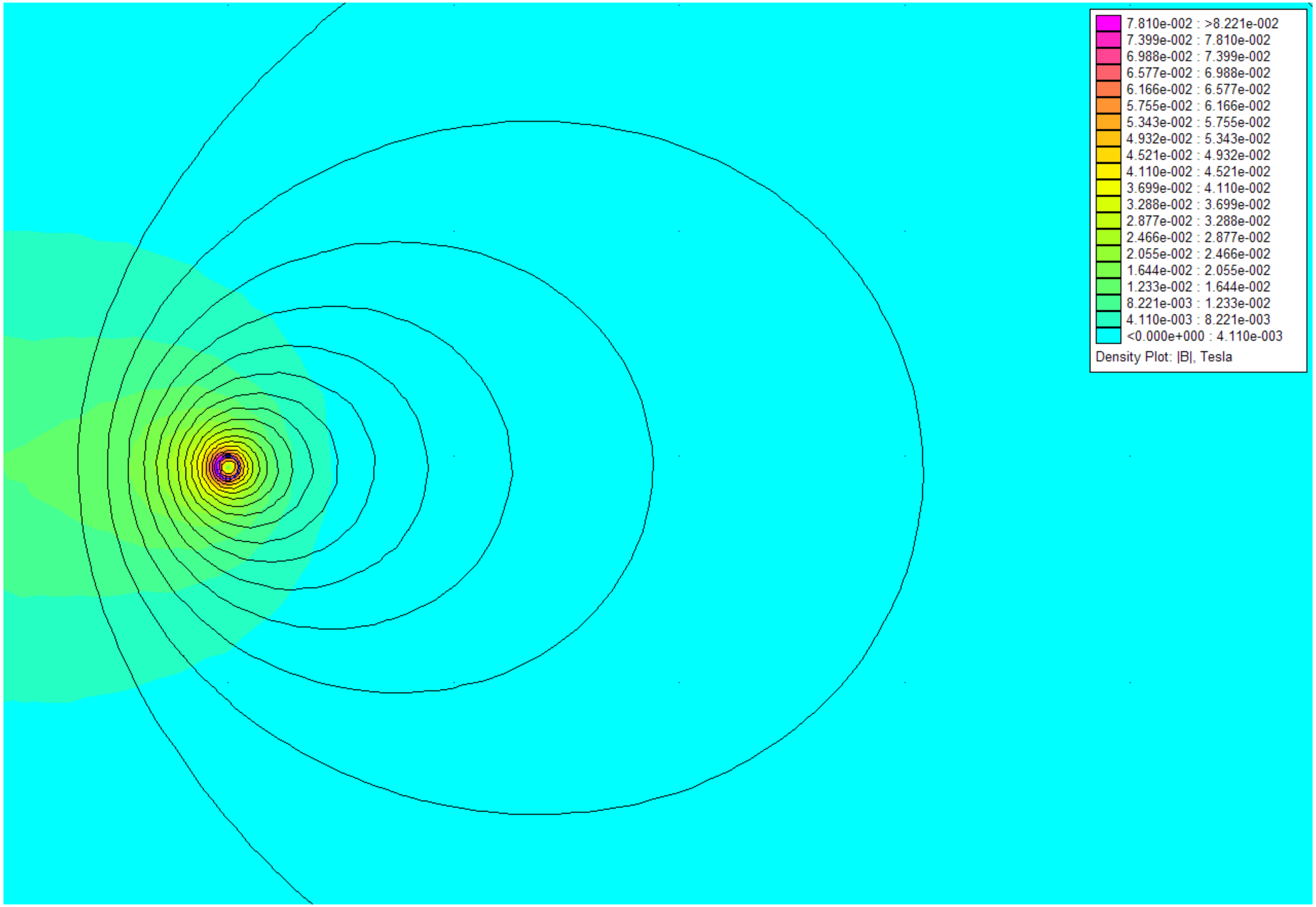
André-Marie Ampère

Finite-Element Analysis (FEA)

- Very useful tool for visualizing and solving shapes and magnitudes of magnetic fields
- FEA is often used to simulate and predict the performance of motors, etc.
- Following we'll see some 2-dimensional (2D) FEA results to help explain Maxwell's equations

Field From Current Loop, NI = 500 A-turns

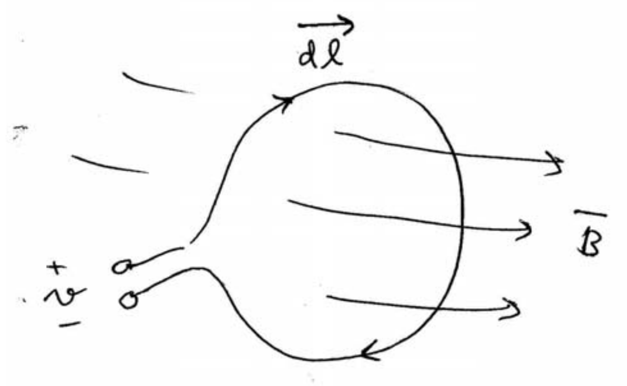
- Coil radius R = 1”; plot from 2D finite-element analysis



Faraday's Law

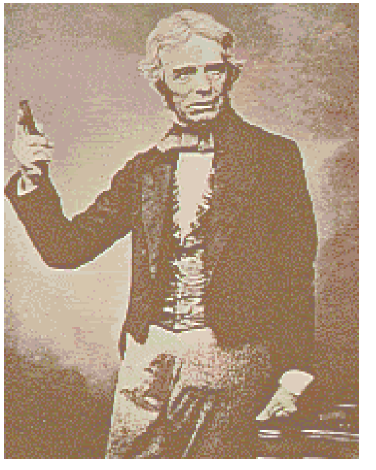
- A changing magnetic flux impinging on a conductor creates an electric field and hence a current (eddy current)

$$\oint_C \vec{E} \cdot d\vec{l} = -\frac{d}{dt}\int_S \vec{B} \cdot d\vec{A}$$



- The electric field integrated around a closed contour equals the net time-varying magnetic flux density flowing through the surface bound by the contour
- In a conductor, this electric field creates a current by:

$$\vec{J} = \sigma\vec{E}$$

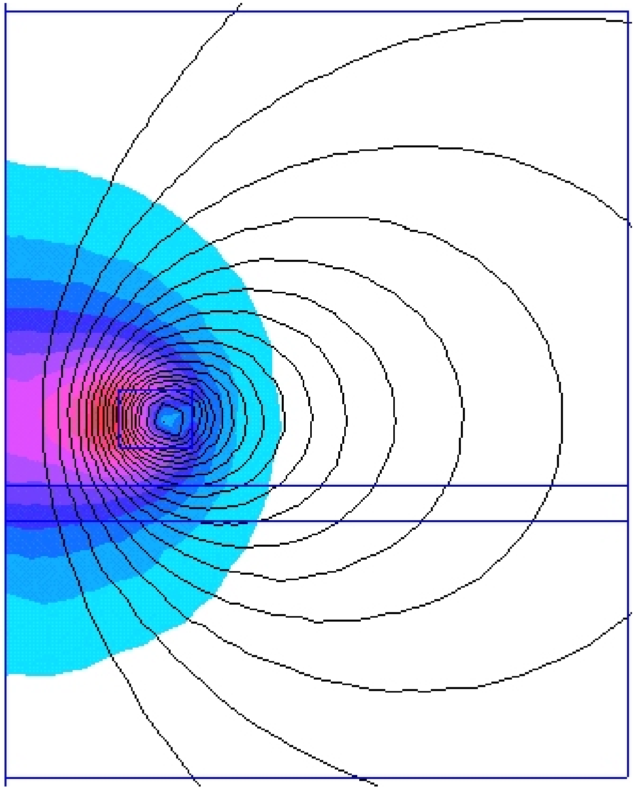


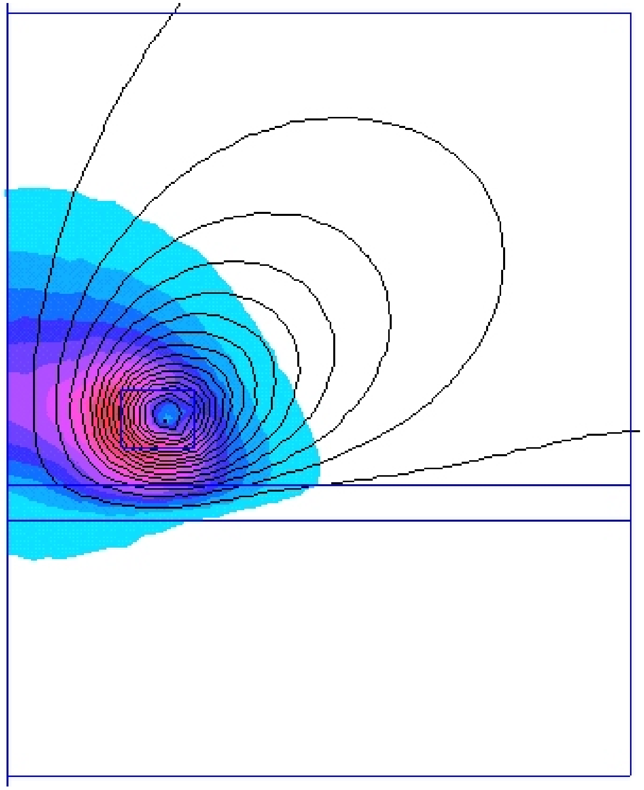
Michael Faraday

- Induction motors, brakes, etc.

Circular Coil Above Conducting Aluminum Plate

- Flux density plots at DC and 60 Hz
- At 60 Hz, currents induced in plate via magnetic induction create lift force





Demonstration of Faraday's Law: Electrodynamic Drag (NdFeB Magnet-in-Tube)

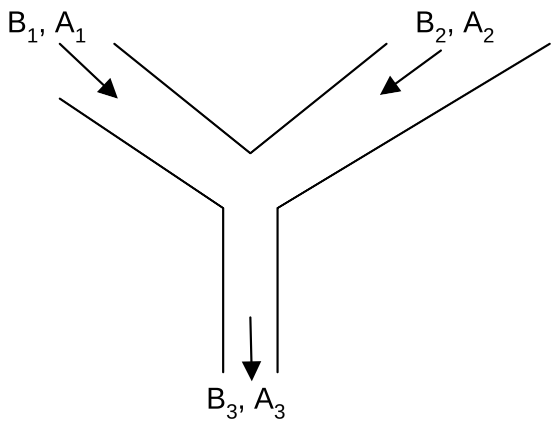
- Process:
  - Moving magnet creates changing magnetic field in copper tube
  - Changing magnetic field creates induced voltage
  - Induced voltage creates current
  - By Lorentz force law, induced current and applied magnetic field create drag force

Gauss' Magnetic Law

- Gauss' magnetic law says that the integral of the magnetic flux density over any closed surface is zero, or:

$$\oint_S B \cdot dA = 0$$

- This law implies that magnetic fields are due to electric currents and that magnetic charges ("monopoles") do not exist.
- Note: similar form to KCL in circuits. (We'll use this analogy later…)



$$B_3 A_3 = B_1 A_1 + B_2 A_2$$

Carl Friedrich Gauss

Gauss' Law --- Continuity of Flux Lines

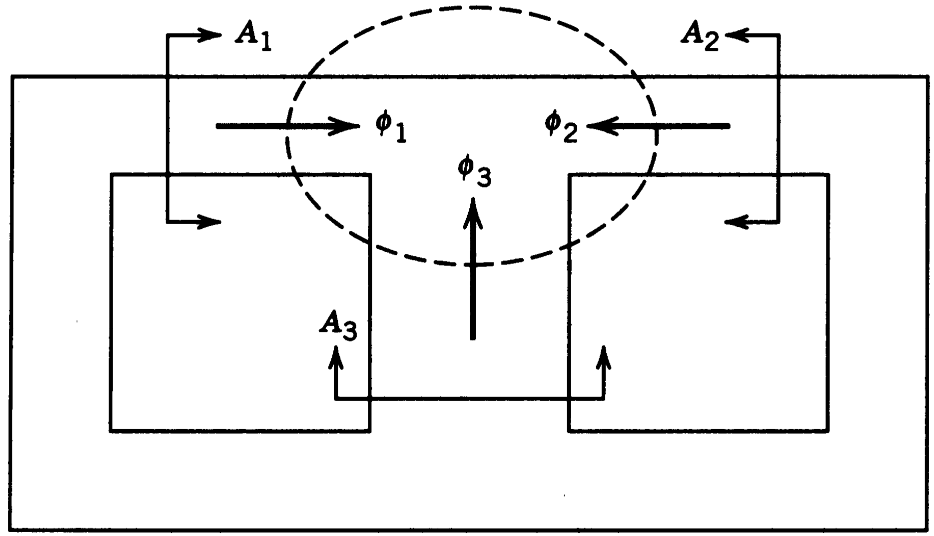


Figure 3-13 Continuity of flux.

$$\phi_1 + \phi_2 + \phi_3 = 0$$

Reference: N. Mohan, et. al., *Power Electronics Converters, Applications and Design*, Wiley, 2003, pp. 48

Lorentz Force Law

- Experimentally derived rule:

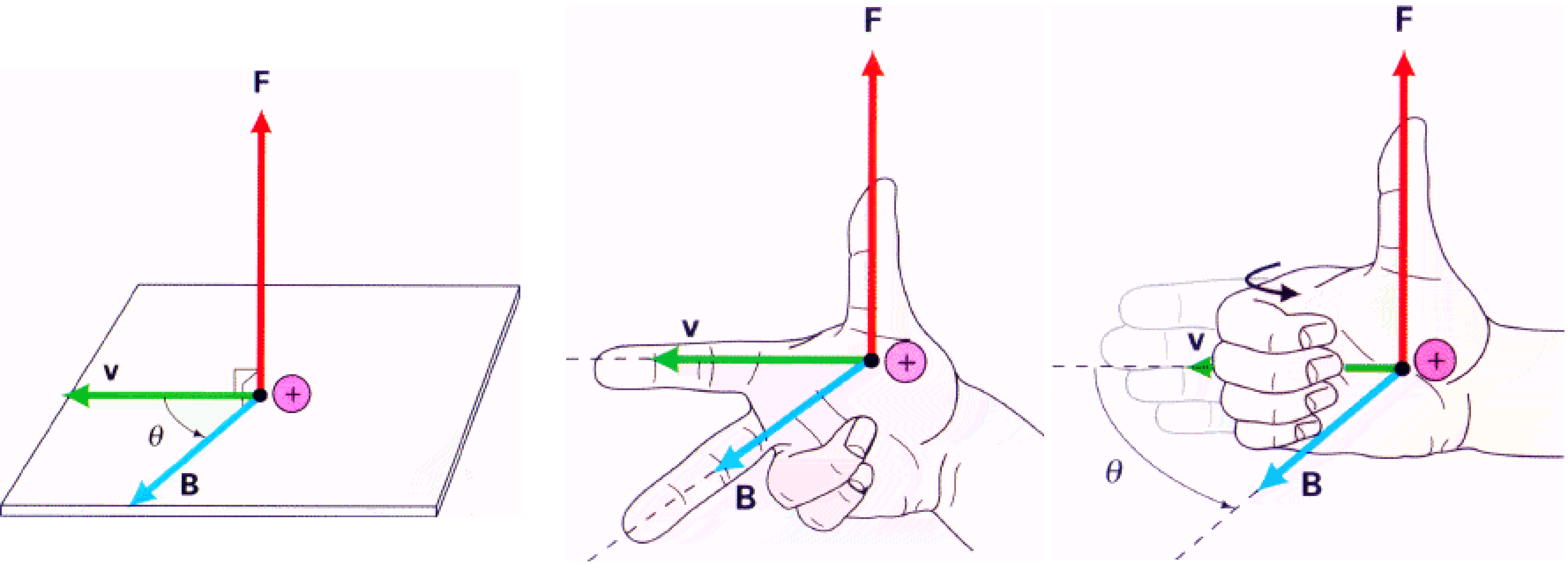
$$F = \int J \times B dV$$

- For a wire of length *l* carrying current *I* perpendicular to a magnetic flux density *B*, this reduces to:

$$F = IBl$$

Lorentz Force Law and the Right Hand Rule

$$F = \int J \times B dV$$



Reference: http://www.physics.brocku.ca/faculty/sternin/120/slides/rh-rule.html

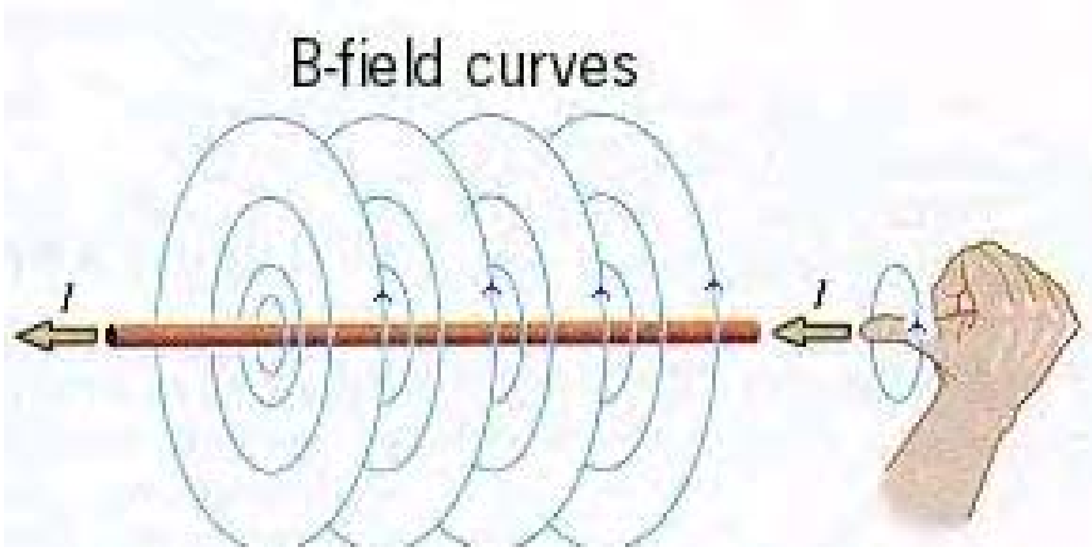
Intuitive Thinking about Magnetics

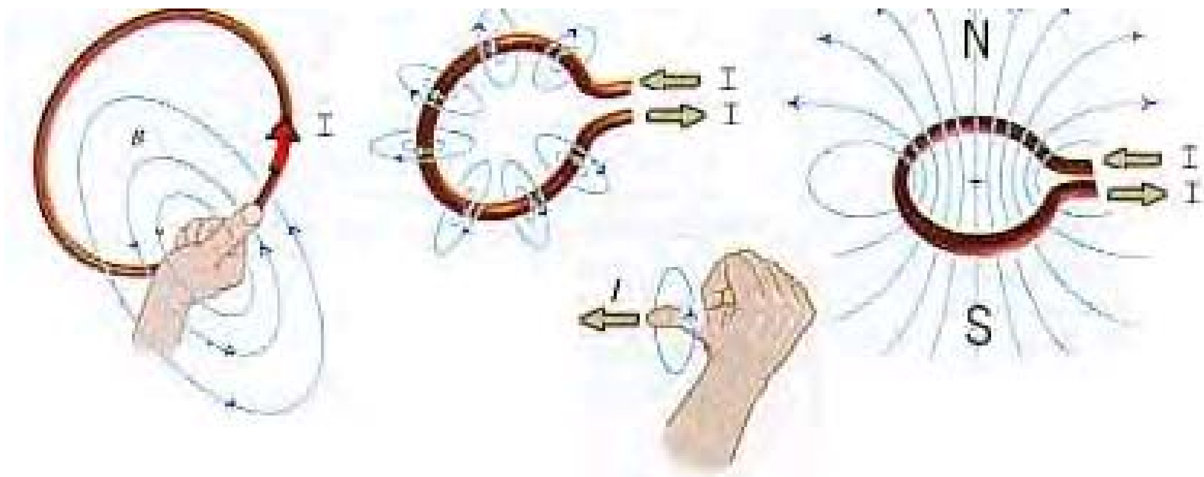
- By Ampere's Law, the current ***J*** and the magnetic field ***H*** are generally at right angles to one another
- By Gauss' law, magnetic field lines loop around on themselves
  – No magnetic monopole
- You can think of high- μ magnetic materials such as steel as an easy conduit for magnetic flux…. i.e. the flux easily flows thru the high- μ material

Magnetic Field (H) and Magnetic Flux Density (B)

- ***H*** is the magnetic field (A/m in SI units) and ***B*** is the magnetic flux density (Weber/m^2, or Tesla, in SI units)
- ***B*** and ***H*** are related by the magnetic permeability μ by ***B*** = μ***H***
- Magnetic permeability μ has units of Henry/meter
- You can think of high- μ magnetic materials such as steel as an easy conduit for magnetic flux…. i.e. the flux easily flows thru the high- μ material
- In free space $\mu_o = 4\pi\times10^{-7}$ H/m
- Note that ***B*** and ***H*** are vectors; they have both a magnitude and a direction

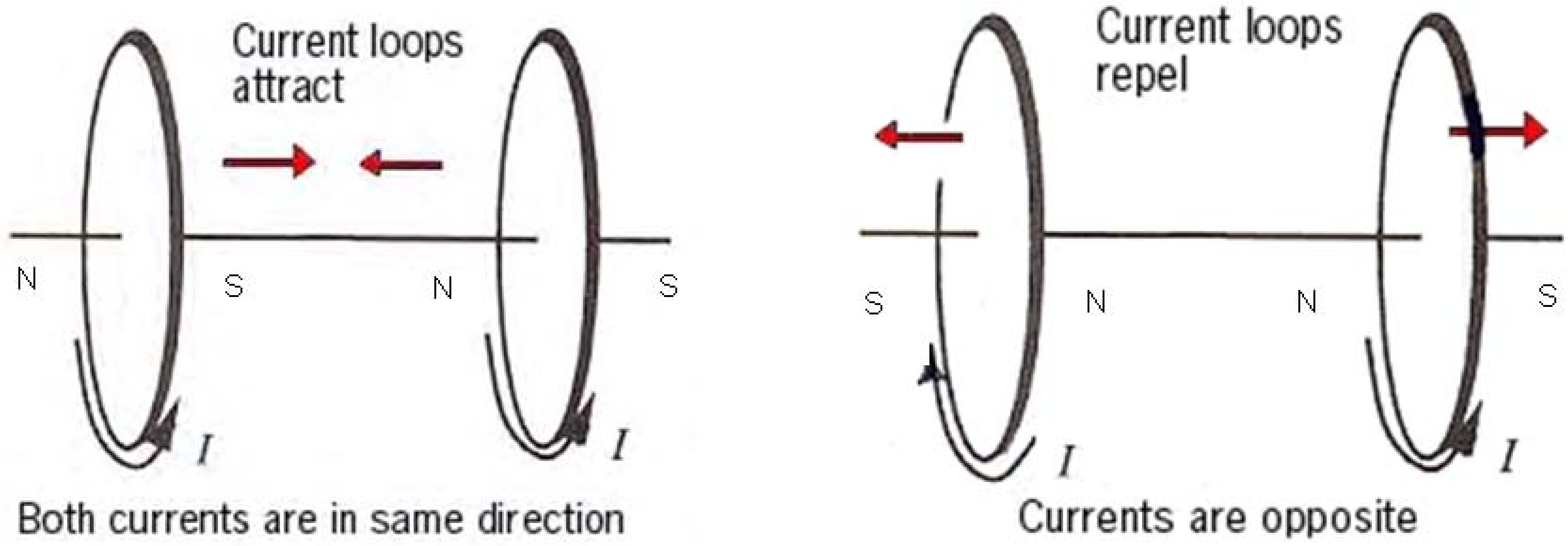
Right Hand Rule and Direction of Magnetic Field





Reference: http://sol.sci.uop.edu/~jfalward/magneticforcesfields/magneticforcesfields.html

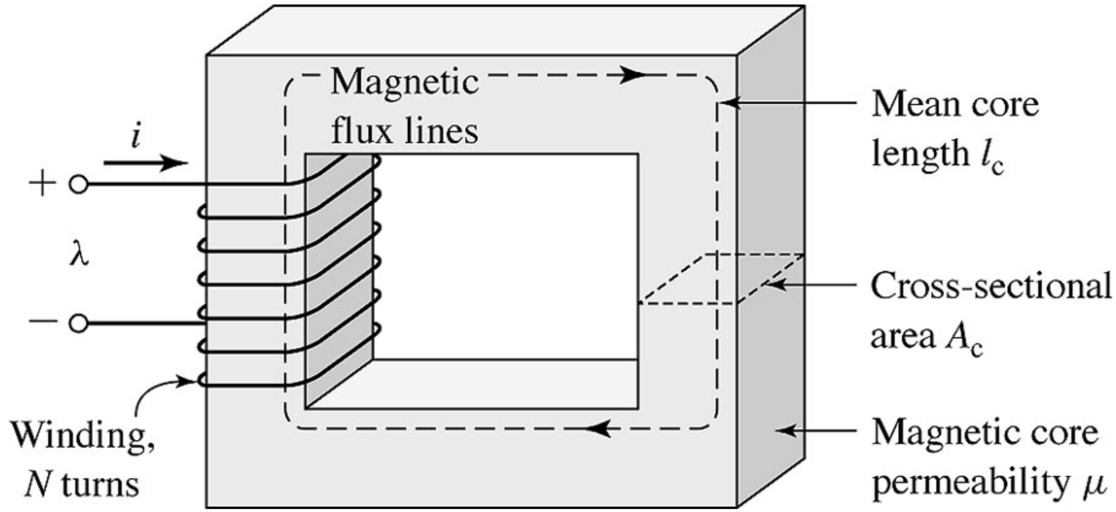
Forces Between Current Loops



Reference: http://sol.sci.uop.edu/~jfalward/magneticforcesfields/magneticforcesfields.html

Inductor Without Airgap

- Magnetic flux is constrained to flow within steel



<u>Constitutive relationships</u>

In free space:

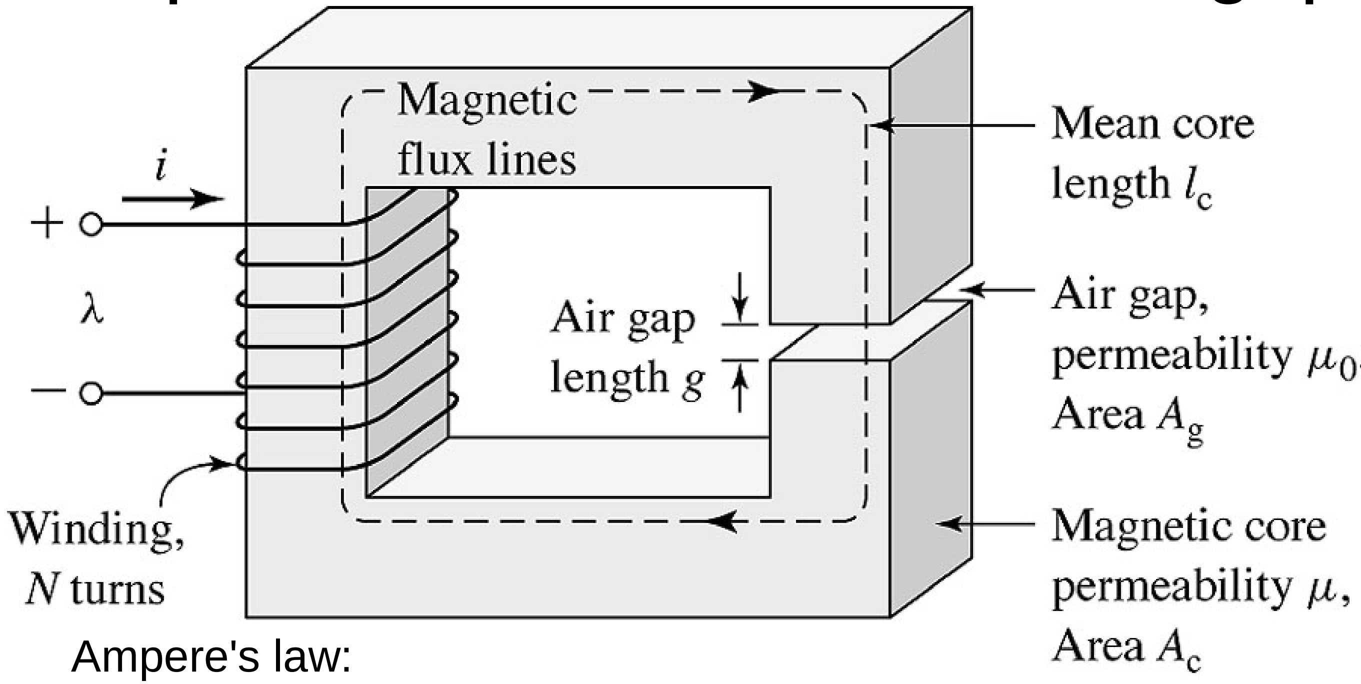
$$B = \mu_o H$$

Magnetic permeability of free space $\mu_o = 4\pi \times 10^{-7}$ Henry/meter.

In magnetic material, magnetic permeability is higher than μ_o:

$$B = \mu H$$

Example: Flux in Inductor with Airgap



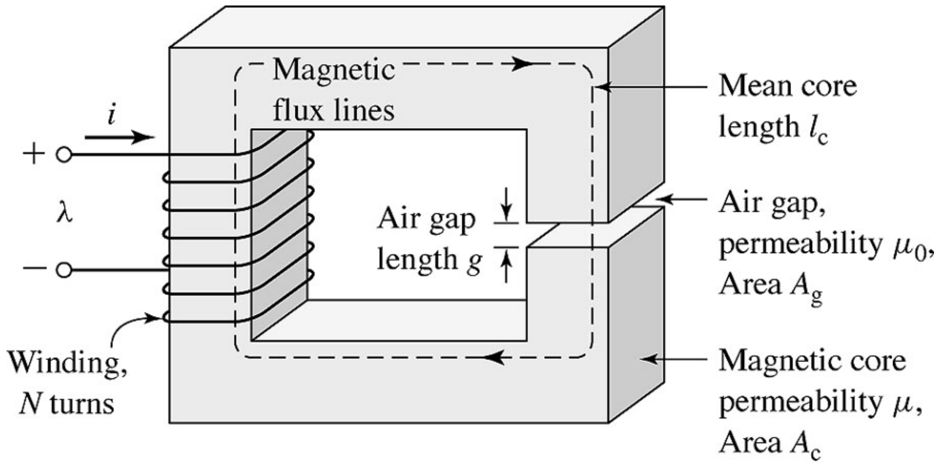
Ampere's law:

$$\oint H \cdot dl = \int_S J \cdot dA \Rightarrow H_c l_c + H_g g = NI$$

Let's use constitutive relationships:

$$\frac{B_c}{\mu_c} l_c + \frac{B_g}{\mu_o} g = NI$$

Example: Flux in Inductor with Airgap



$$B_c = \frac{\Phi}{A_c}$$

$$B_g = \frac{\Phi}{A_c}$$

Put this into previous expression:

$$NI = \Phi\left(\frac{l_c}{\mu A_c} + \frac{g}{\mu_o A_g}\right)$$

Solve for flux:

$$\Phi = \frac{NI}{\left(\frac{l_c}{\mu A_c} + \frac{g}{\mu_o A_g}\right)}$$

If $g/\mu_o A_g >> l_c/\mu A_c$,

$$\Phi \approx \frac{NI}{\frac{g}{\mu_o A_g}}$$

Magnetic Circuits

- Use Ohm's law analogy to model magnetic circuits

$$V \Leftrightarrow NI$$

$$I \Leftrightarrow \Phi$$

$$R \Leftrightarrow \Re$$

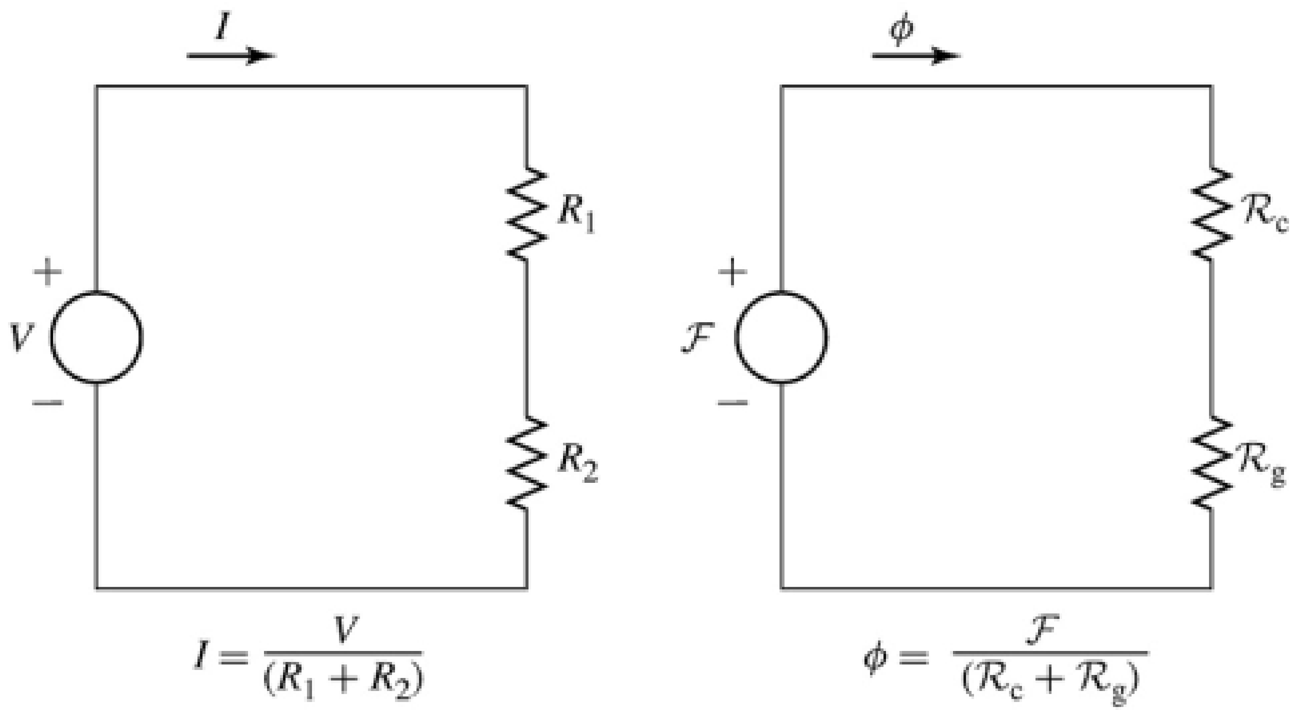
- Use magnetic "reluctance" instead of resistance

$$R = \frac{l}{\sigma A} \Leftrightarrow \Re = \frac{l}{\mu A}$$

- This is a <u>very</u> powerful method to get approximate answers in magnetic circuits

Magnetic-Electric Circuit Analogy

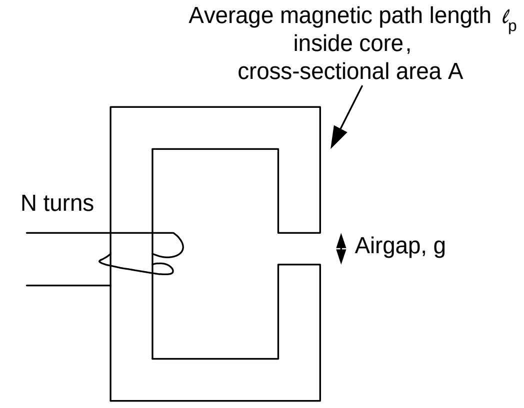
- In an electric circuit, voltage V forces current I to flow through resistances R
- In a magnetic circuit, MMF NI forces flux Φ to flow through reluctances $\Re$



C-Core with Gap --- Using Magnetic Circuits

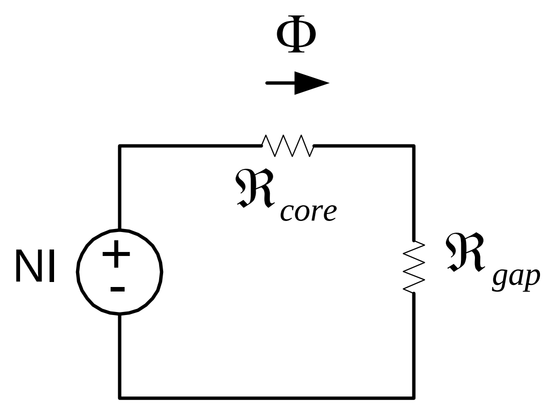
- Flux in the core is easily found by:

$$\Phi = \frac{NI}{\Re_{core} + \Re_{gap}} = \frac{NI}{\dfrac{l_p}{\mu_c A_c} + \dfrac{g}{\mu_o A_c}}$$



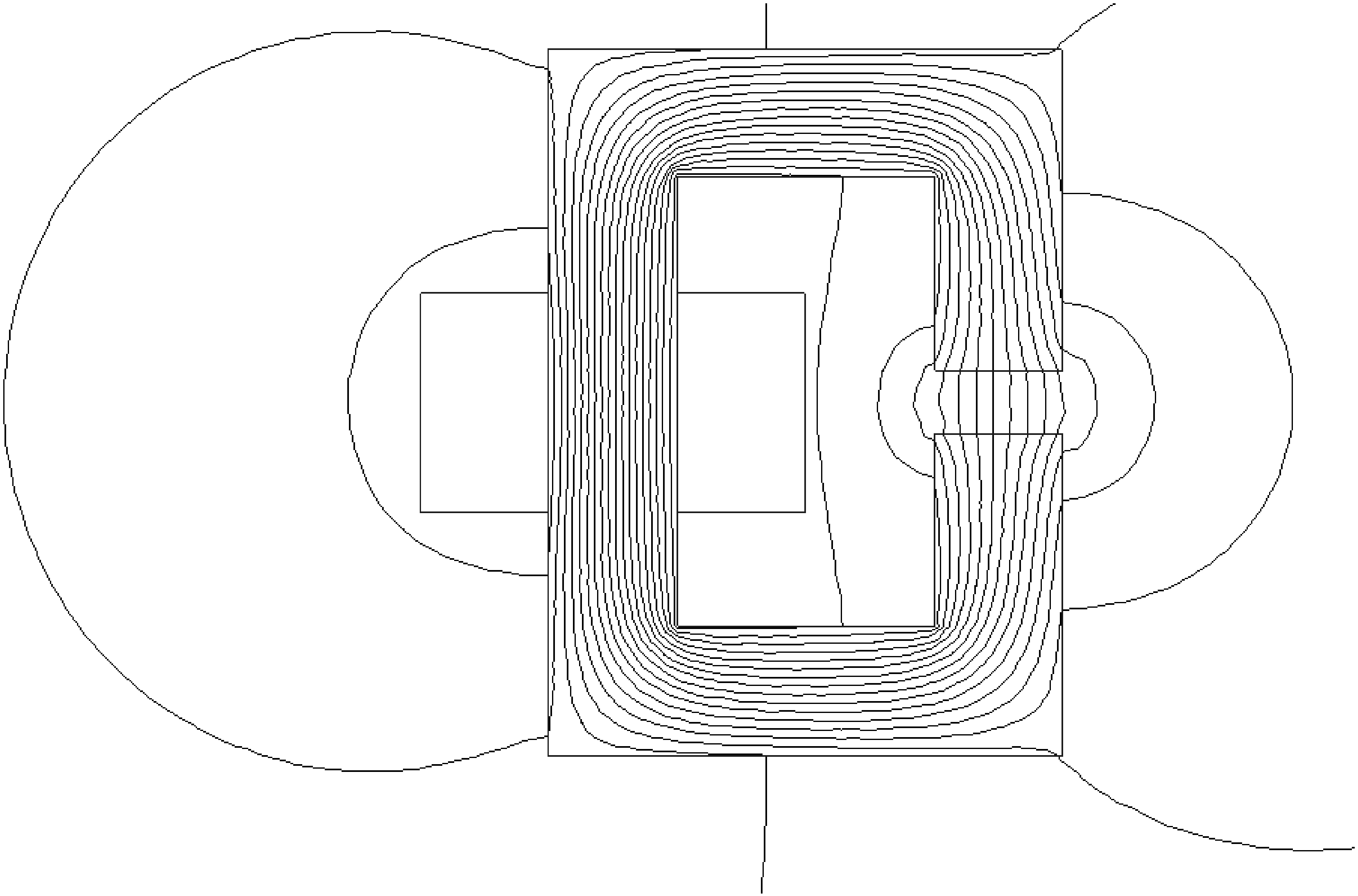
- Now, note what happens if $g/\mu_o >> l_p/\mu_c$: The flux in the core is now approximately independent of the core permeability, as:

$$\Phi \approx \frac{NI}{\Re_{gap}} \approx \frac{NI}{\dfrac{g}{\mu_o A_c}}$$



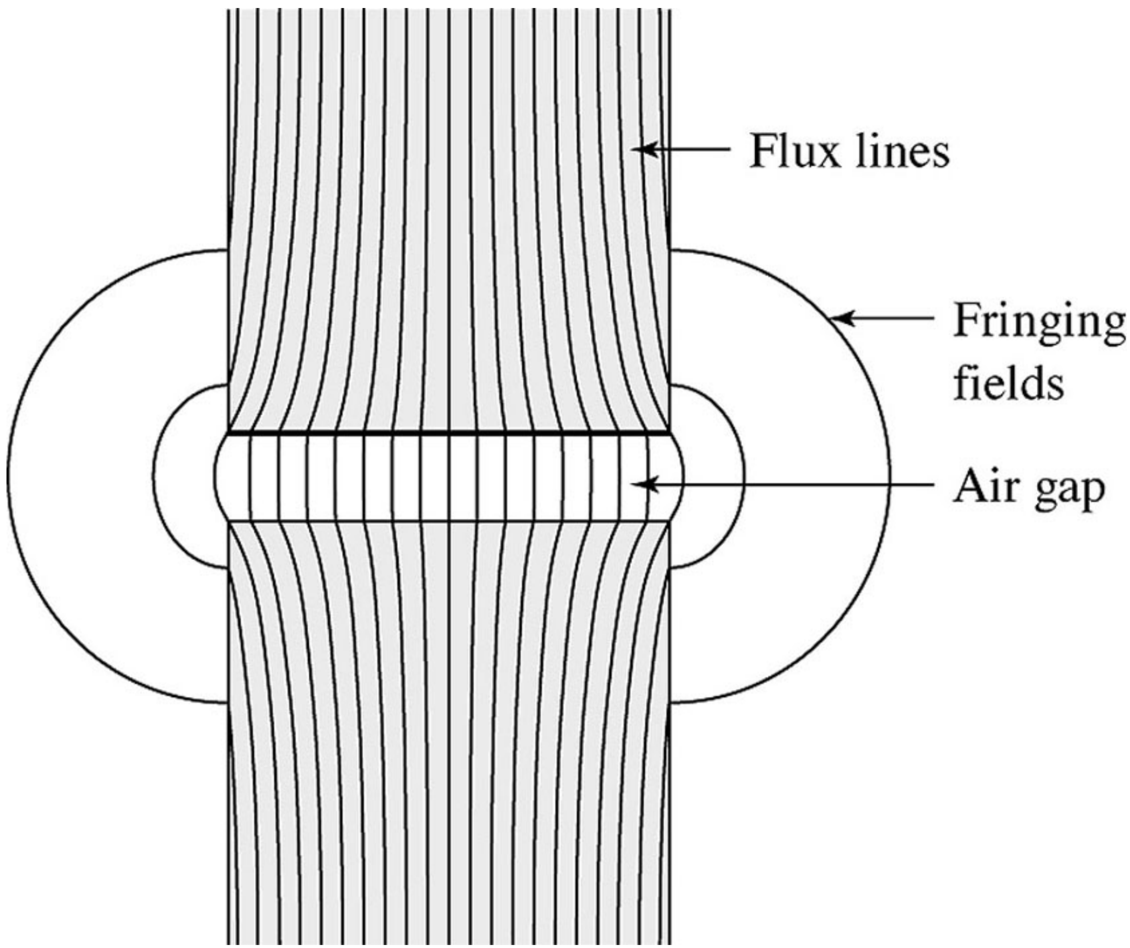
- Inductance: $L = \dfrac{N\Phi}{I} \approx \dfrac{N^2}{\Re_{gap}} \approx \dfrac{N^2}{\dfrac{g}{\mu_o A_c}}$

C-Core with Gap --- FEA



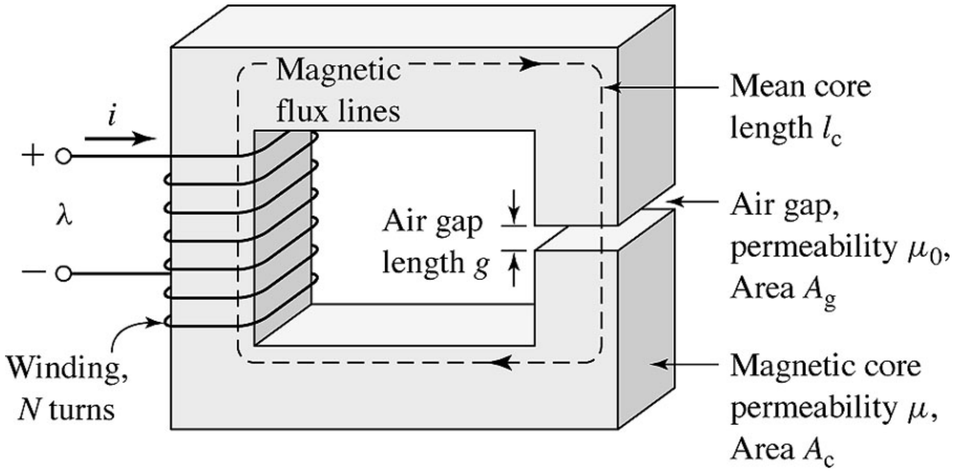
Fringing Fields in Airgap

- If fringing is negligible, $A_c = A_g$



Example: C-Core with Airgap

- *Fitzgerald*, Example 1.1; with B_c = 1.0T, find reluctances, flux and coil current



$A_c = A_g = 9\ \text{cm}^2$

$g = 0.05$ cm

$l_c = 30$ cm

$N = 500$

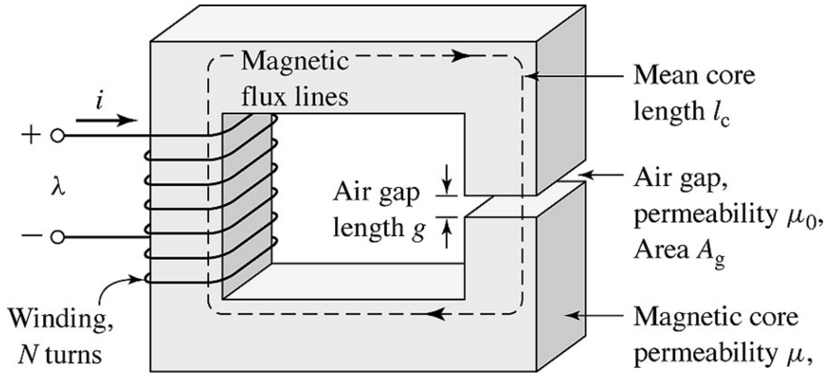
$\mu_r = 70{,}000$

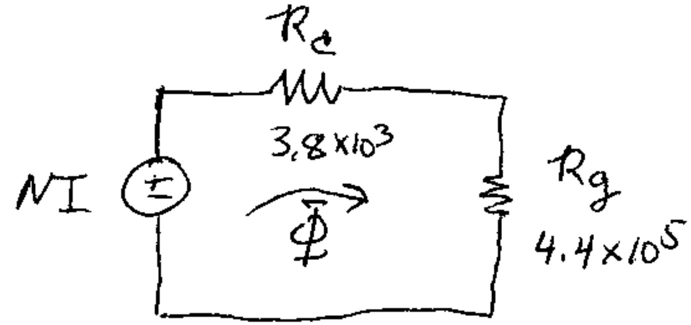
$$\Re_c = \frac{l_c}{A_c \mu_c} = \frac{0.3}{(70{,}000)(4\pi\times 10^{-7})(9\times 10^{-4})} = 3.8\times 10^3\ \frac{A-turns}{Wb}$$

$$\Re_g = \frac{g}{A_g \mu_o} = \frac{5\times 10^{-4}}{(4\pi\times 10^{-7})(9\times 10^{-4})} = 4.4\times 10^5\ \frac{A-turns}{Wb}$$

Note that $R_g >> R_c$

Example: C-Core with Airgap





Flux:

$$\Phi = B_c A_c = (1.0)(9\times10^{-4}) = 9\times10^{-4}\ Wb$$

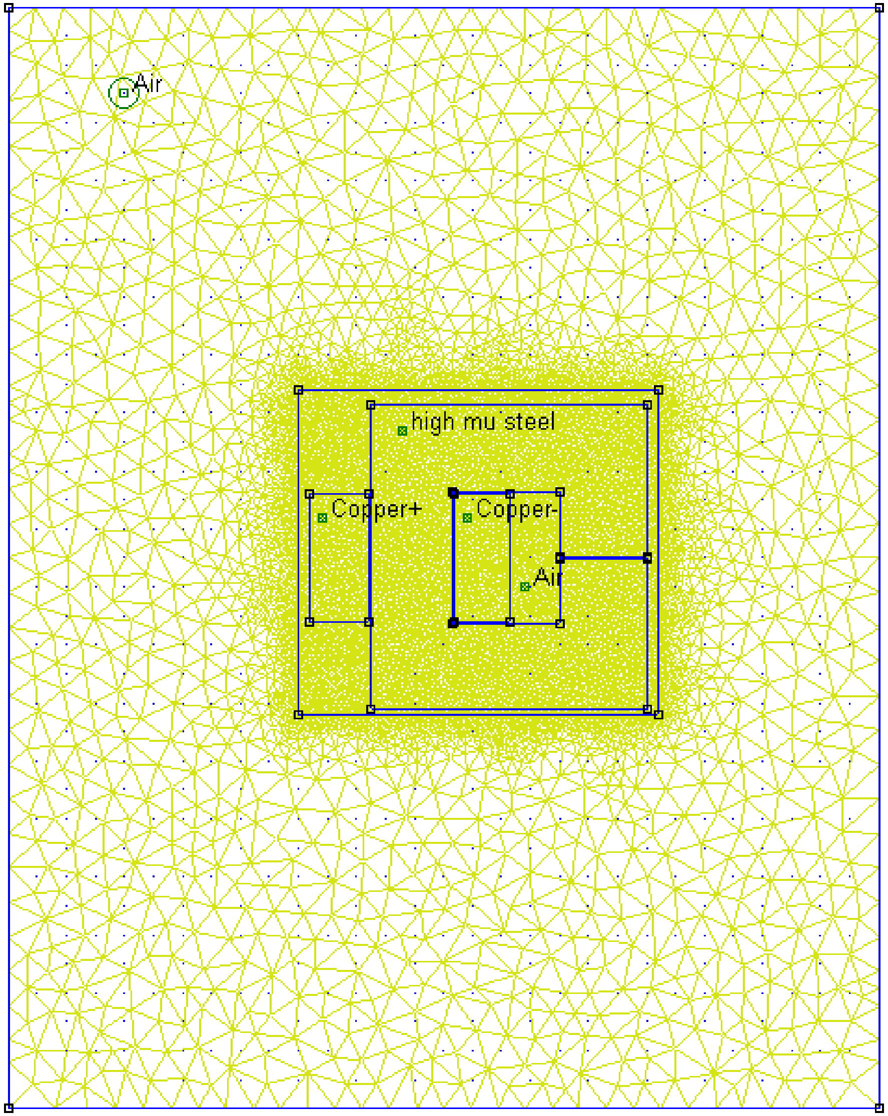
Coil current:

$$NI = \Phi(\Re_c + \Re_g)$$

$$\Rightarrow I = \frac{\Phi(\Re_c + \Re_g)}{N} = \frac{(9\times10^{4})(4.46\times10^{5})}{500} = 0.8\ A$$

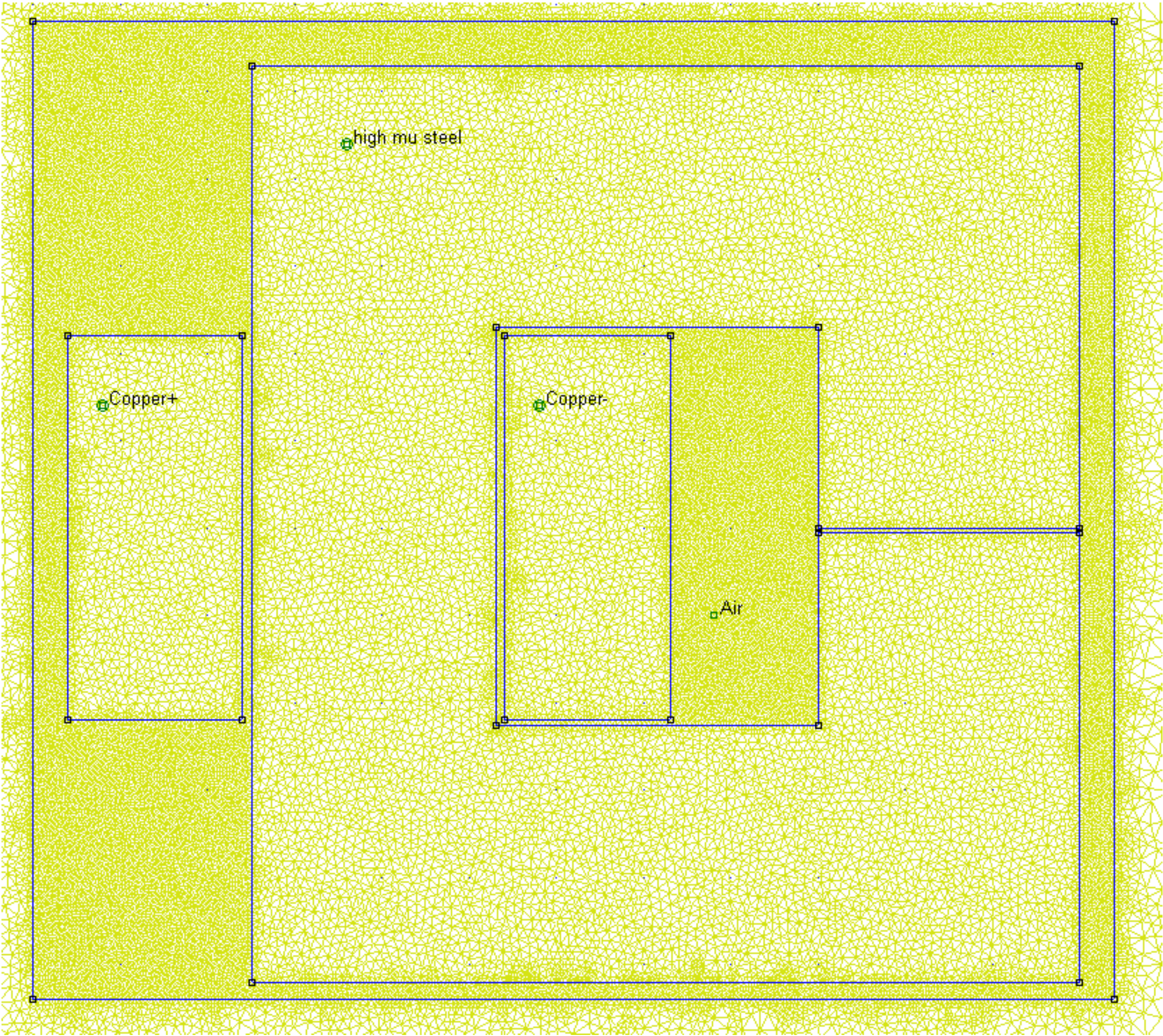
Example: C-Core with Airgap --- FEA

- NI = 400 A-turns



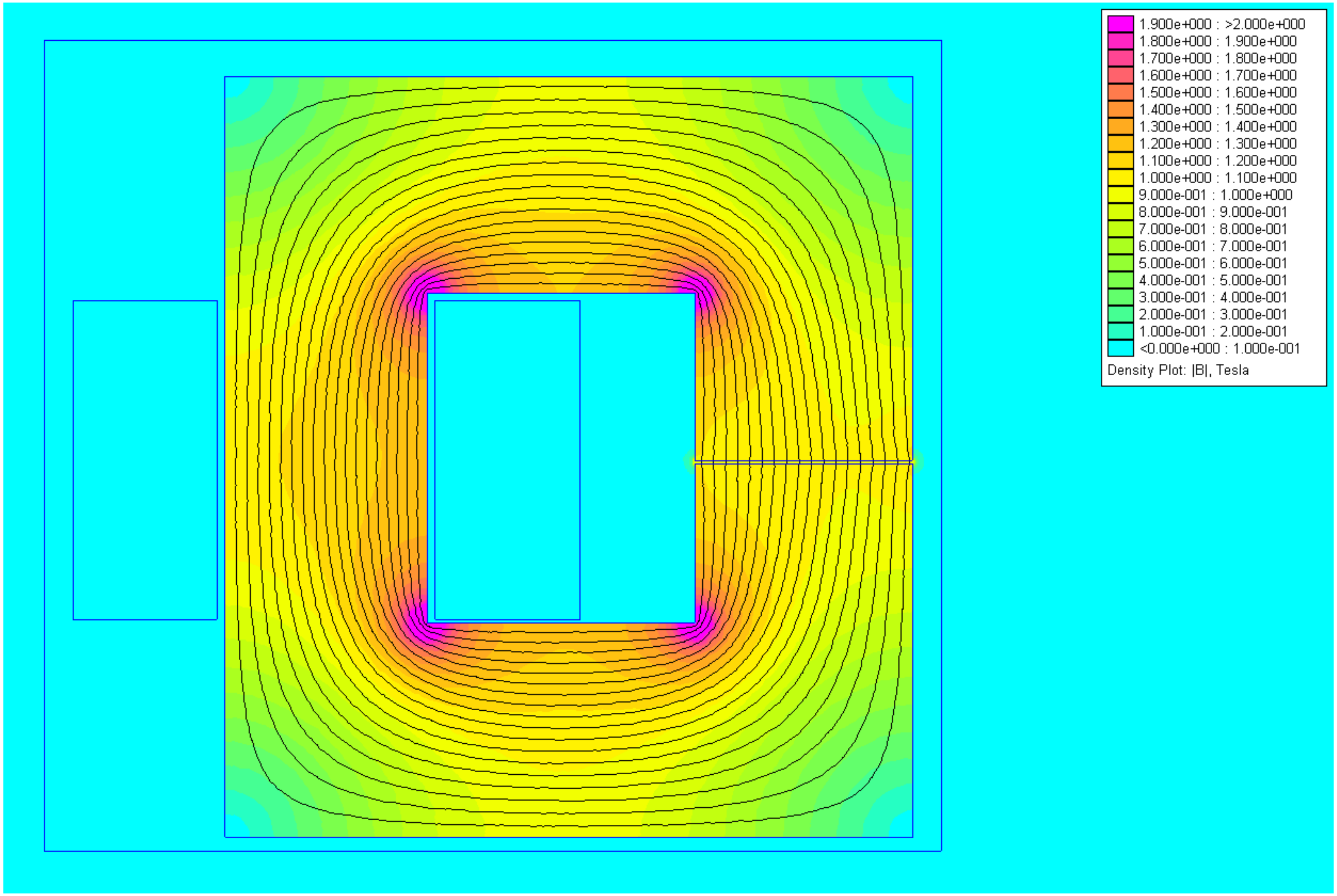
Example: C-Core with Airgap --- FEA

- NI = 400 A-turns, close up near the core

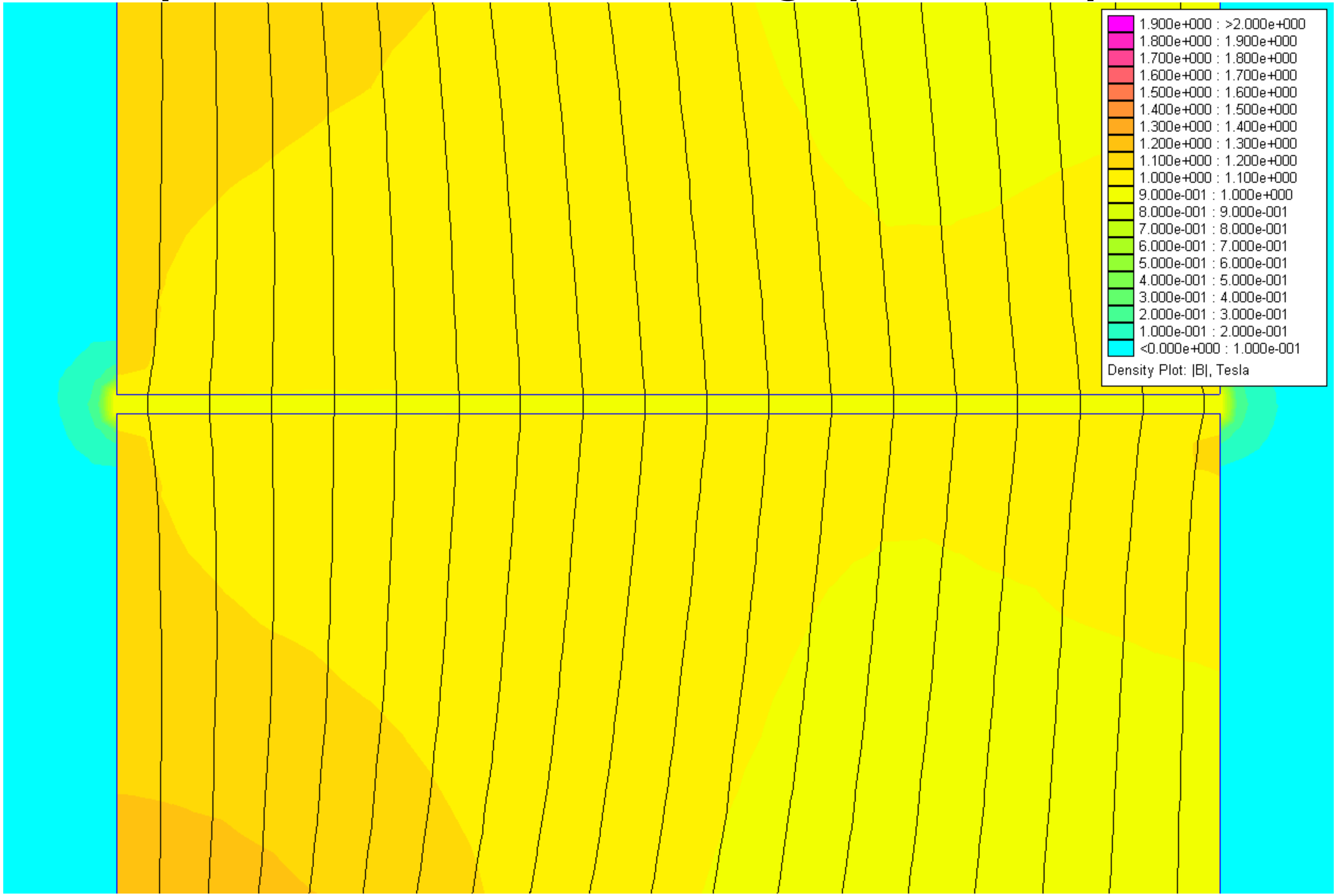


Example: C-Core with Airgap --- FEA Result

- Flux density in the core is approximately 1 Tesla

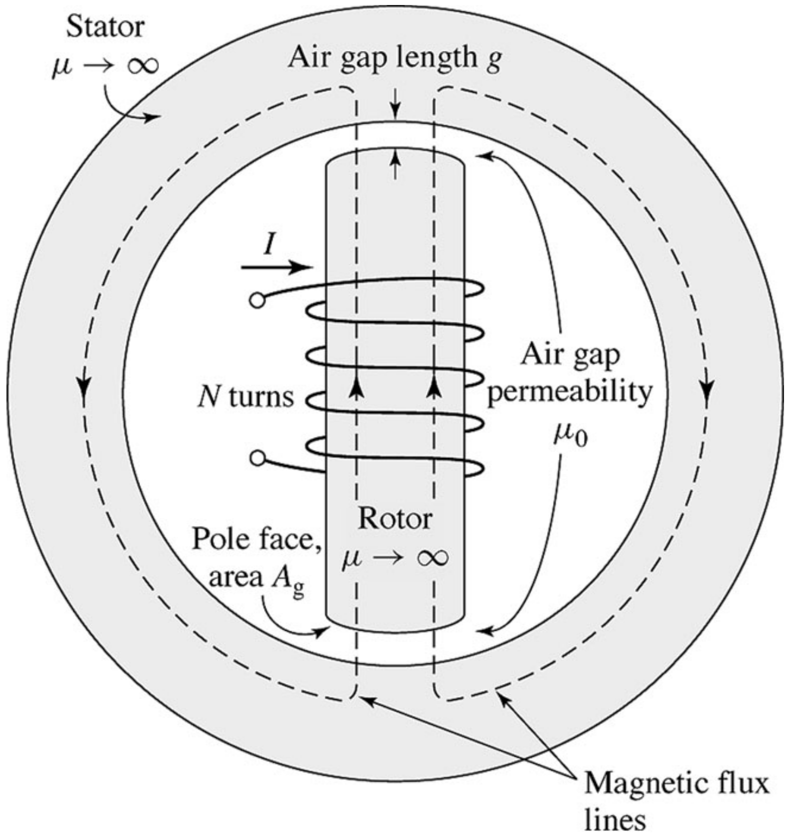


Example: C-Core with Airgap --- Gap Detail

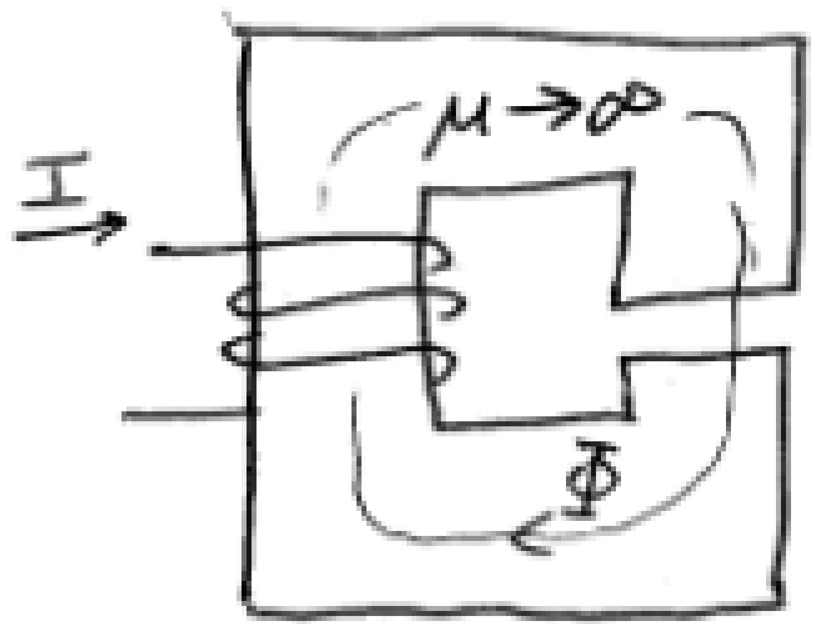


Example: Simple Synchronous Machine

- *Fitzgerald*, Example 1.2
- Assuming $\mu \rightarrow \infty$, find airgap flux Φ and flux density B_g
  Assume I = 10A, N = 1000 turns, g = 1 cm and A_g = 2000 cm^2



Aside: What Does Infinite Core μ Imply?



In core:

$$\Phi_c = B_c A_C$$

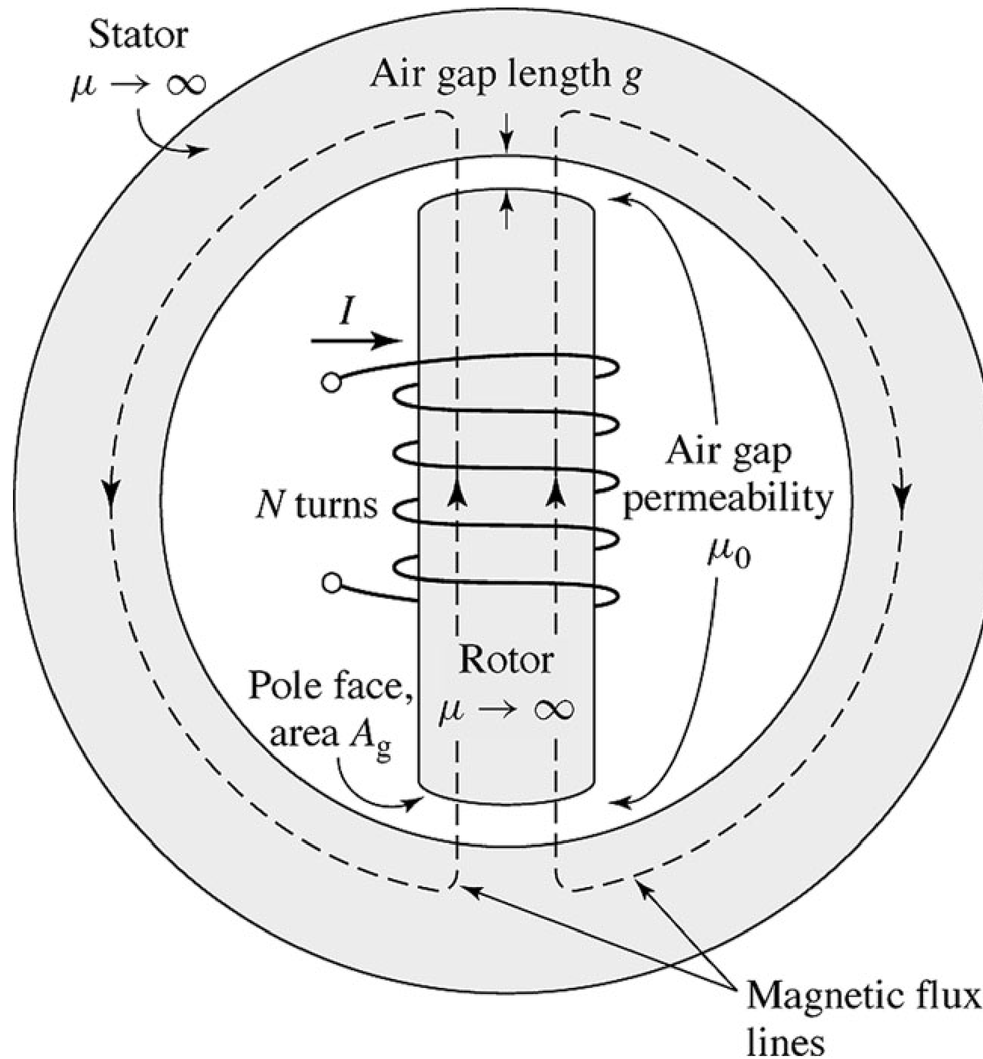
In airgap:

$$\Phi_g = B_g A_g$$

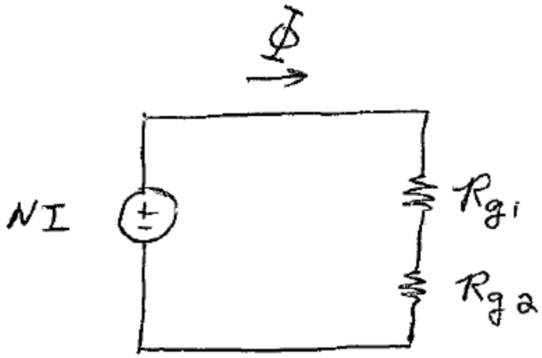
In core, B_c is finite; this means that if $\mu \rightarrow \infty$, then $H_c \rightarrow 0$ for finite B_c. Also, infinite μ implies zero reluctance

Example: Simple Synchronous Machine

- Some initial thoughts, before doing any equations:
  - By symmetry, airgap flux and flux density are the same in both gaps
  - Since permeability is infinite, H inside steel is zero



Example: Simple Synchronous Machine

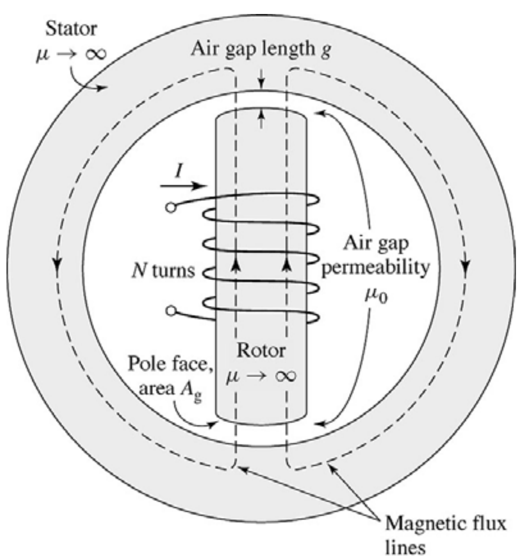


Reluctances:

$$\Re_{g1} = \Re_{g2} = \frac{g}{\mu_o A_g} = \frac{(0.01)}{(4\pi \times 10^{-7})(2000)(0.01^2)} = 39789\ \frac{A-turns}{Wb}$$

Flux:

$$\Phi = \frac{NI}{\Re_{g1} + \Re_{g2}} = \frac{(1000)(10)}{(2)(39789)} = 0.126\ Wb$$



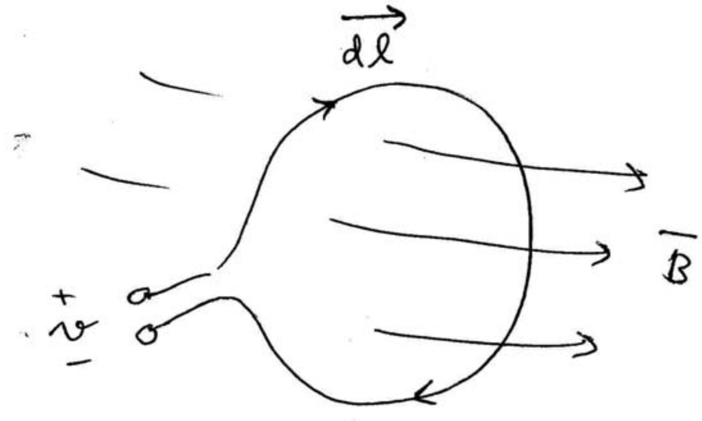
Magnetic flux density:

$$B = \frac{\Phi}{A_g} = \frac{0.126\ Wb}{(2000)(0.01^2)} = 0.63\ \frac{Wb}{m^2} = 0.63\ T$$

Flux Linkage, Voltage and Inductance

- By Faraday's law, changing magnetic flux density creates an electric field (and a voltage)

$$\oint_C \vec{E} \cdot d\vec{l} = -\frac{d}{dt}\int_S \vec{B} \cdot d\vec{A}$$



- Induced voltage:

$$v = N\frac{d\Phi}{dt} = \frac{d\lambda}{dt}$$

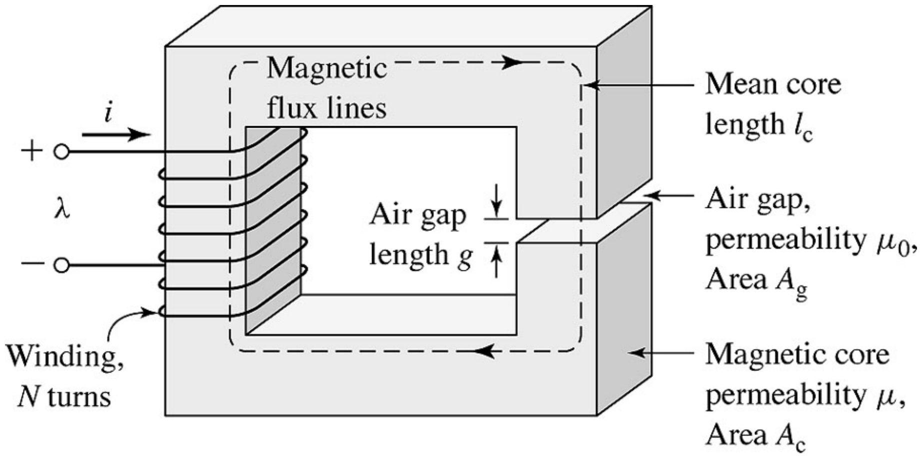
λ = "flux linkage" = $N\Phi$

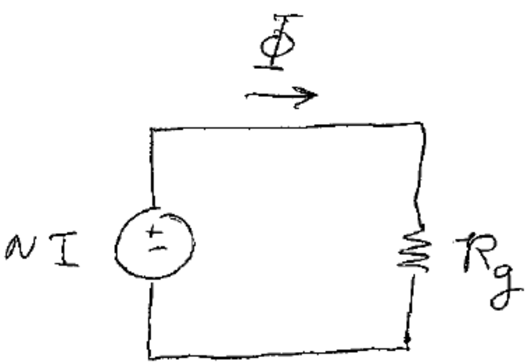
Inductance relates flux linkage to current

$$L = \frac{\lambda}{I}$$

Finding Inductance Using Magnetic Circuits

- Let's at first assume infinite core permeability; this means that the reluctance of the core is zero
- Note that inductance always scales as N^2 (why is that?)



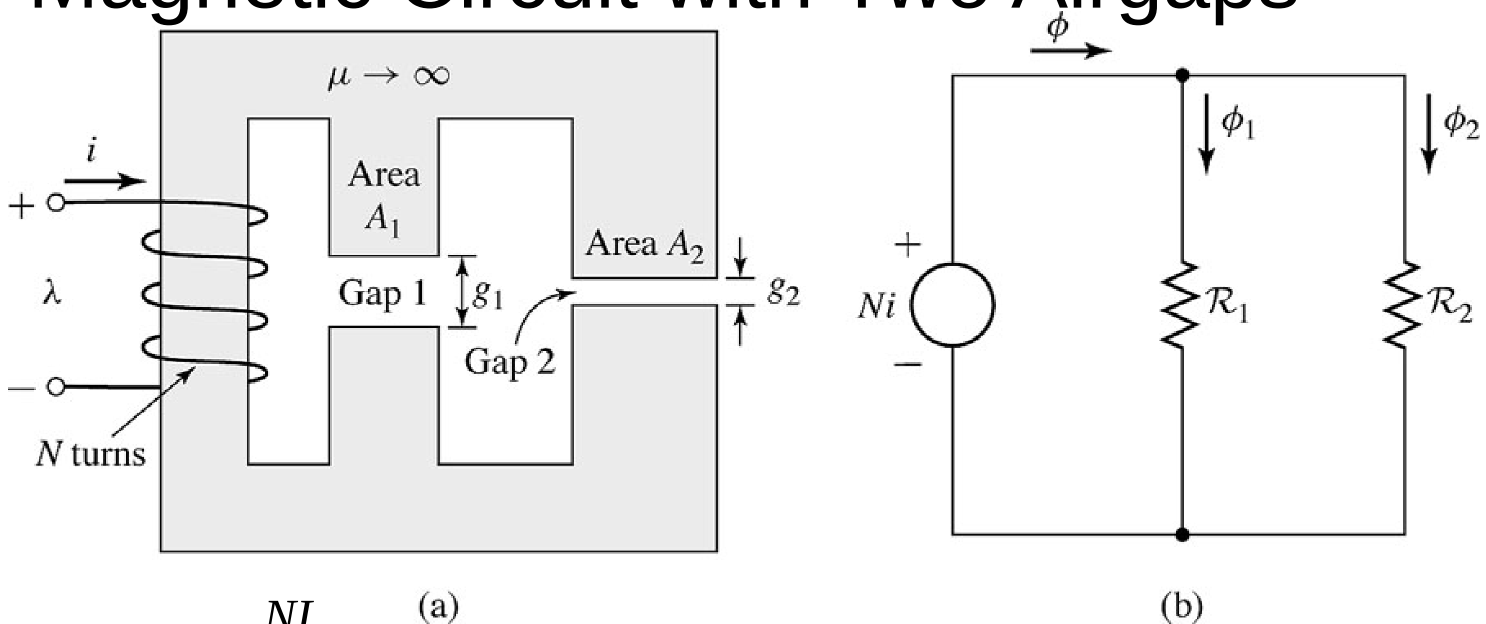


$$\Phi \approx \frac{NI}{\dfrac{g}{\mu_o A_g}}$$

$$\lambda \approx N\Phi = \frac{\mu_o A_g N^2 I}{g}$$

$$L \approx \frac{\lambda}{I} = \frac{\mu_o A_g N^2}{g}$$

Magnetic Circuit with Two Airgaps



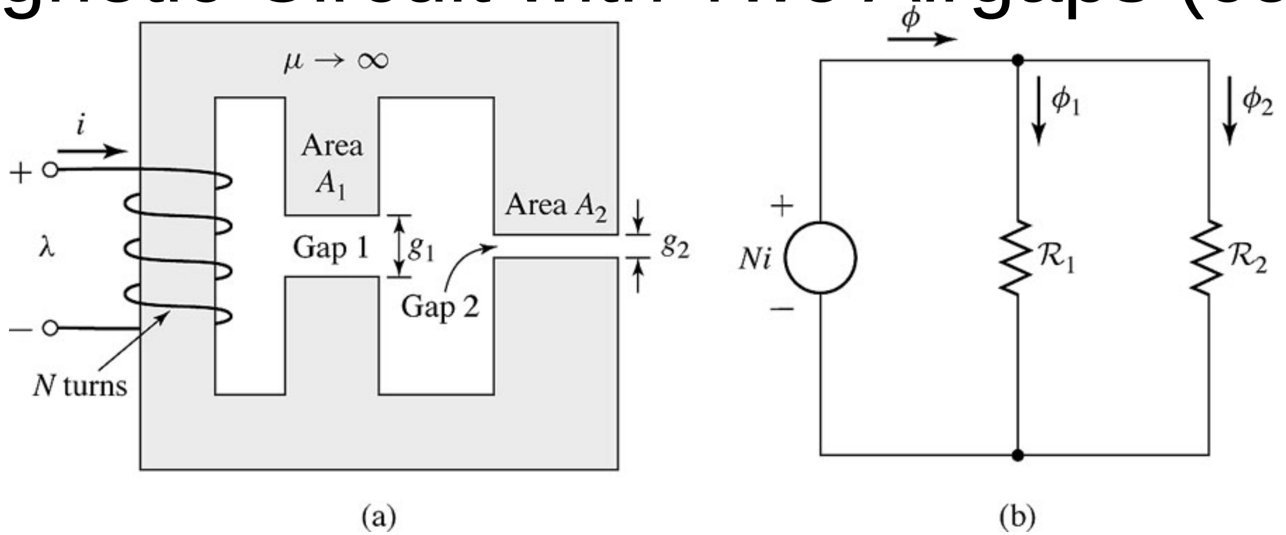
Total flux: $\Phi = \dfrac{NI}{\Re_1 \| \Re_2}$

Reluctances: $\Re_1 = \dfrac{g_1}{\mu_o A_1}$ and $\Re_2 = \dfrac{g_2}{\mu_o A_2}$

Inductance:

$$L = \frac{\lambda}{I} = \frac{N\Phi}{I} = N^2\left(\frac{\Re_1 + \Re_2}{\Re_1 \Re_2}\right) = \mu_o N^2\left(\frac{A_1}{g_1} + \frac{A_2}{g_2}\right)$$

Magnetic Circuit with Two Airgaps (cont.)



Flux in leg#1: $\Phi_1 = \dfrac{NI}{\Re_1} = \dfrac{\mu_o A_1 NI}{g_1}$

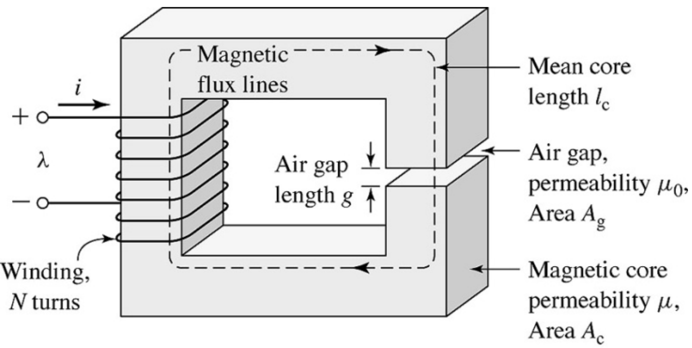
Flux in leg#2: $\Phi_2 = \dfrac{NI}{\Re_2} = \dfrac{\mu_o A_2 NI}{g_2}$

Flux density in leg#1: $B_1 = \dfrac{\Phi_1}{A_1} = \dfrac{\mu_o NI}{g_1}$

Flux density in leg#2: $B_2 = \dfrac{\Phi_2}{A_2} = \dfrac{\mu_o NI}{g_2}$

Inductance vs. Relative Permeability

- What happens if we assume finite core permeability?
- Reluctance of the core is now finite as well



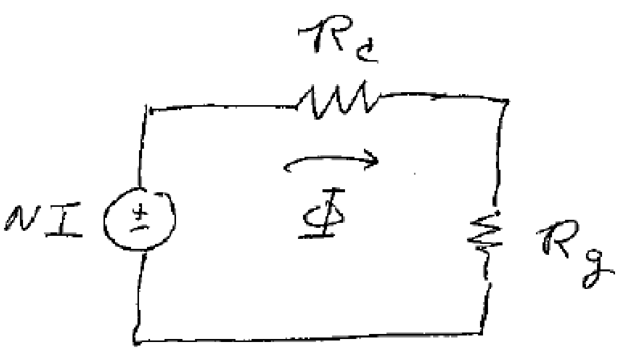
Reluctances: $\Re_c = \dfrac{l_c}{\mu A_c}$ and $\Re_g = \dfrac{g}{\mu_o A}$

Let's assume that $A_c = A_g = A$

Flux: $\Phi = \dfrac{NI}{\Re_c + \Re_g}$

Flux linkage: $\lambda = N\Phi = \dfrac{N^2 I}{\Re_c + \Re_g}$

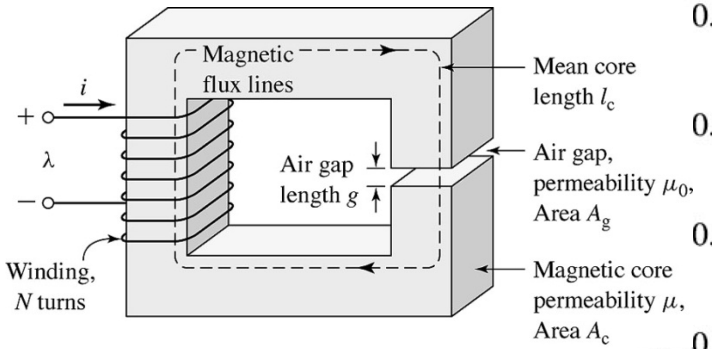
Inductance: $N = \dfrac{\lambda}{I} = \dfrac{N^2}{\Re_c + \Re_g} = \dfrac{N^2}{\left(\dfrac{l_c}{\mu A}\right) + \left(\dfrac{g}{\mu_o A}\right)} = \dfrac{N^2 \mu_o A}{g + \left(\dfrac{l_c \mu_o}{\mu}\right)}$

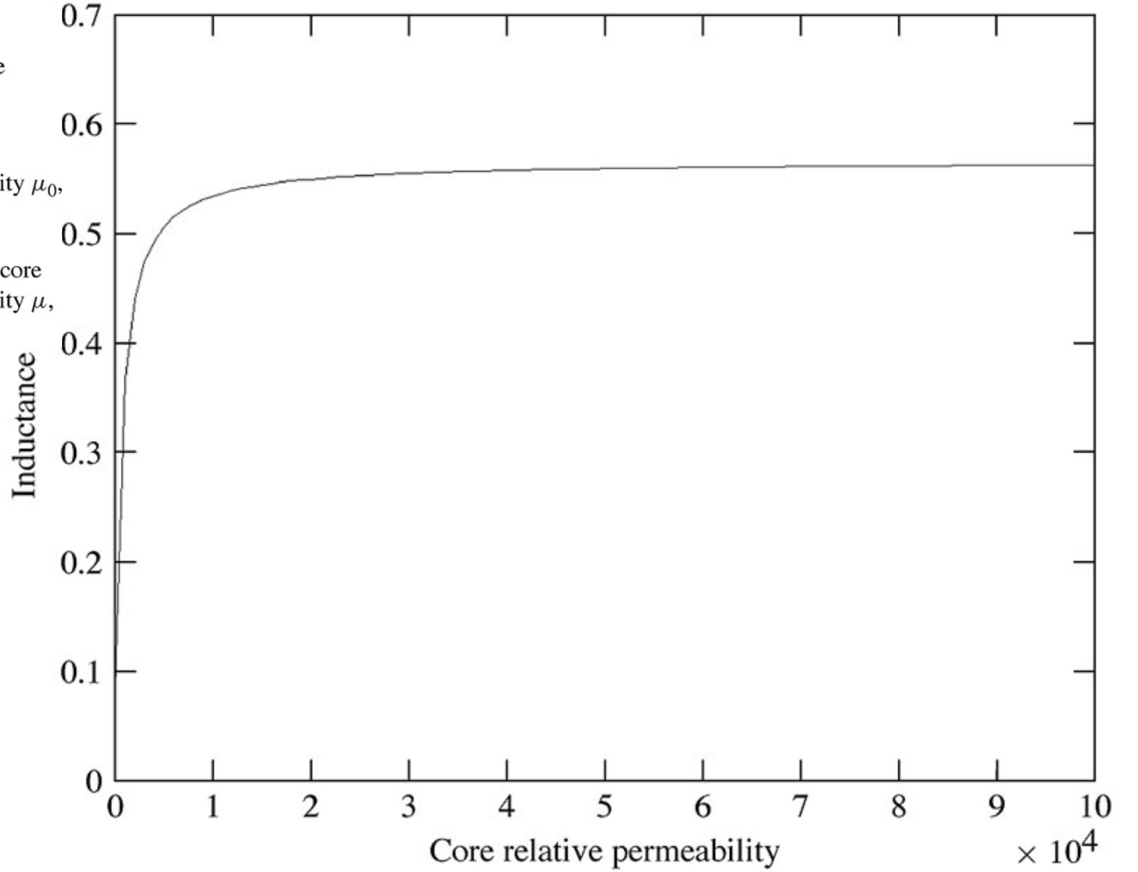


Inductance is independent of core permeability if:

$$g >> \frac{l_c \mu_o}{\mu}$$

Inductance vs. Relative Permeability

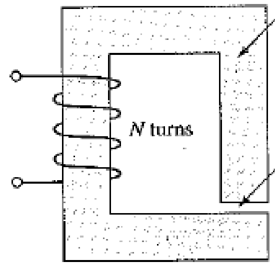




Example: Effects of Finite Permeability

- *Fitzgerald*, problem 1.5

Cross-sectional area $A_c = 1.8 \times 10^{-3}\ \text{m}^2$
Mean core length $l_c = 0.6$ m
Gap length $g = 2.3 \times 10^{-3}$ m
$N = 83$ turns



1.5 The magnetic circuit of Problem 1.1 has a nonlinear core material whose permeability as a function of B_m is given by

$$\mu = \mu_0 \left(1 + \frac{3499}{\sqrt{1 + 0.047(B_m)^{7.8}}} \right)$$

where B_m is the material flux density.

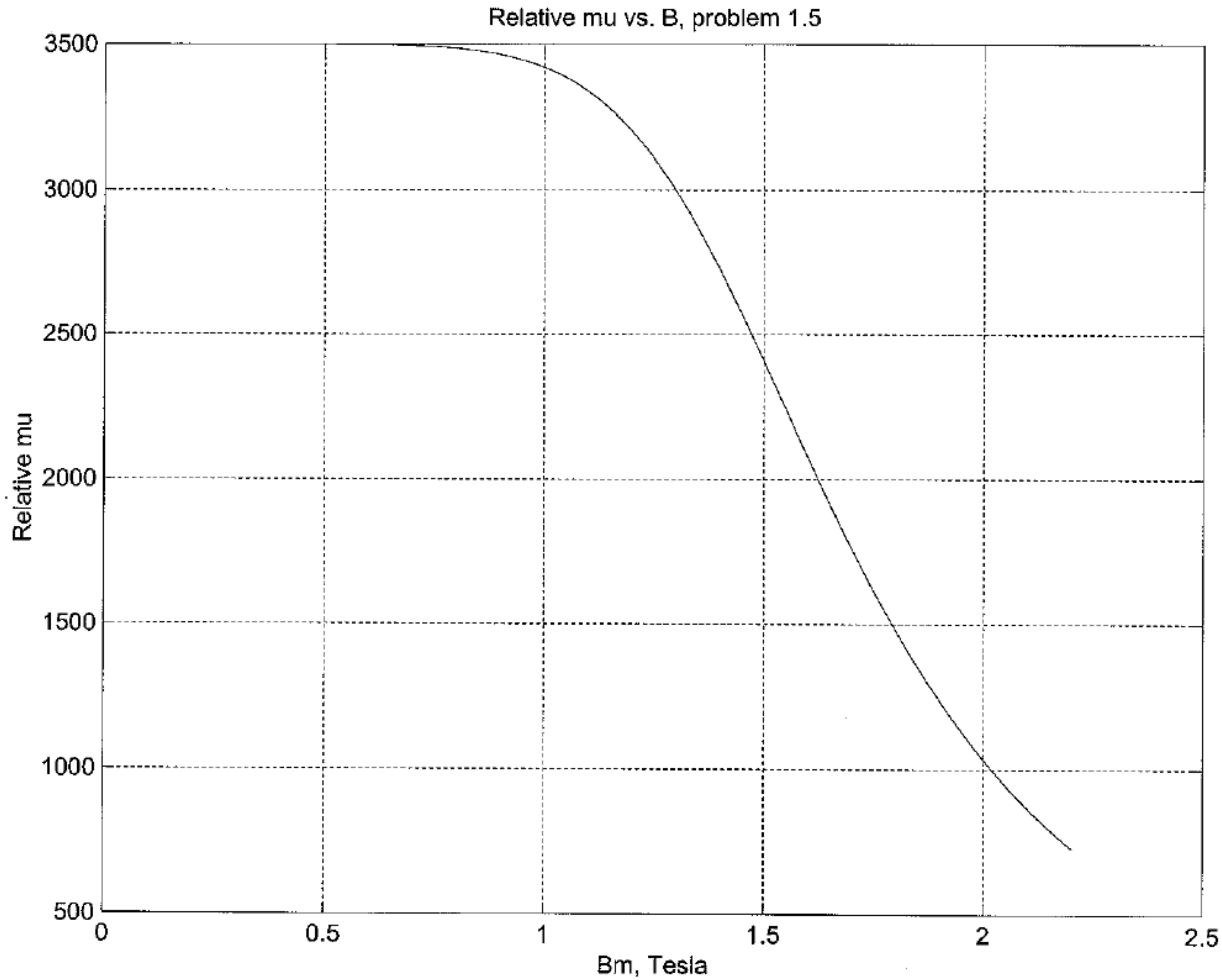
a. Using MATLAB, plot a dc magnetization curve for this material (B_m vs. H_m) over the range $0 \le B_m \le 2.2$ T.

b. Find the current required to achieve a flux density of 2.2 T in the core.

c. Again, using MATLAB, plot the coil flux linkages as a function of coil current as the current is varied from 0 to the value found in part (b).

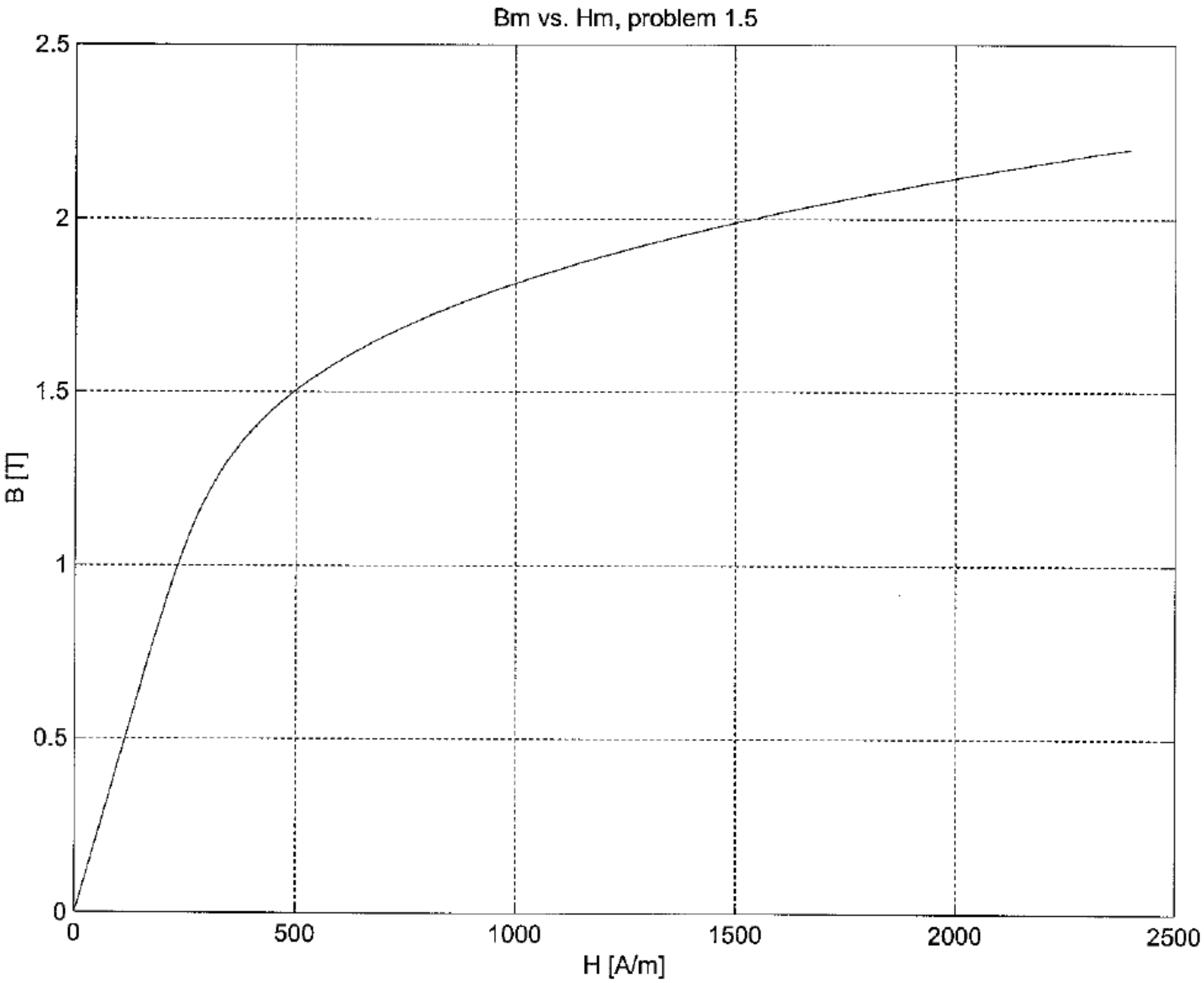
Example: Effects of Finite Permeability

- Relative μ vs. B



Example: Effects of Finite Permeability

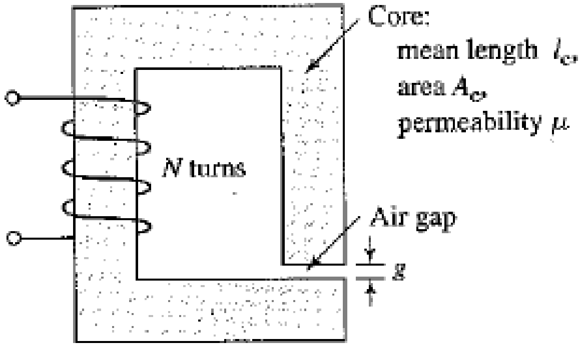
- B/H curve



Example: Effects of Finite Permeability

- Current calculation

Cross-sectional area $A_c = 1.8 \times 10^{-3}\ \mathrm{m}^2$
Mean core length $l_c = 0.6$ m
Gap length $g = 2.3 \times 10^{-3}$ m
$N = 83$ turns



From Ampere's law:

$$H_c l_c + H_g g = NI$$

In core:

$$H_c = \frac{B_c}{\mu}$$

In gap (let's assume $B_c = B_g = B$):

$$H_g = \frac{B}{\mu_o}$$

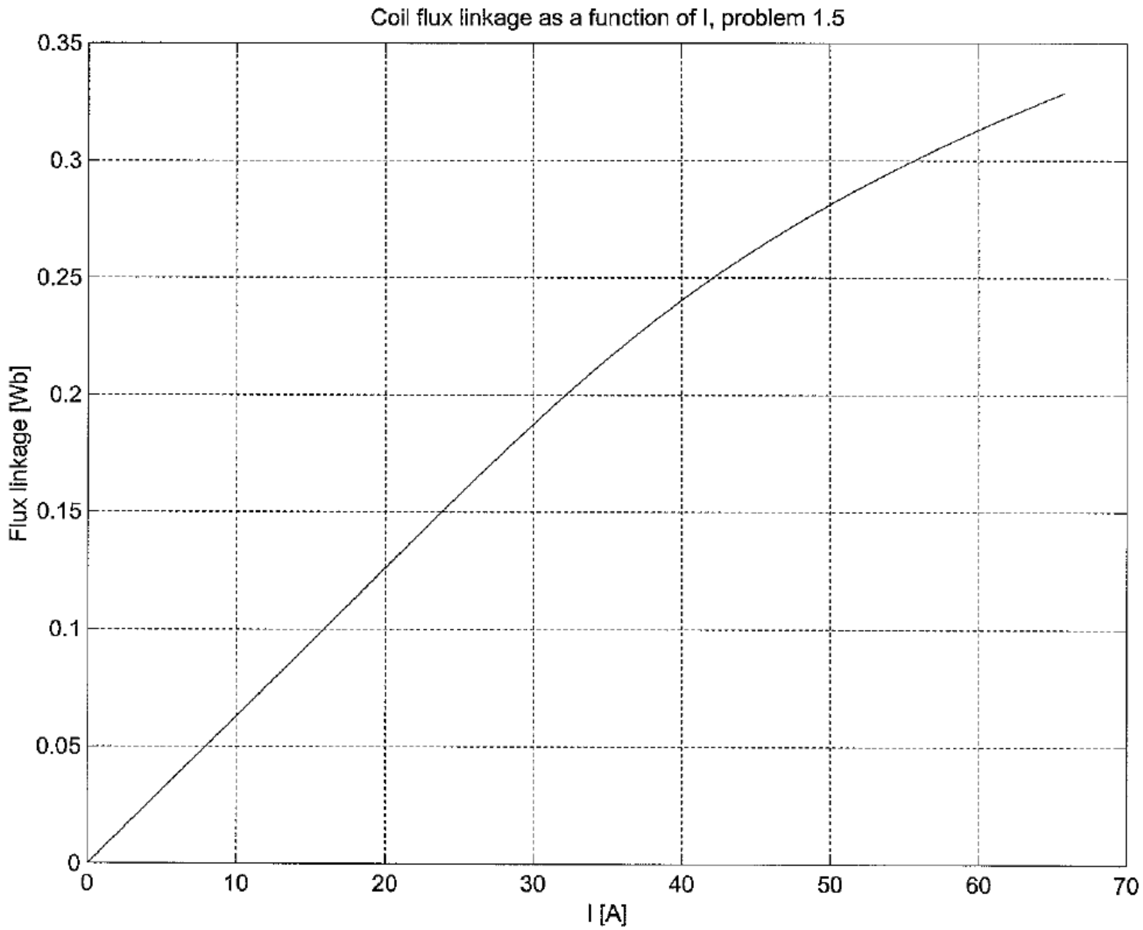
Put this back into Ampere's law:

$$\frac{Bl_c}{\mu} + \frac{Bg}{\mu_o} = NI$$

$$\therefore I = \left(\frac{B}{\mu_o N}\right)\left(\frac{l_c}{\mu_r} + g\right) = 65.8A$$

Example: Effects of Finite Permeability

- Coil flux linkage λ as a function of coil current
- Note that at low current, λ-I curve is linear, indicating constant inductance



Example: MATLAB Script

```
% Fitzgerald, problem 1.5
% Constants
munot=4*pi*1e-7;

%Part a
Bm=0:0.01:2.2;
mu=munot*(1+3499./(sqrt(1+0.047*Bm.^7.8)))
plot(Bm,mu/munot,'k')
xlabel('Bm, Tesla')
ylabel('Relative mu')
grid
title('Relative mu vs. B, problem 1.5')
mu_rel=mu(length(mu))/munot

figure
Hm=Bm./mu;
plot(Hm,Bm,'k')
xlabel('H [A/m]')
ylabel('B [T]')
grid;
title('Bm vs. Hm, problem 1.5')

% Part b
B=2.2;
N=83;
lc=0.6;
g=2.3e-3;
I=(B/(munot*N))*(lc/mu_rel + g)

% part c
Ac=1.8e-3;
figure
I=(Bm/(munot*N)).*(lc./(mu/munot)+g);
flux_linkage=N*Bm*Ac;
plot(I,flux_linkage);
xlabel('I [A]')
ylabel('Flux linkage [Wb]')
title('Coil flux linkage as a function of I, problem 1.5')
grid
```

Inductance and Energy

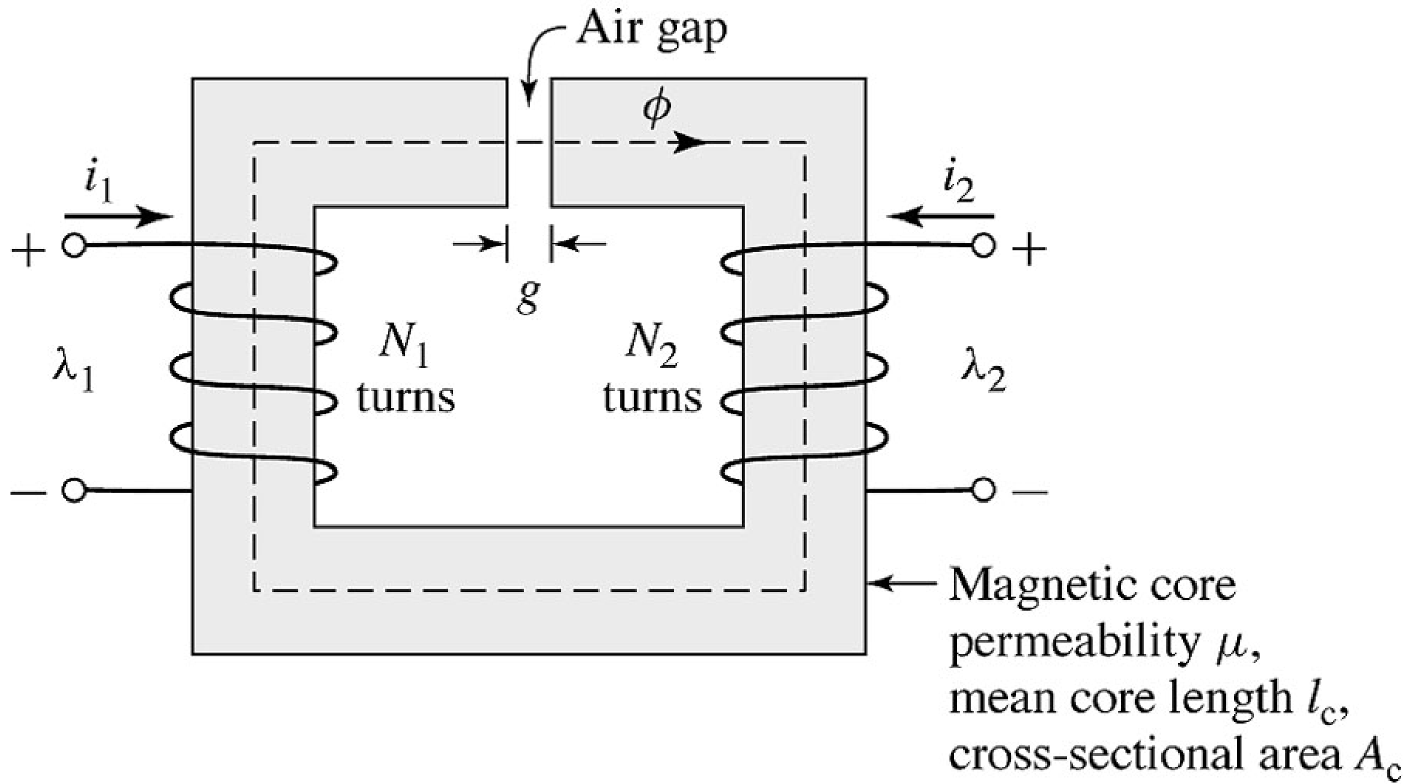
- Magnetic stored energy (in Joules) is:

$$W = \frac{1}{2} L I^2$$

- This is a good thing to remember

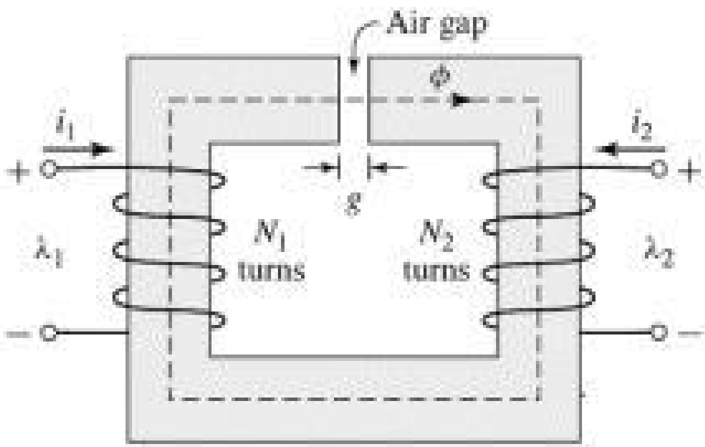
Magnetic Circuit with Two Windings

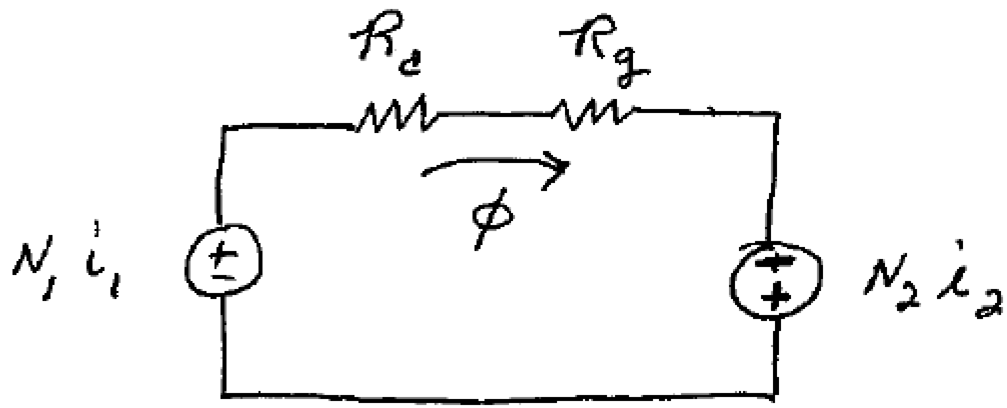
- Note that flux is the sum of flux due to i_1 and that due to i_2



Magnetic Circuit with Two Windings

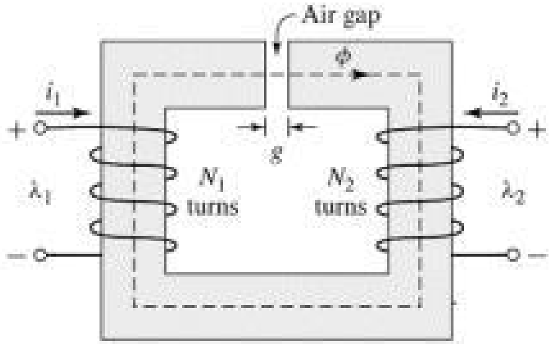
- Note that by the right-hand rule the flux due to i_1 and i_2 are additive given the current directions shown





Magnetic Circuit with Two Windings

- Note that by the right-hand rule the flux due to i_1 and i_2 are additive given the current directions shown



Flux: $\Phi = (N_1 i_1 + N_2 i_2)\left(\frac{\mu_o A_c}{g}\right)$

Flux linkage for coil #1:

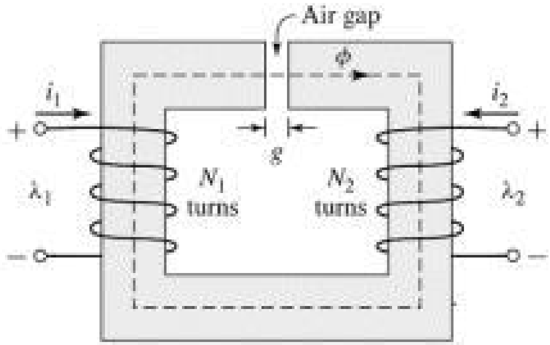
$$\lambda_1 = N_1 \Phi = N_1^2 i_1 \left(\frac{\mu_o A_c}{g}\right) + N_1 N_2 i_2 \left(\frac{\mu_o A_c}{g}\right)$$

We can write this as:

$$\lambda_1 = L_{11} i_1 + L_{12} i_2$$

L_{11} is “self inductance” of coil #1
L_{12} is “mutual inductance” between coils #1 and #2

Magnetic Circuit with Two Windings



Flux linkage for coil #2:

$$\lambda_2 = N_2\Phi = N_1 N_2 i_1\left(\frac{\mu_o A_c}{g}\right) + N_2^2 i_2\left(\frac{\mu_o A_c}{g}\right)$$

Or rewriting:

$$\lambda_2 = L_{21} i_1 + L_{22} i_2$$

Or, in matrix form:

$$\begin{bmatrix}\lambda_1 \\ \lambda_2\end{bmatrix} = \begin{bmatrix} L_{11} & L_{12} \\ L_{21} & L_{22}\end{bmatrix}\begin{bmatrix} i_1 \\ i_2\end{bmatrix}$$

“Soft” Magnetic Materials

- Materials with a small B/H curve, such as steels, etc.
- Much of the previous analysis assumed that steel had infinite permeability ($\mu \rightarrow \infty$) or that permeability was constant and large.
- However, soft magnetic materials exhibit both saturation and losses

B-H Curve and Saturation

- Definition of magnetic permeability: slope of B-H curve

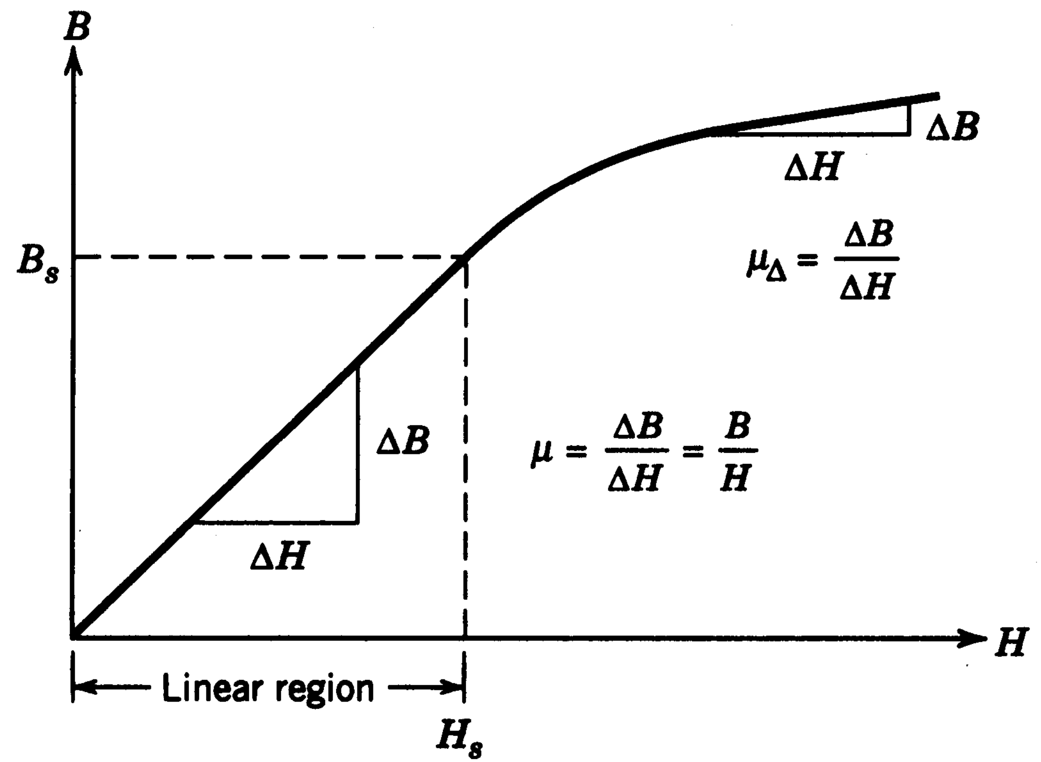
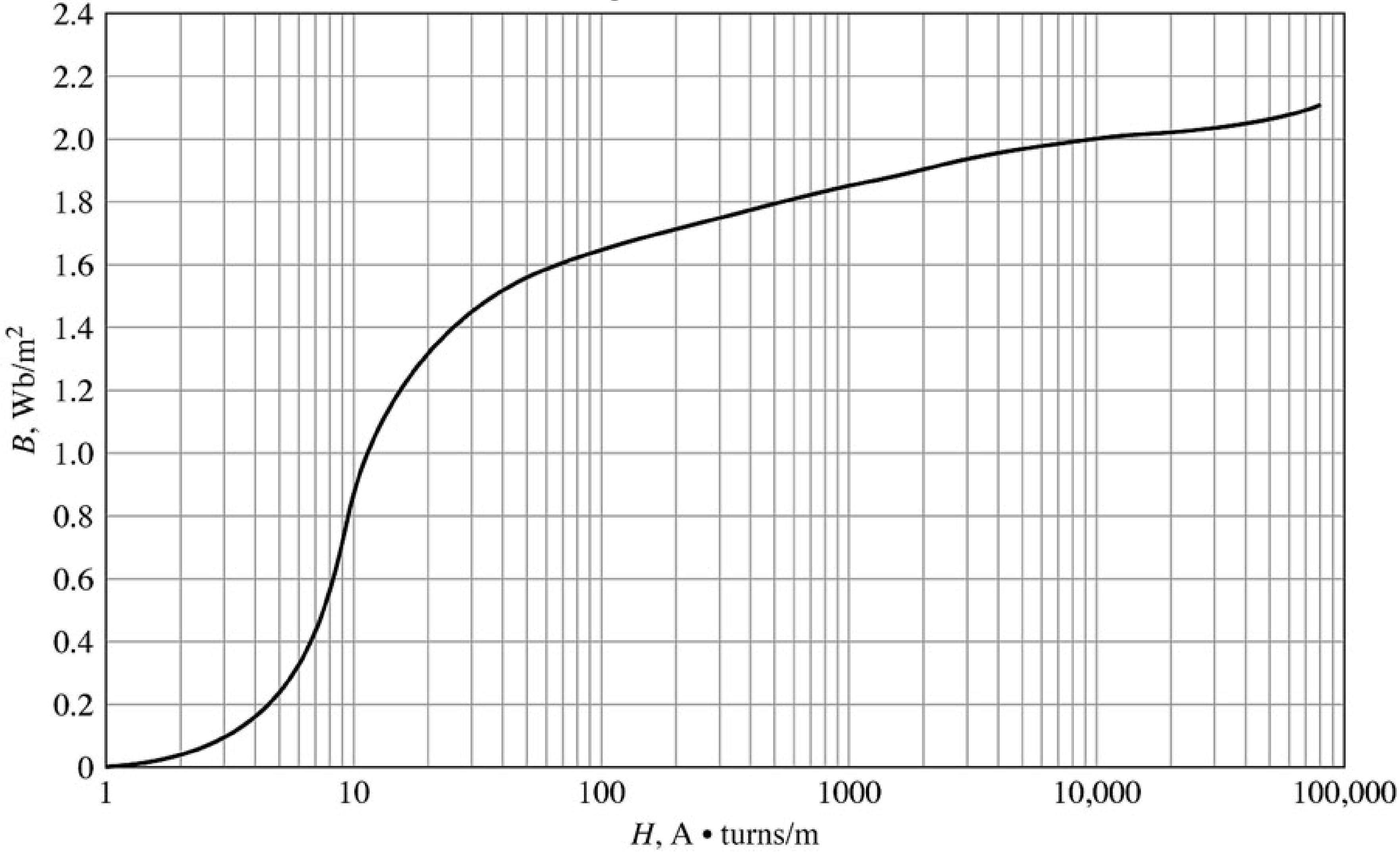


Figure 3-12 Relation between B- and H-fields.

Reference: N. Mohan, et. al., _Power Electronics Converters, Applications and Design_, Wiley, 2003, pp. 48

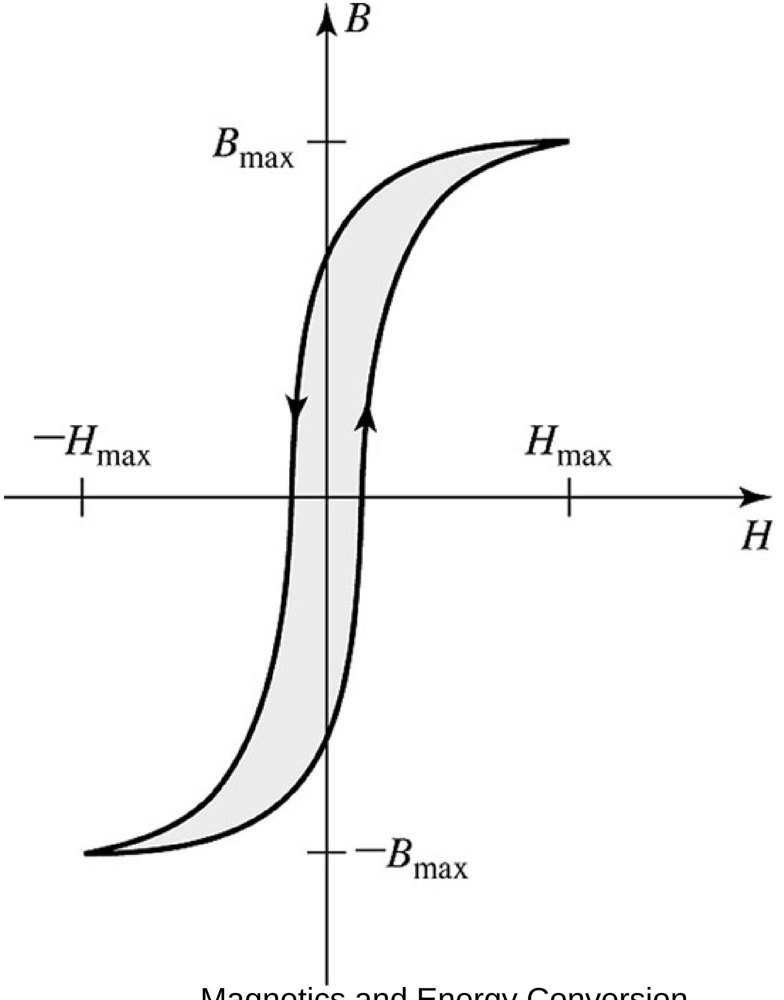
BH Curve for M-5 Steel

- Note horizontal scale is logarithmic



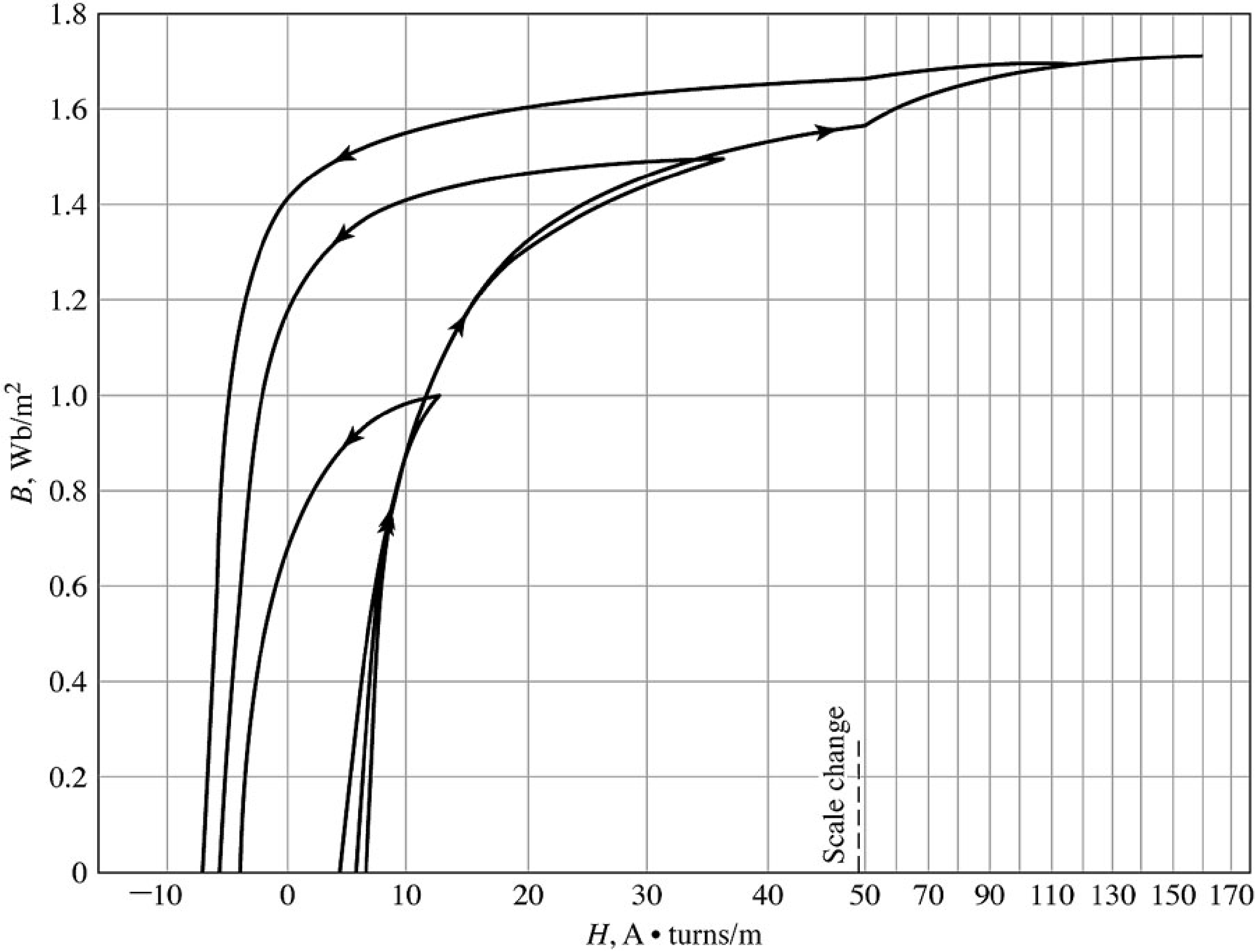
Hysteresis Loop

- Real-world magnetic materials have a “hysteresis loop”
- Hysteresis loss is proportional to shaded area



B-H Loop for M-5 Grain-Oriented Steel

- Only the top half of the loops shown for steel 0.012” thick



BH Curves for Various Soft Magnetic Materials

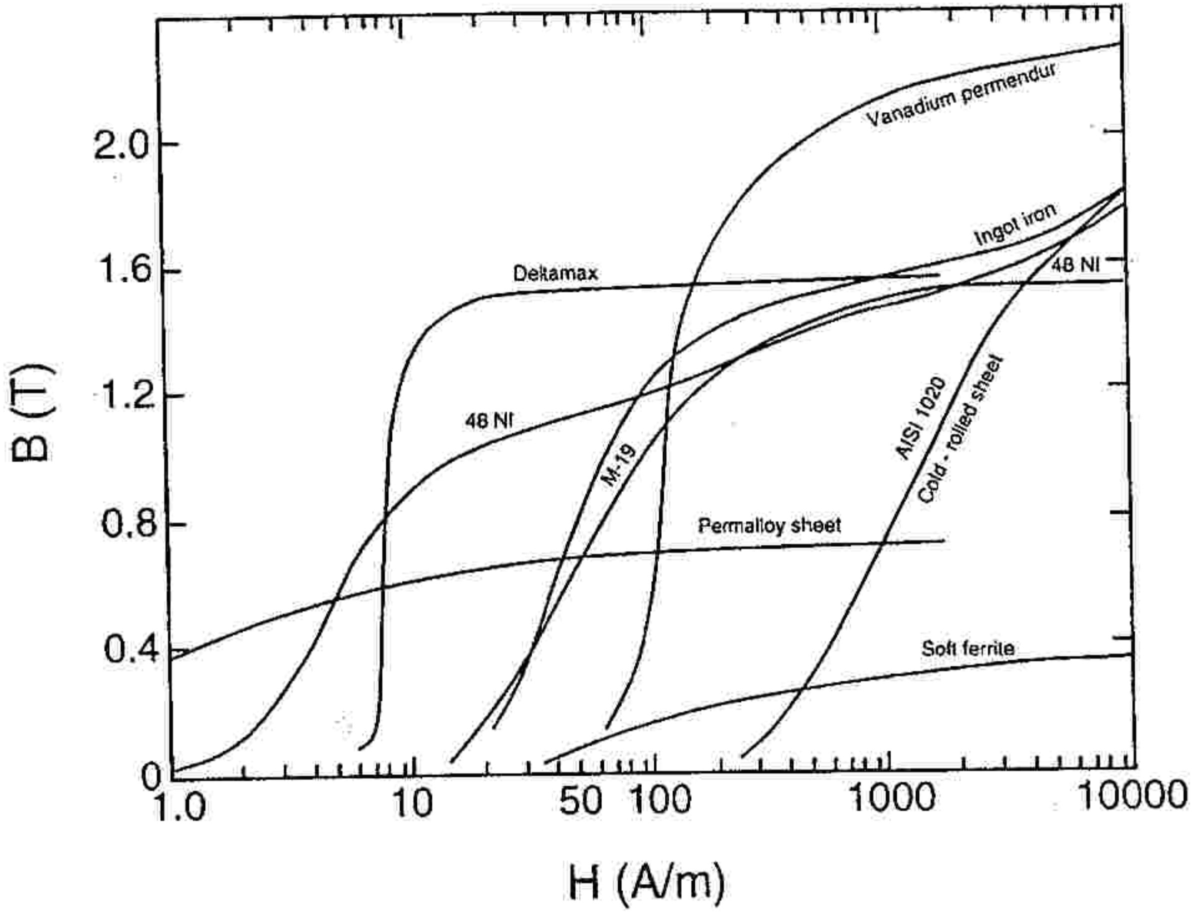
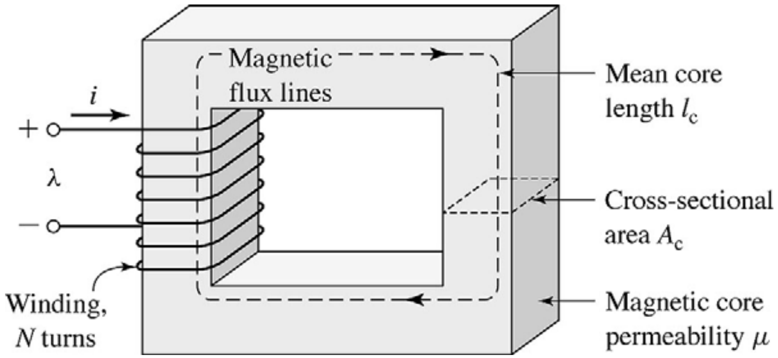
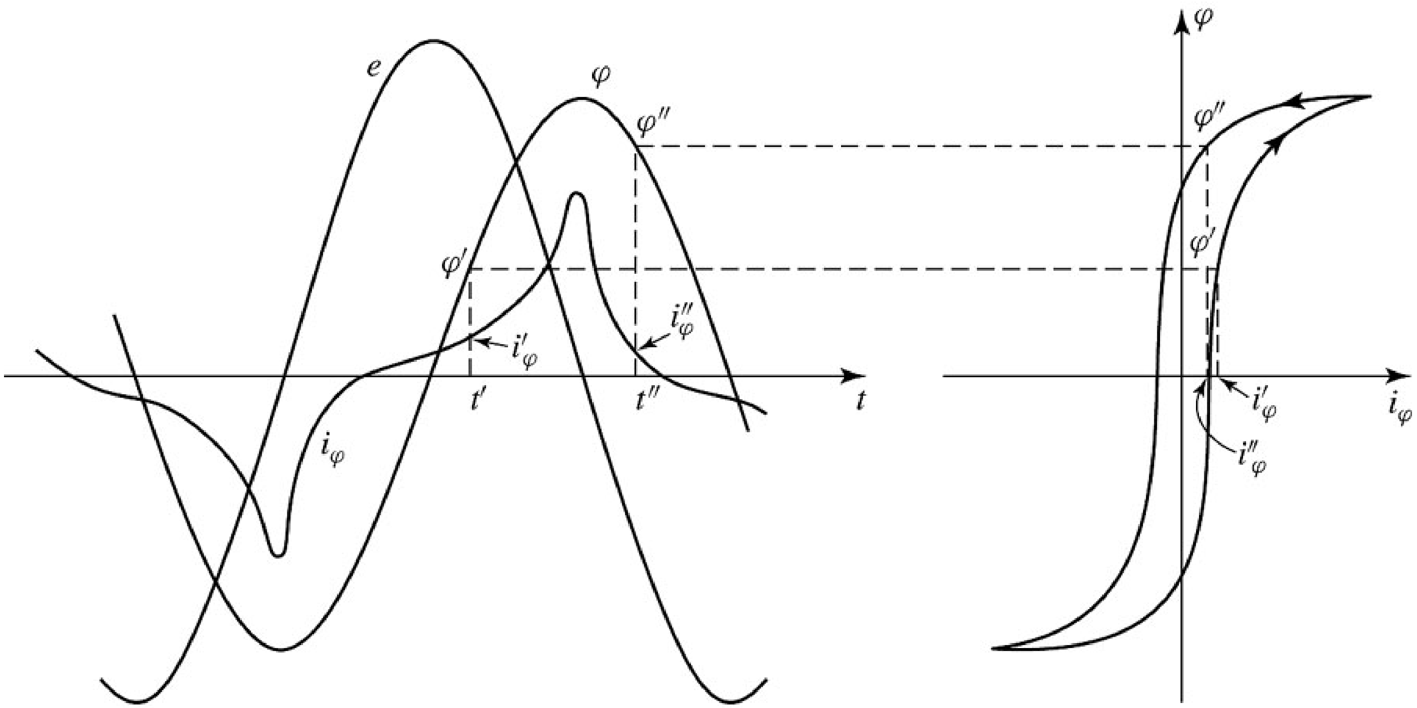


FIGURE 1.27 The *B-H* curves of various soft magnetic materials.

Reference: E. Furlani, *Permanent Magnet and Electromechanical Devices*, Academic Press, 2001, pp. 41

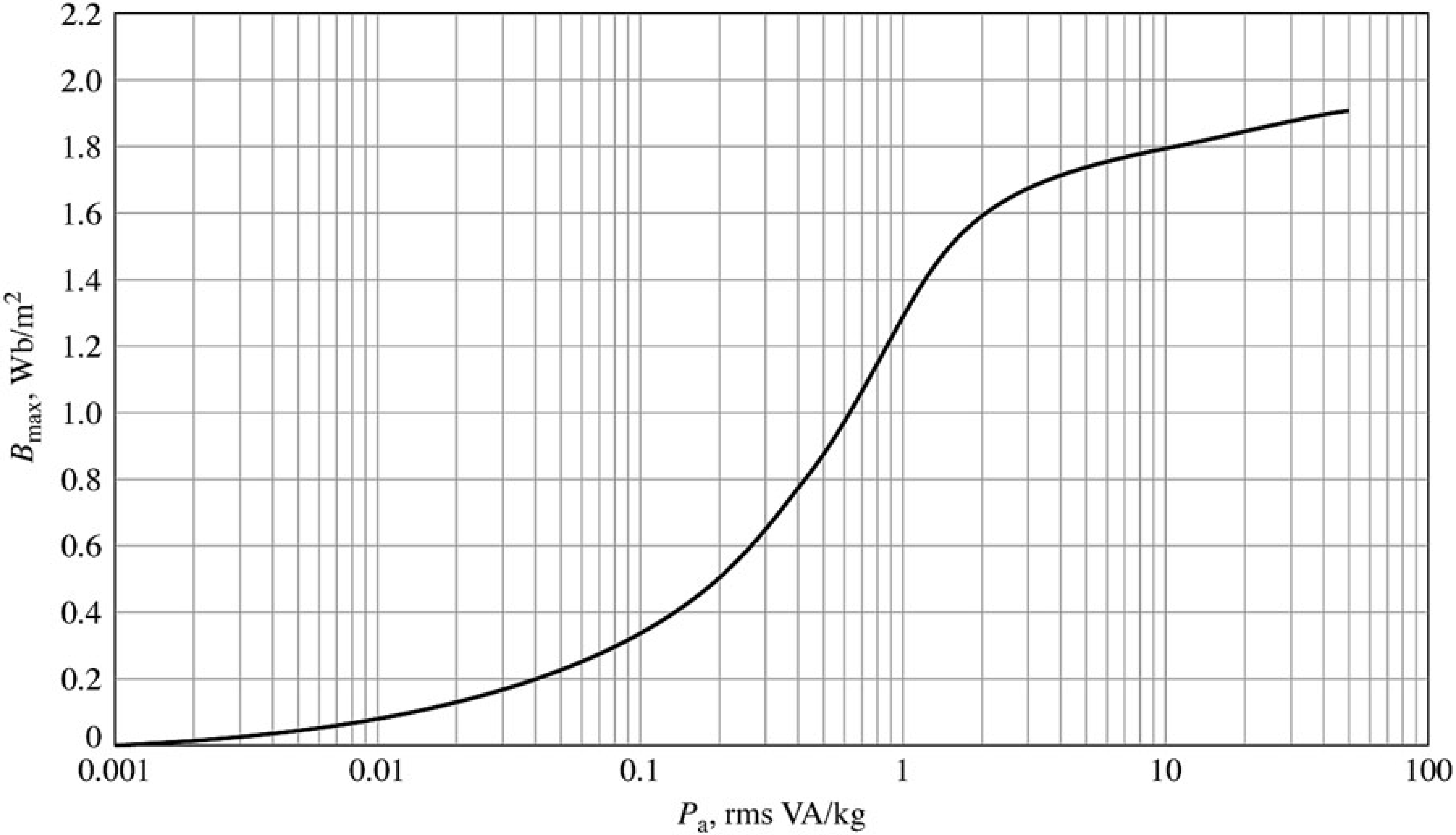
Relationship Between Voltage, Flux and Current





(a) (b)

Exciting RMS VA per kg at 60 Hz



Hysteresis Loop Size Increases with Frequency

- Hysteresis loss increases as frequency increases

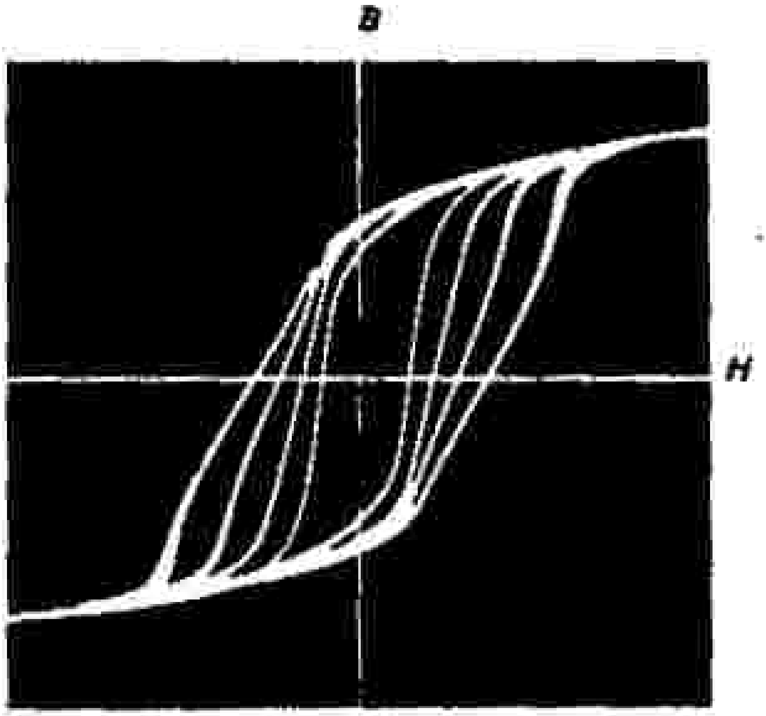


Fig. 2.3
Hysteresis loops at different frequencies

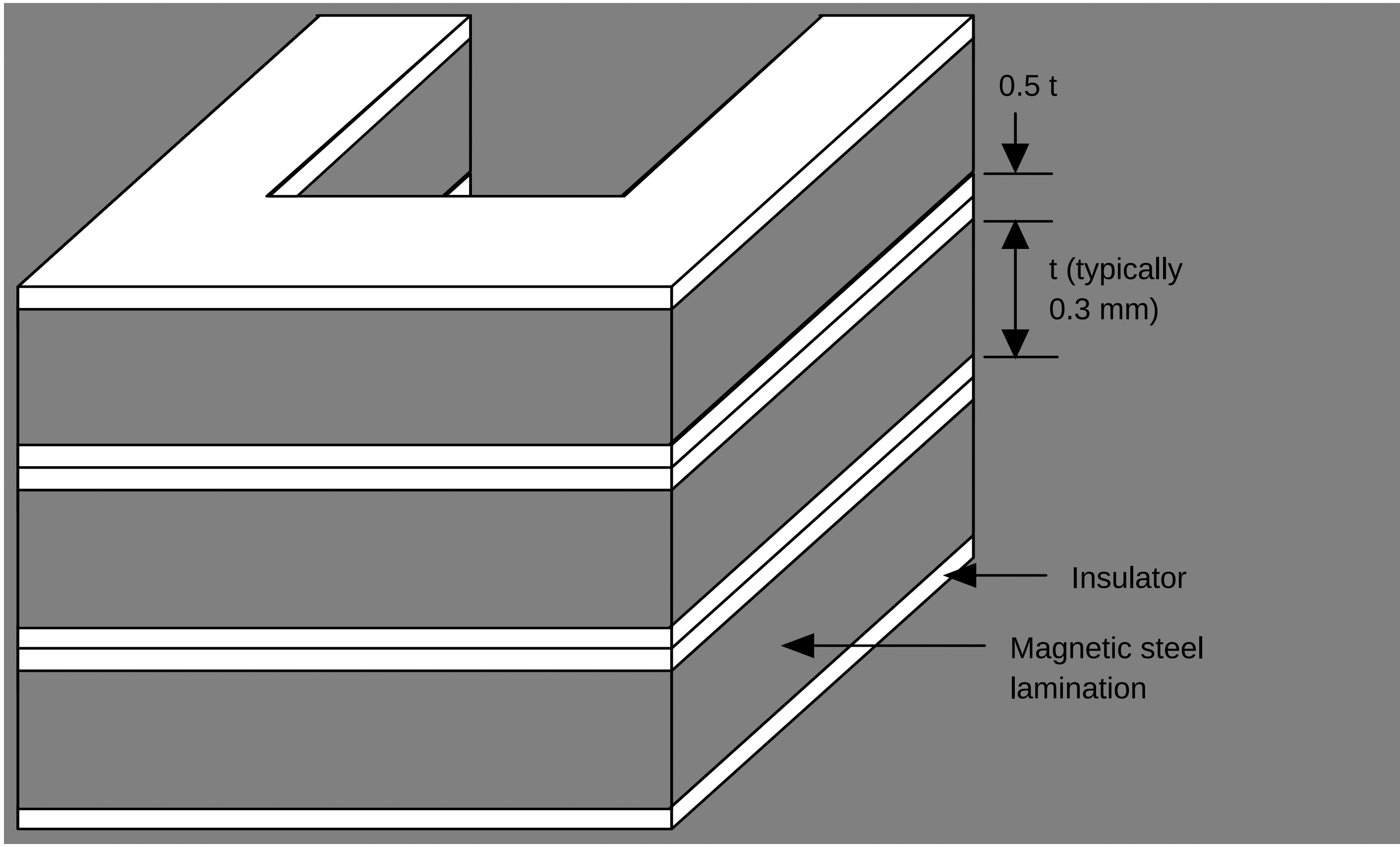
Reference: Siemens, _Soft Magnetic Materials_ (Vacuumschmelze Handbook), pp. 30

Total Core Loss

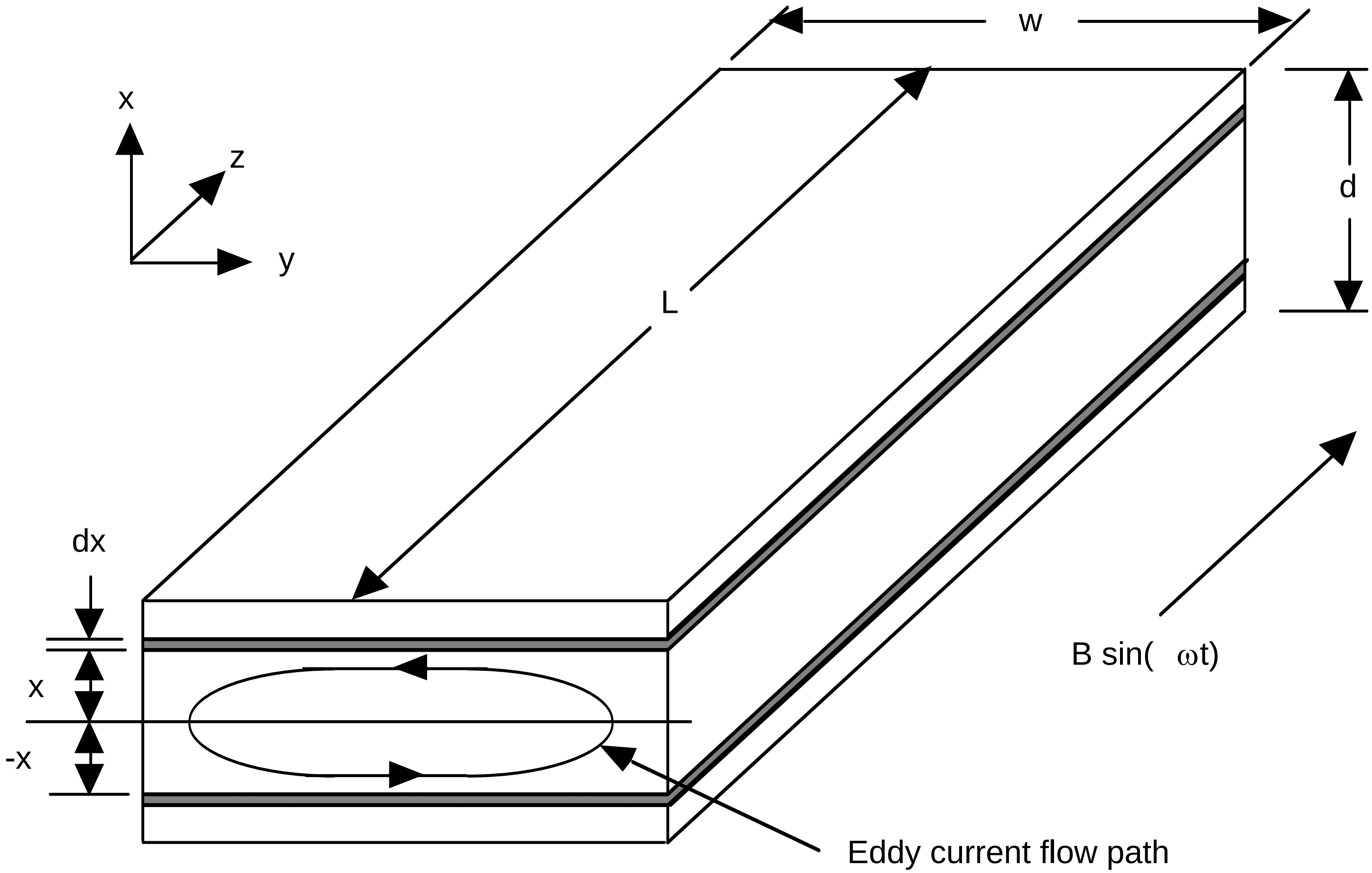
- Total core loss is the sum of:
  - Hysteresis loss
  - Eddy current losses
- Eddy current losses are due to induced currents (via Faraday's law)
- Eddy current losses are minimized by <u>laminating</u> magnetic cores

Laminated Core

- Cores made from conductive magnetic materials must be made of many thin laminations. Lamination thickness < skin depth.

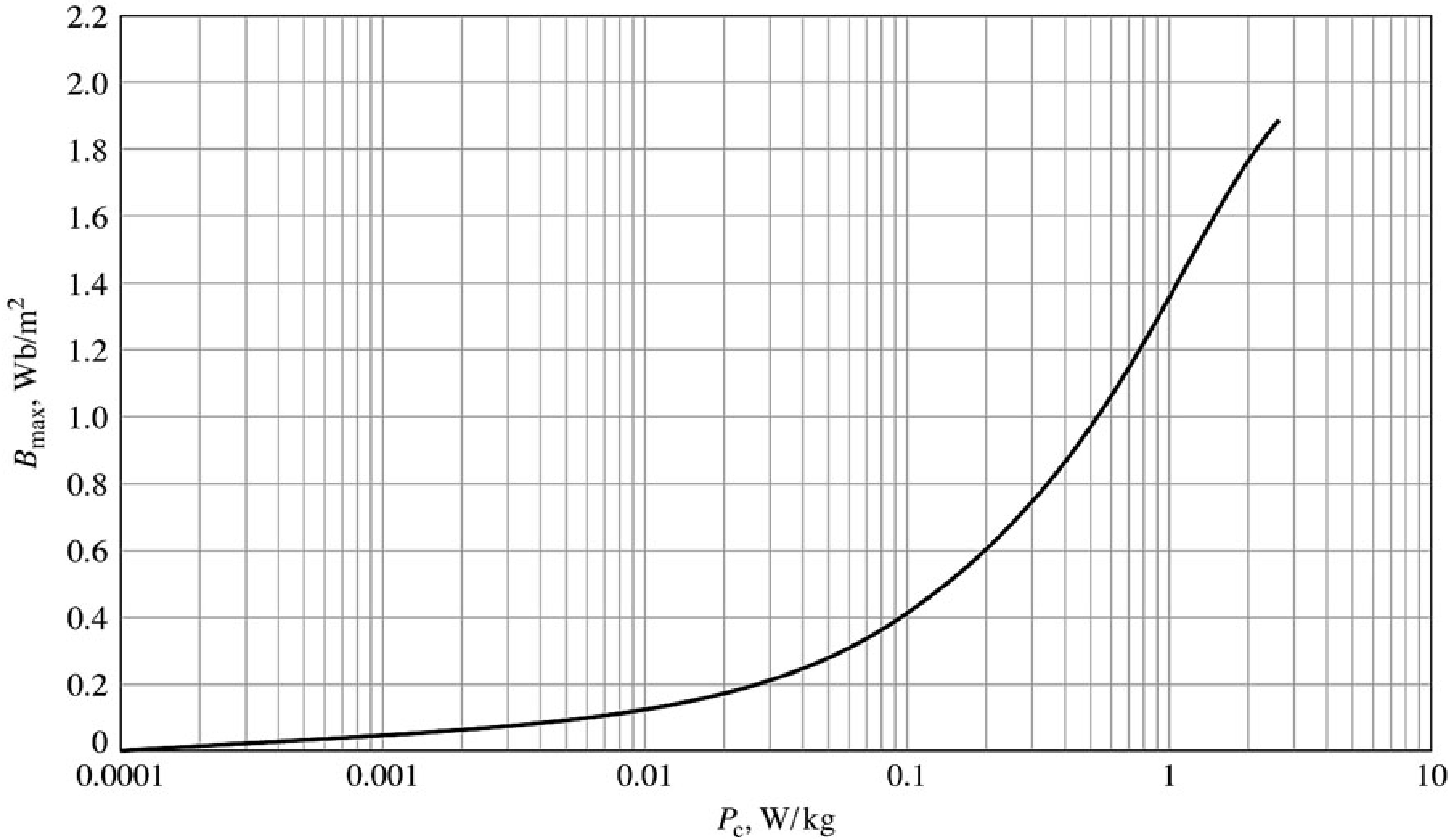


Eddy Current Loss in Lamination



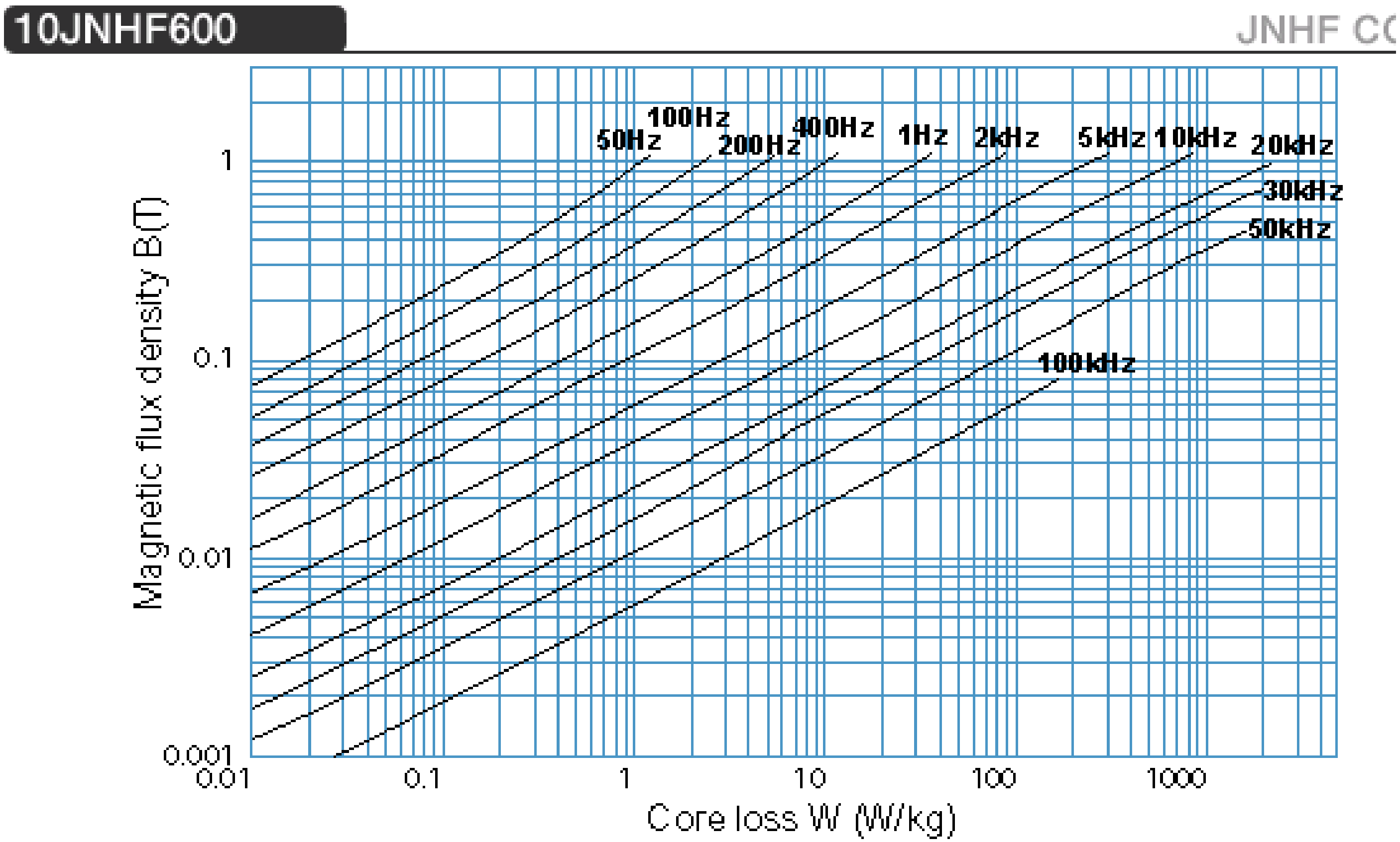
Total Core Loss

- M-5 steel at 60 Hz



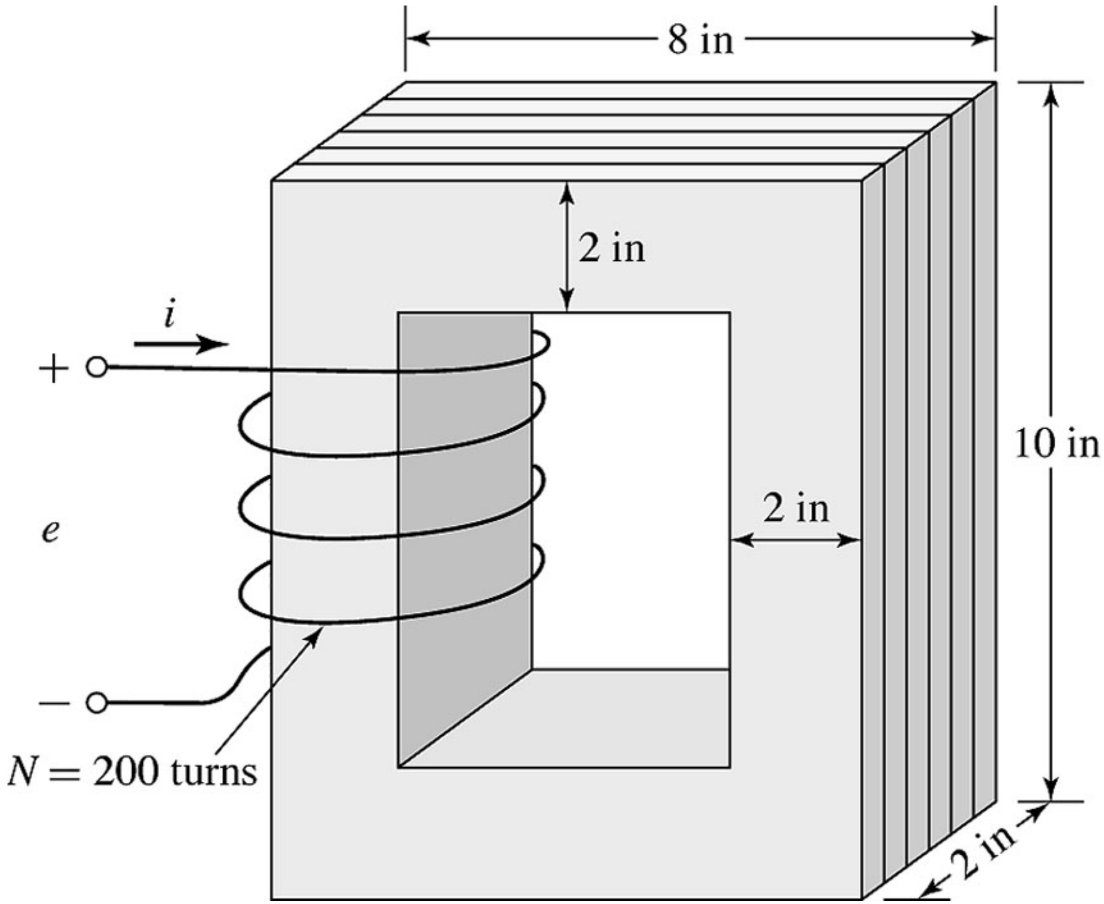
Total Core Loss vs. Frequency and B_{max}

- Core loss depends on peak flux density and excitation frequency
- This is the curve for a high frequency core material



Reference: http://www.jfe-steel.co.jp/en/products/electrical/jnhf/02.html

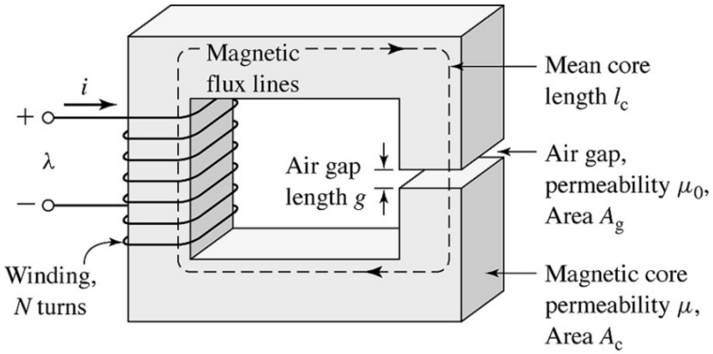
Process to Find Core Loss



- Find maximum B
- From this B and switching frequency, find core loss per kg
- Total loss is power density × mass of core

Example: C-Core with Airgap --- Current

- *Fitzgerald*, Example 1.7; find the current necessary to produce B_c = 1T



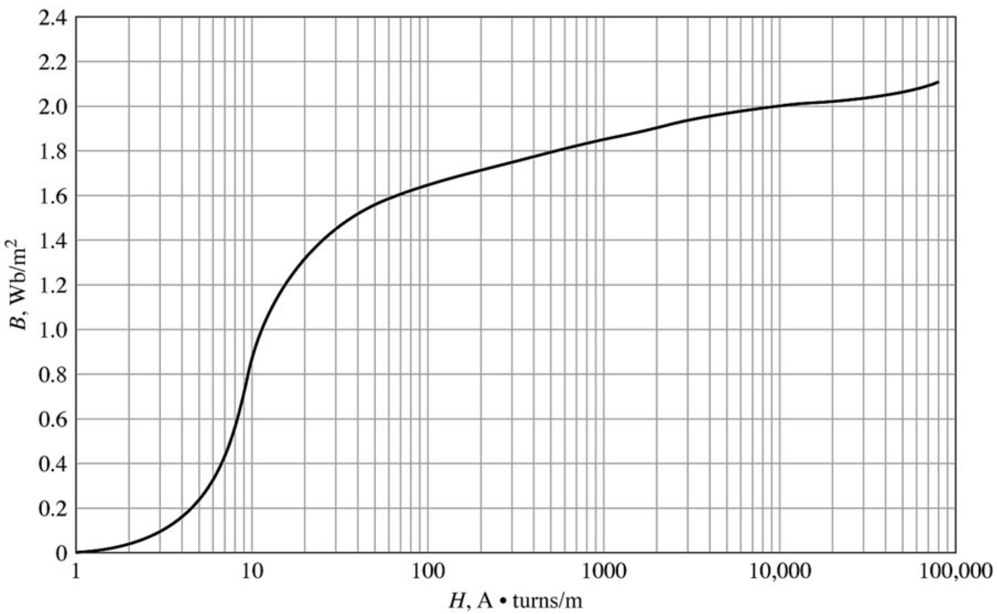
$$A_c = A_g = 9\ \mathrm{cm}^2$$

$$g = 0.05\ \mathrm{cm}$$

$$l_c = 30\ \mathrm{cm}$$

$$N = 500$$

$$\mu_r = 70{,}000$$



Example: C-Core with Airgap --- Solution

The value of H_c needed for B_c = 1 Tesla is read from the chart:

$$H_c = 11\ A - turns / meter$$

The MMF drop in the core is:

$$H_c l_c = (11)(0.3) = 33\ A - turns$$

The MMF drop in the airgap is:

$$H_g g = \frac{B_g g}{\mu_o} = \frac{(1.0)(5 \times 10^{-4})}{4\pi \times 10^{-7}} = 396\ A - turns$$

The winding current is:

$$I = \frac{\sum MMF}{N} = \frac{33 + 396}{500} = 0.8A$$

Example: Inductor

- *Fitzgerald*, Example 1.8

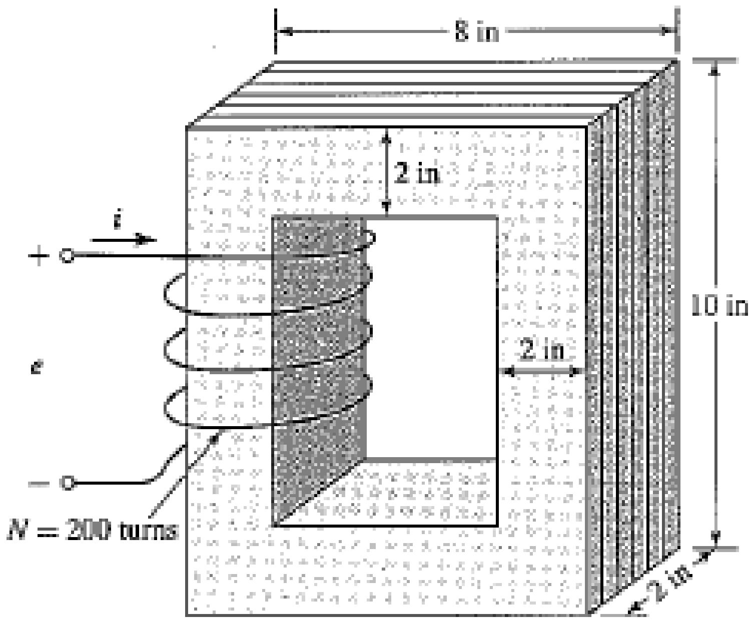


Figure 1.15 Laminated steel core with winding for Example 1.8.

Material: M-5 steel
f = 60 Hz
N=200
B_c = 1.5 sinωt Tesla
Steel is 94% of cross section
Density of steel = 7.65 g/cm^3

Find:

(a) Applied voltage
(b) Peak current
(c) RMS current
(d) Core loss

Example: Excitation Voltage

From Faraday's law:

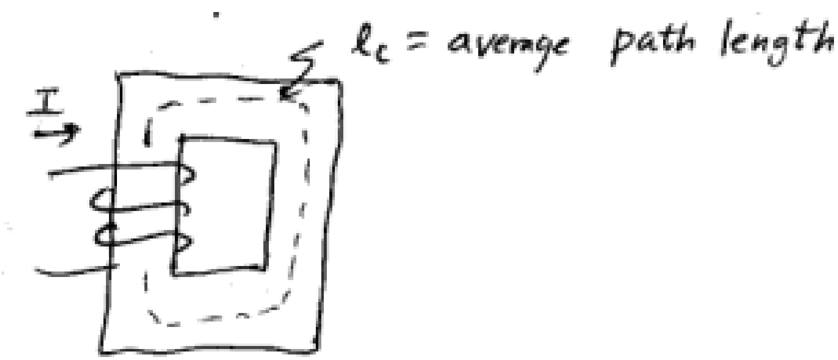
$$e = \frac{d\lambda}{dt} = N\frac{d\Phi}{dt} = NA_c\frac{dB_c}{dt}$$

$$A_c = 2in \times 2in \times 0.94 = 3.76in^2 = 2.4 \times 10^{-3}\ m^2$$

$$\frac{dB_c}{dt} = (1.5\omega)\cos(\omega t) = 565\cos(\omega t)$$

$$e = (200)(2.4 \times 10^{-3})(565\cos(\omega t)) = 274\cos(\omega t)$$

Example: Peak Current in Winding



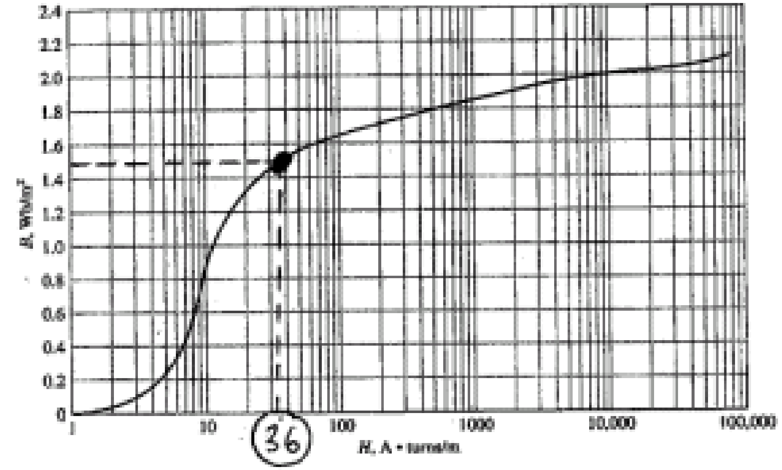


Figure 1.10 Dc magnetization curve for M-5 grain-oriented electrical steel 0.012 in thick. (Armco Inc.)

From Figure 1.10, B = 1.5T requires H = 36 A-turns/m

From Ampere's law, $Hl_c = NI$

$l_c = 2\times(8"+6") = 28" = 0.71\text{m}$

$$I = \frac{Hl_c}{N} = \frac{(36)(0.71)}{200} = 0.13A$$

Example: RMS Winding Current

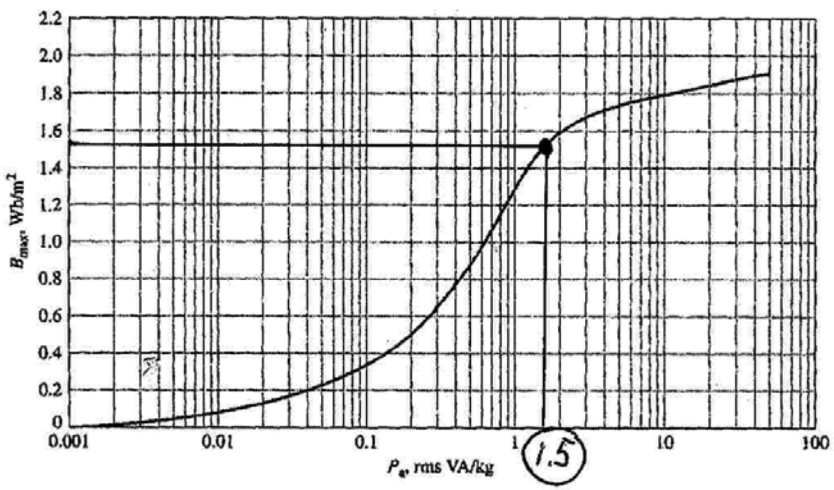


Figure 1.12 Exciting rms voltamperes per kilogram at 60 Hz for M-5 grain-oriented electrical steel 0.012 in thick. *(Armco Inc.)*

From Figure 1.12, at B_{max} = 1.5T, P_a = 1.5 VA/kg

Core volume:

$$V_c = (4in^2)(0.94)(28in) = 105.5\ in^3 = 1.7\times10^{-3}\,m^3$$

Core mass:

$$M = V_c\rho_c = (1.7\times10^{-3}\,m^3)(7650\ \frac{kg}{m^3}) = 13.2\ kg$$

Core Volt-

Amperes: $P_c = 1.5\,\frac{VA}{kg}\times 13.2\ kg = 19.8\,VA$

Current: $I_{RMS} = \frac{VA}{E_{RMS}} = \frac{19.7}{\left(\frac{274}{\sqrt{2}}\right)} = 0.10A$

Example: Core Loss

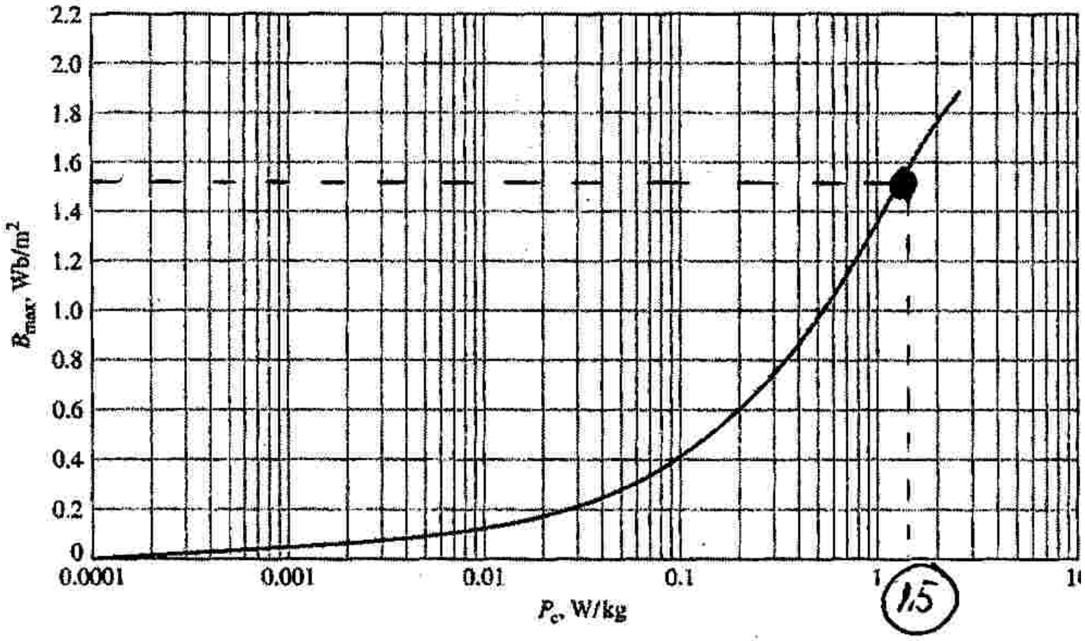


Figure 1.14 Core loss at 60 Hz in watts per kilogram for M-5 grain-oriented electrical steel 0.012 in thick. (*Armco Inc.*)

From Figure 1.14, core loss density = 1.5 W/kg at B_{max} = 1.5 Tesla. Total core loss is:

$$P_c = M \times 1.5 \frac{W}{kg} = (13.2)(1.5) = 20W$$

Permanent Magnets

- “Soft” magnetic materials such as magnetic steel can behave as very weak permanent magnets
- Permanent magnets, or “hard” magnetic materials, have a high coercive force H_c and can produce significant flux in an airgap; they also have a “wide” hysteresis loop

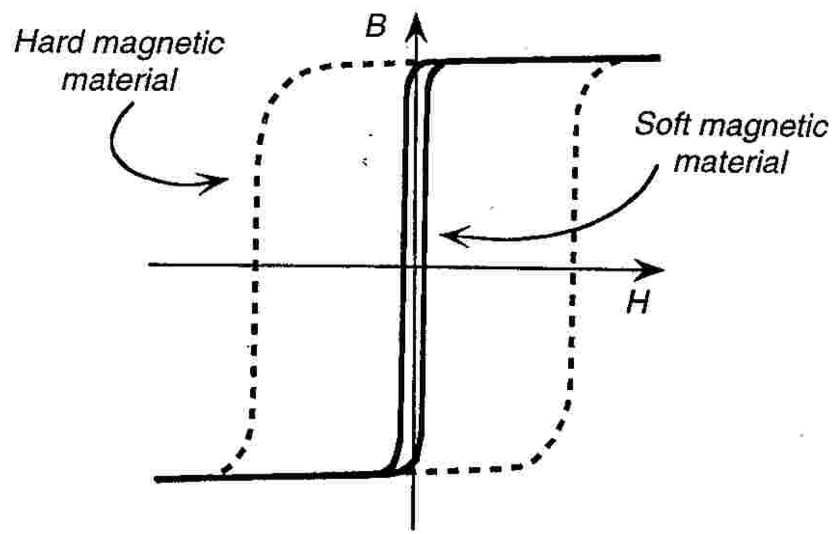


FIGURE 1.25 The *B-H* loops for soft and hard magnetic materials.

Reference: E. Furlani, *Permanent Magnet and Electromechanical Devices*, Academic Press, 2001, pp. 39

Brief History of Permanent Magnets

- c. 1000 BC: Chinese compasses using lodestone
  - Later used to cross the Gobi desert

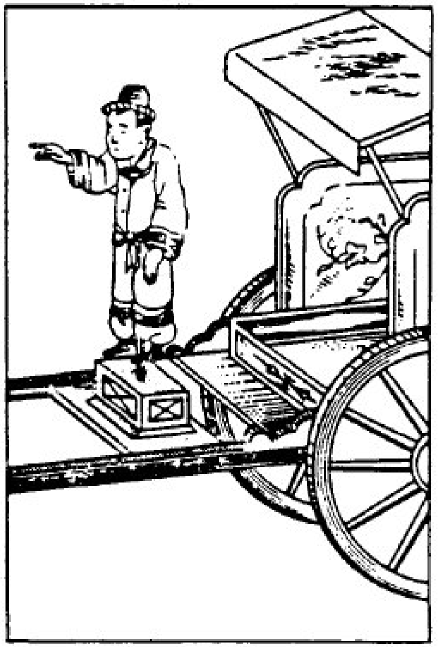


Fig. 2 *Man-shaped compass mounted on a chariot, after Kartsev [2]*

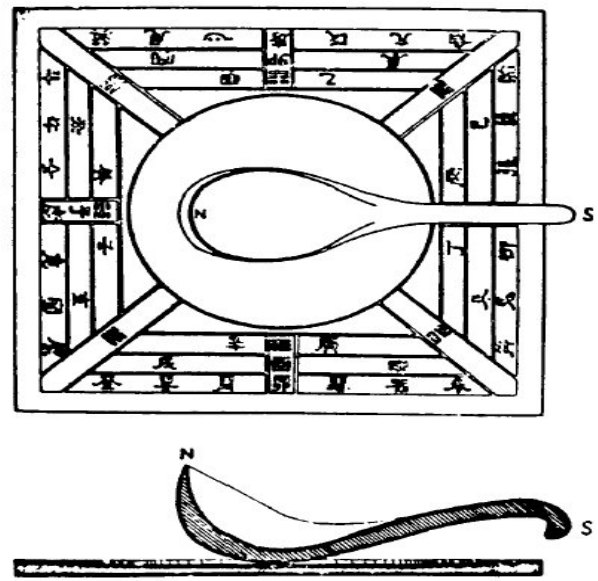


Fig. 3 *Chinese spoon compass*

Reference: K. Overshott, “Magnetism: it is permanent,” *IEE Proceedings-A*, vol. 138, no. 1, Jan. 1991, pp. 22-31

Brief History of Permanent Magnets

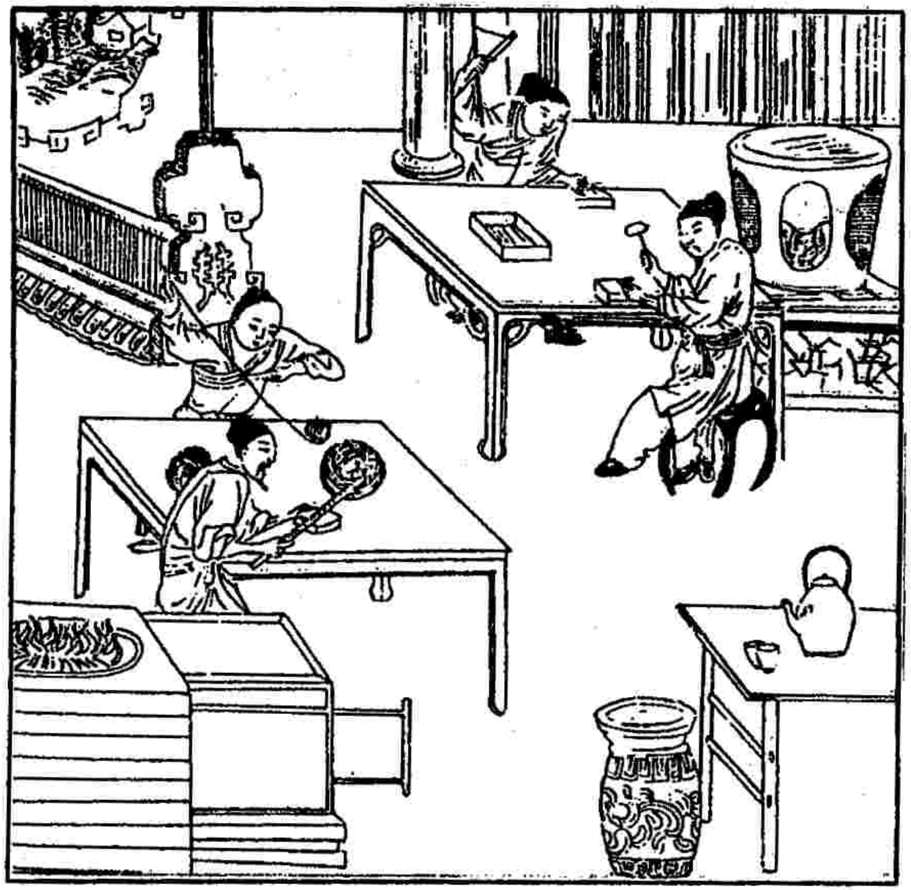


Figure 1.1 Magnetic needles for compasses are being made by craftsmen in this print of 1637. Good steel was manufactured in China from 500 A.D. onwards.

Reference: R. Parker, *Advances in Permanent Magnetism*, John Wiley, 1990, pp. 3

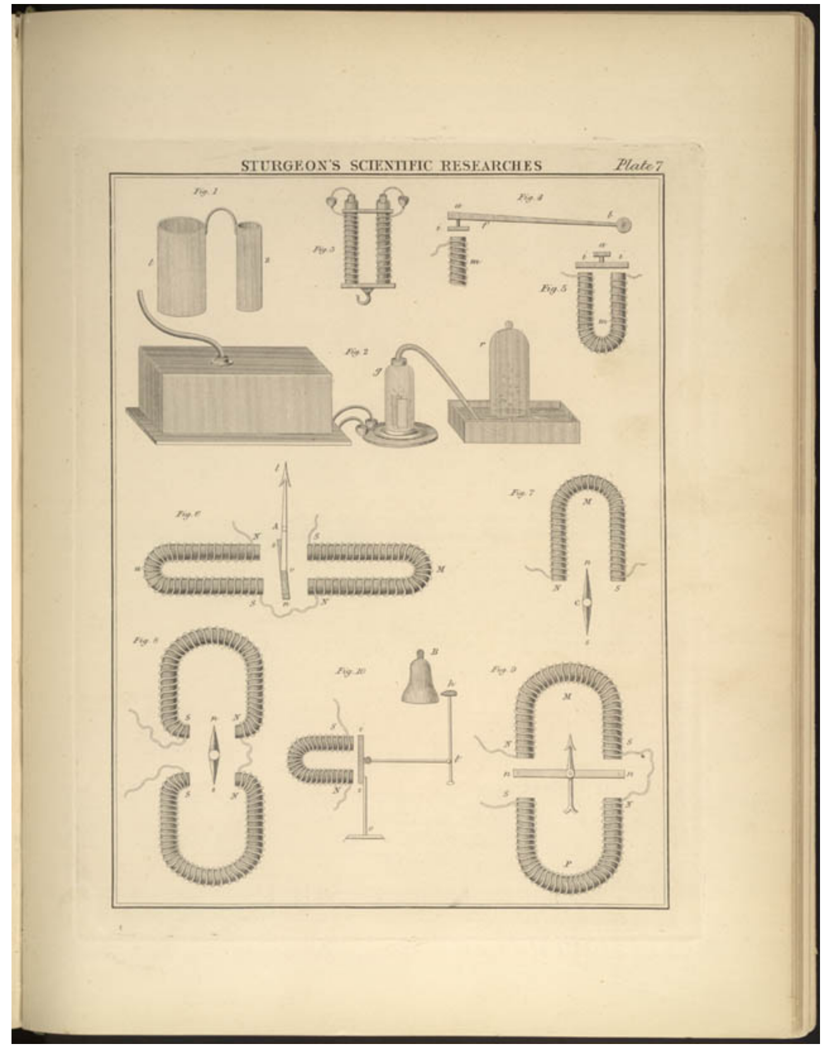
Brief History of Permanent Magnets (cont.)

- c. 200 BC: Lodestone (magnetite) known to the Greeks
  - Touching iron needles to magnetite magnetized them
- 1200 AD: French troubadour de Provins describes use of a primitive compass to magnetize needles
- 1600: William Gilbert publishes first journal article on permanent magnets
- 1819: Oersted reports that an electric current moves compass needle

References:
1. K. Overshott, “Magnetism: it is permanent,” *IEE Proceedings-A*, vol. 138, no. 1, Jan. 1991, pp. 22-31
2. R. Petrie, “Permanent Magnet Material from Loadstone to Rare Earth Cobalt,” *Proc. 1995 Electronics Insulation and Electrical Manufacturing and Coil Winding Conf*., pp. 63-64
3. Rollin Parker, *Advances in Permanent Magnetism*, John Wiley, 1990
4. E. Hoppe, “Geshichte des Physik,” Vieweg, Braunshweig, 1926, pp. 339
5. W. Gilbert, “De Magnete 1600,” translation by S. P. Thompson, 1900, republished by Basic Books, Inc., New York, 1958

Brief History of Permanent Magnets (cont.)

- c. 1825: Sturgeon invents the electromagnet, resulting in a way to artificially magnetize materials
- 7-ounce magnet was able to lift 9 pounds



References:
1. W. Sturgeon, *Mem. Manchester Lit. Phil. Soc.*, 1846, vol. 7, pp. 625
2. Britannica Online

Brief History of Permanent Magnets (cont.)

- c. 1830: Joseph Henry (U.S.) constructs electromagnets

Joseph Henry

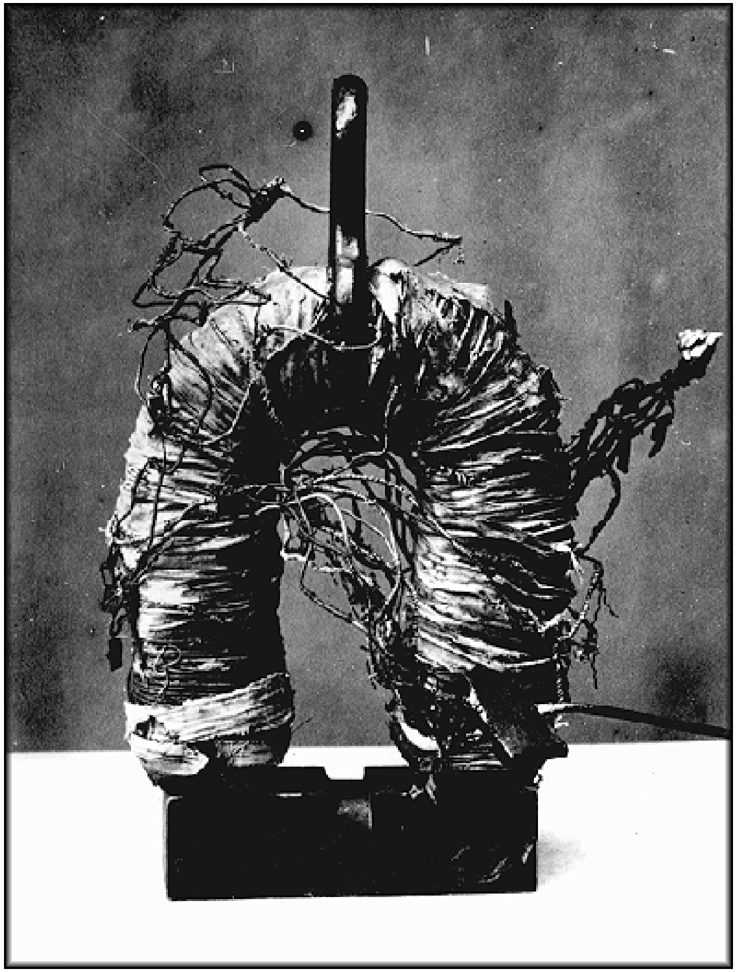


Photo reference: Smithsonian Institute archives

Brief History of Permanent Magnets (cont.)

- 1917: Cobalt magnet steels developed by Honda and Takagi in Japan
- 1940: Alnico --- first “modern” material still in common use
  - Good for high temperatures
- 1960: SmCo (samarium cobalt) rare earth magnets
  - Good thermal stability
- 1983: GE and Sumitomo develop neodymium iron boron (NdFeB) rare earth magnet
  - Highest energy product, but limited temperature range

References:
1. K. Overshott, “Magnetism: it is permanent,” *IEE Proceedings-A*, vol. 138, no. 1, Jan. 1991, pp. 22-31
2. R. Petrie, “Permanent Magnet Material from Loadstone to Rare Earth Cobalt,” *Proc. 1995 Electronics Insulation and Electrical Manufacturing and Coil Winding Conf*., pp. 63-64

Magnetizing Permanent Magnets

- Material is placed inside magnetizing fixture
- Magnetizing coil is energized with a current producing sufficient field to magnetize the PM material

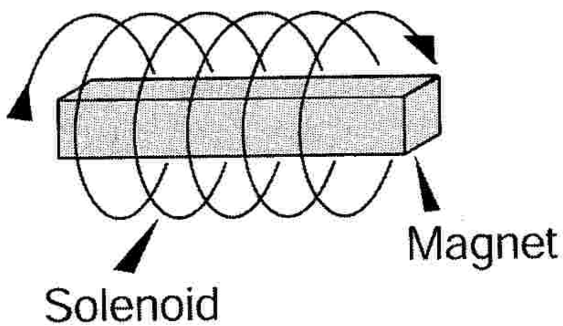


FIGURE 1.33 Magnetization of a bar magnet using a solenoid.

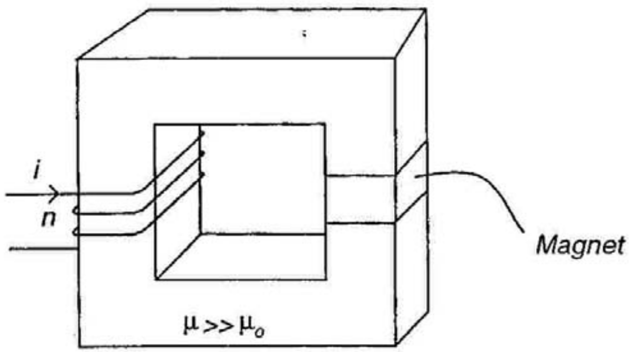


FIGURE 1.34 Conventional magnetizing fixture.

Reference: E. Furlani, *Permanent Magnet and Electromechanical Devices*, Academic Press, 2001, pp. 57

Pictorial View of Magnetization Process

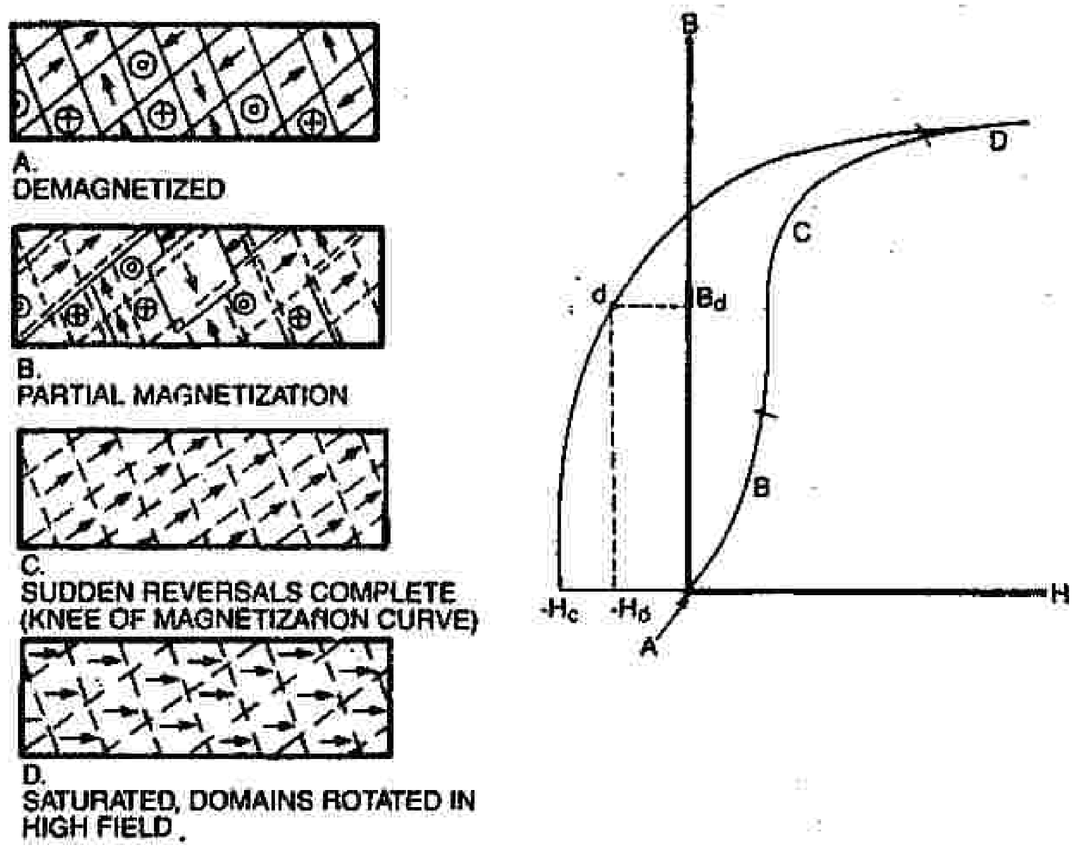


Figure 3.6 Pictorial explanation of magnetization curve in a ferromagnetic bar.

Reference: R. Parker, *Advances in Permanent Magnetism*, John Wiley, 1990, pp. 49

Permanent Magnets

- External effects of permanent magnets can be modeled as surface current

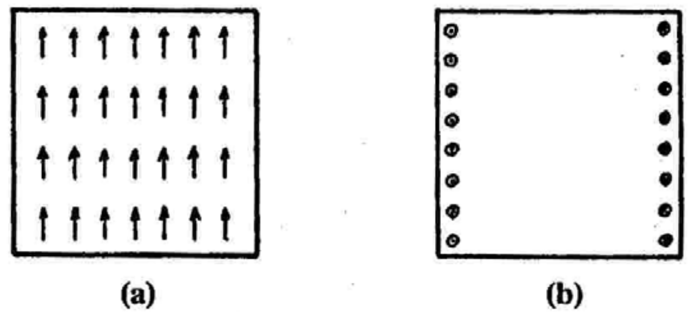
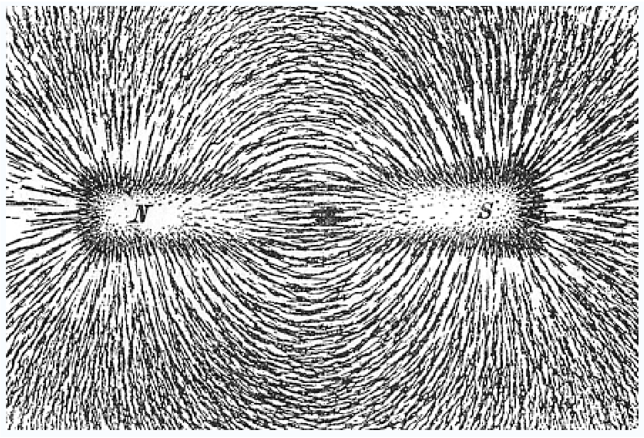


Figure 1.4. Uniformly magnetized magnet (*a*), which may be modeled by a current density over its boundary (*b*).



Reference: P. Campbell, *Permanent Magnet Materials and their Applications*, Cambridge University Press, 1994, pp. 7

Permanent Magnets

- After magnetization, **M** has values of either +Msat or -Msat

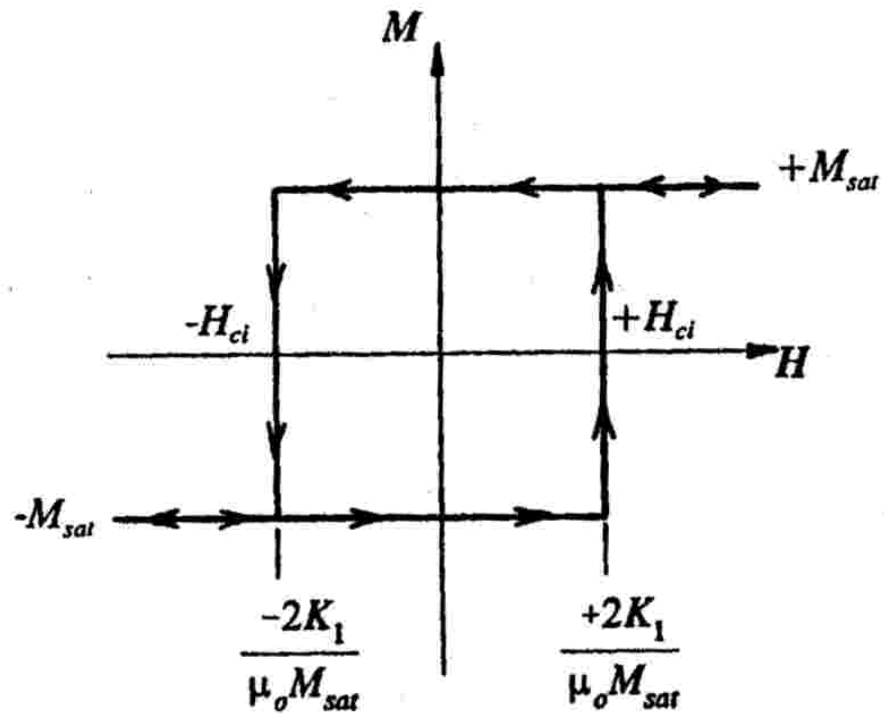


Figure 1.10. Intrinsic magnetization characteristic for an elemental volume of a magnet.

Reference: P. Campbell, *Permanent Magnet Materials and their Applications*, Cambridge University Press, 1994, pp. 14-15

Permanent Magnets

- By a constitutive relationship, $\mathbf{B} = \mu_o(\mathbf{H}+\mathbf{M})$
- Since **M** has values of either +Msat or -Msat, it follows that the slope of the BH curve for the permanent magnet is μ_o

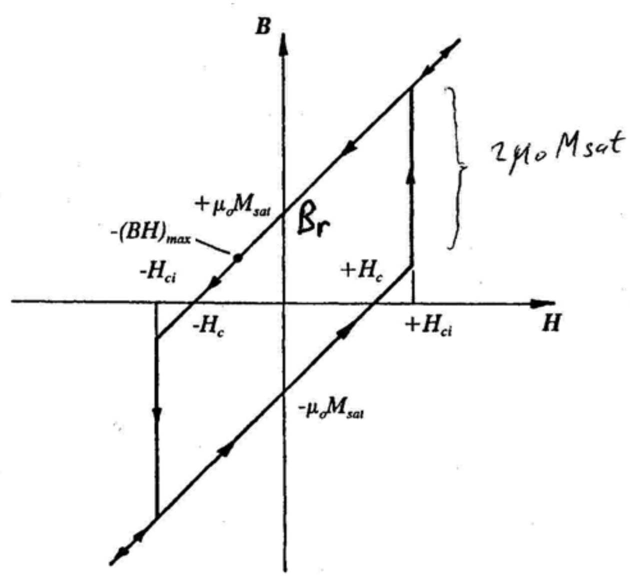


Figure 1.11. *B versus H* characteristic for a magnet. (with $H_{ci} > M_{sat}$)

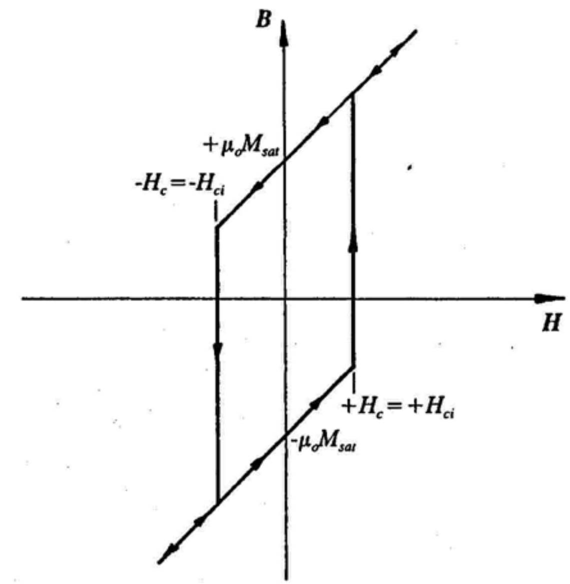


Figure 1.17. *B versus H* characteristic for a magnet with $|H_{ci}| < M_{sat}$.

Reference: P. Campbell, *Permanent Magnet Materials and their Applications*, Cambridge University Press, 1994, pp. 15, 23

Demagnetization Curves of Ceramic 8

- Typical sintered ceramic magnet

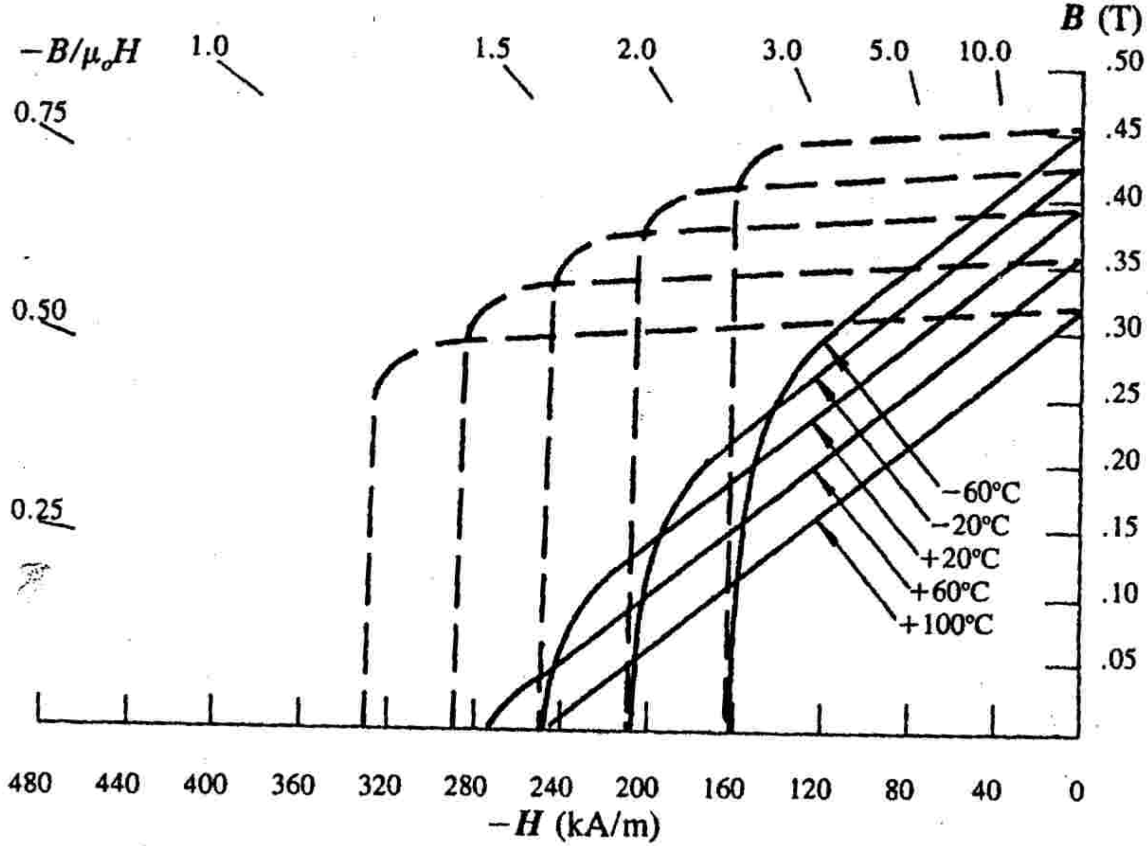


Figure 3.3. Demagnetization curves of Ceramic 8 at various temperatures.

Reference: P. Campbell, *Permanent Magnet Materials and their Applications*, Cambridge University Press, 1994, pp. 62

Demagnetization Curves for NdFeB

- Strong neodymium-iron-boron

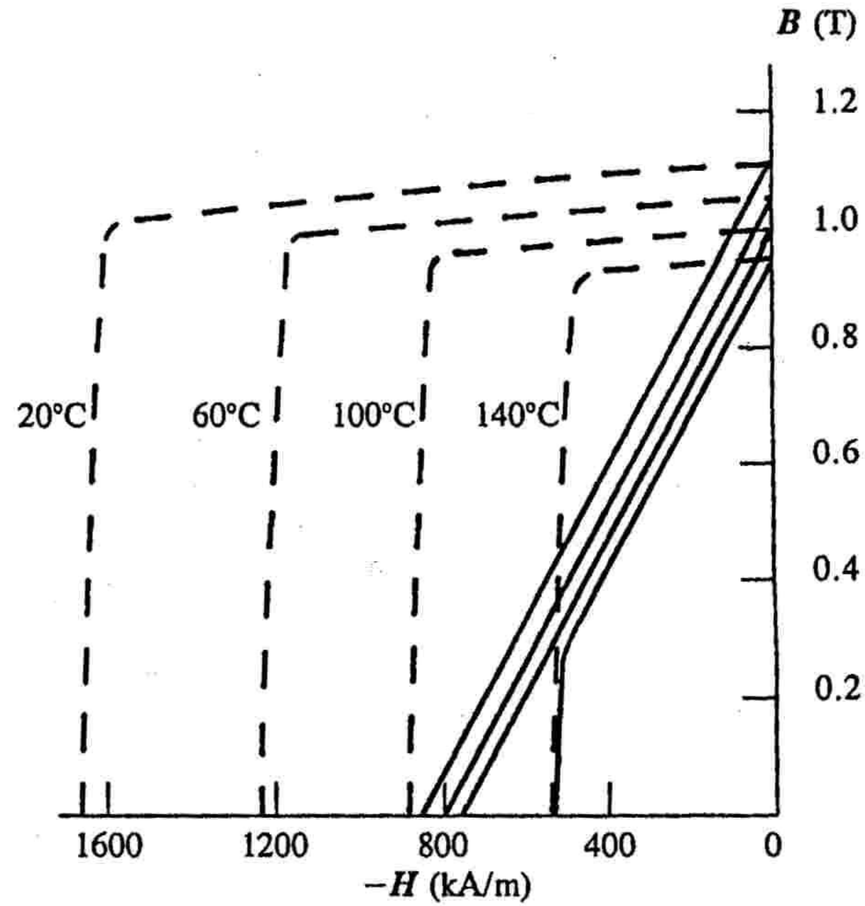


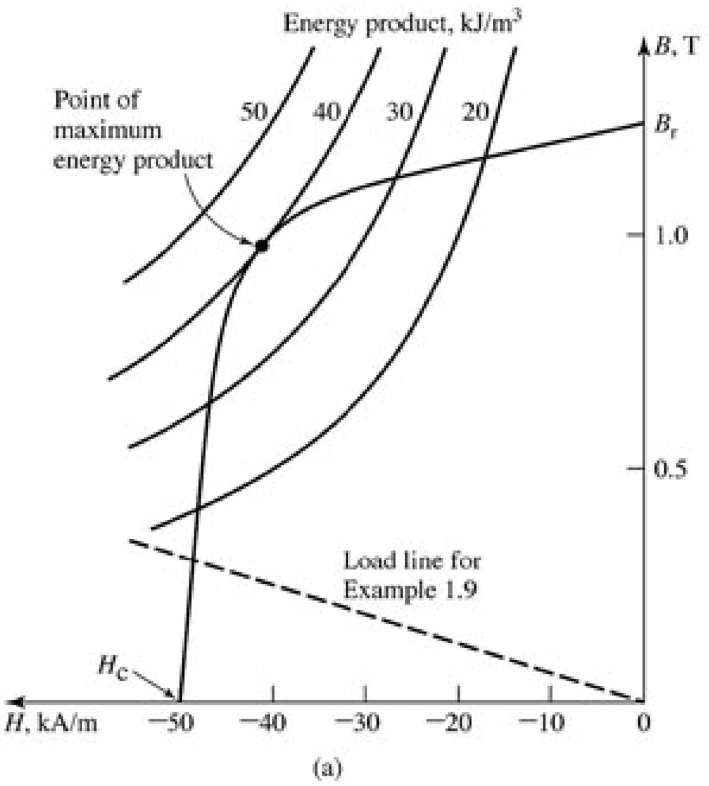
Figure 3.14. Demagnetization curves of (Nd, Dy)–Fe–B at various temperatures.

Reference: P. Campbell, *Permanent Magnet Materials and their Applications*, Cambridge University Press, 1994, pp. 74

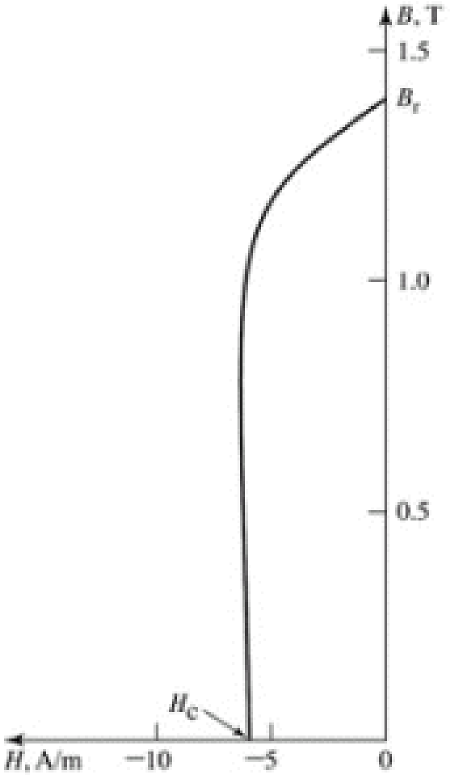
Permanent Magnets vs. Steel

- Note that PM has much higher coercive force

Permanent magnet: Alnico 5

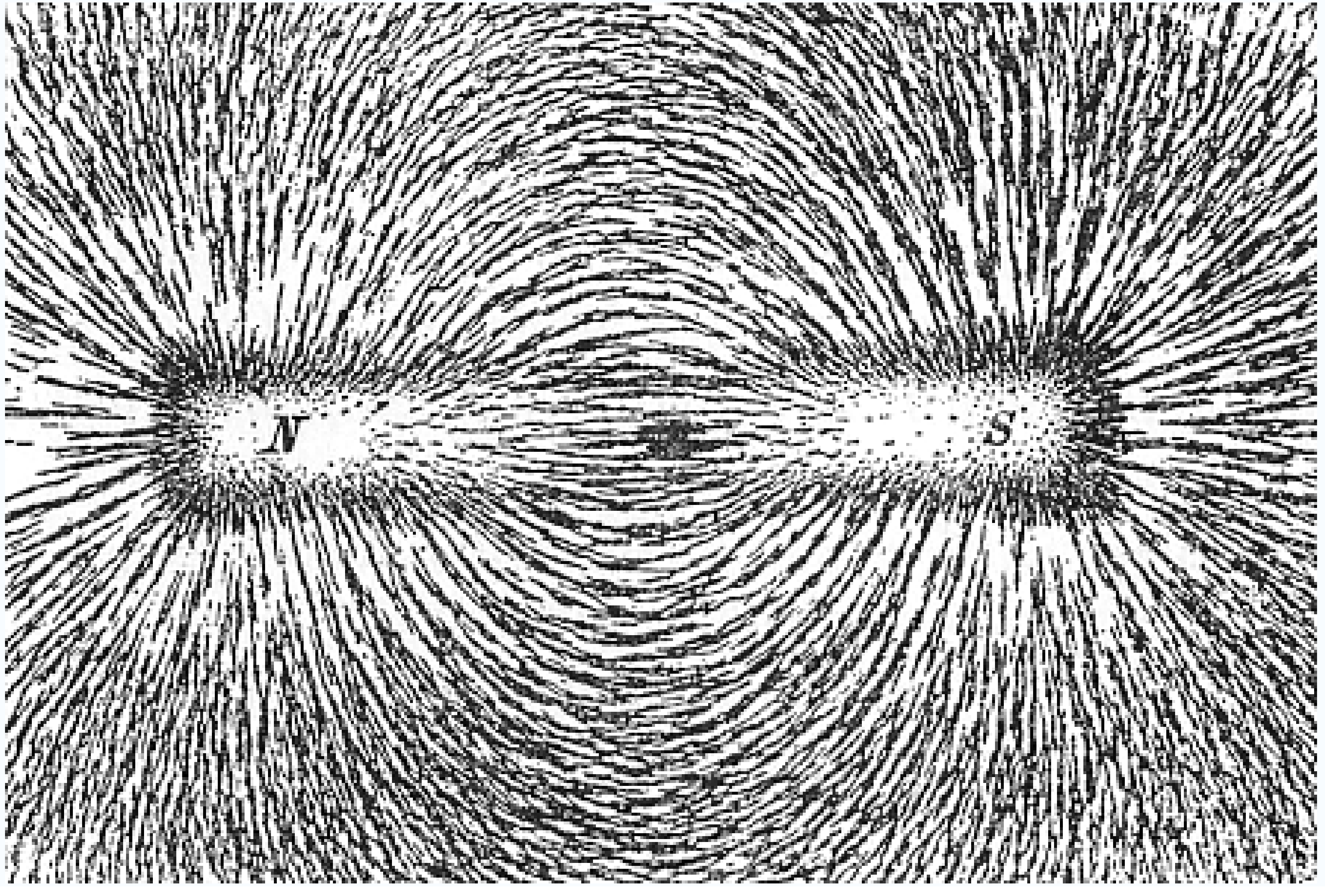


M-5 steel



Lines of Force

- Iron filings follow magnetic field lines



Cast Alnico

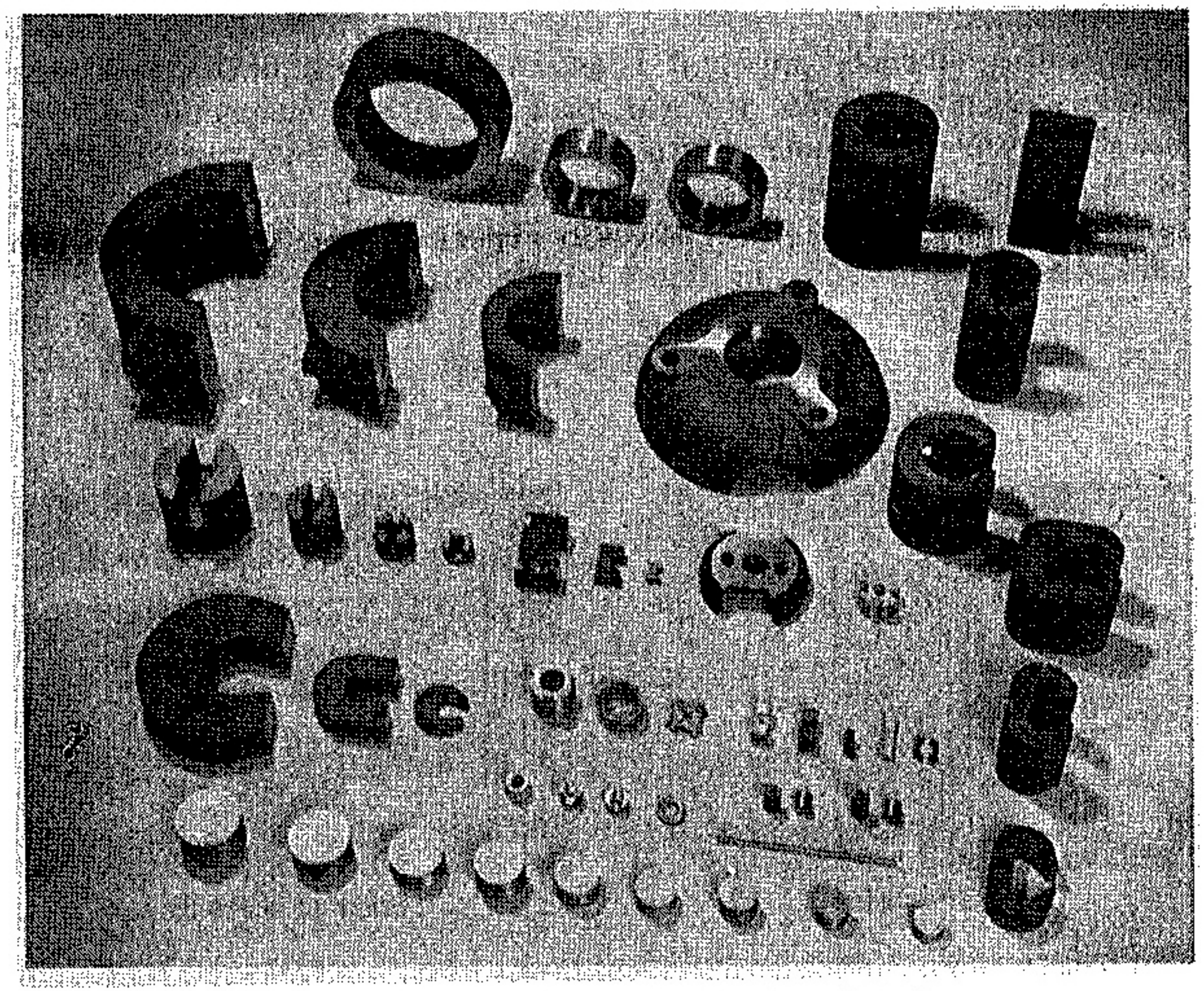
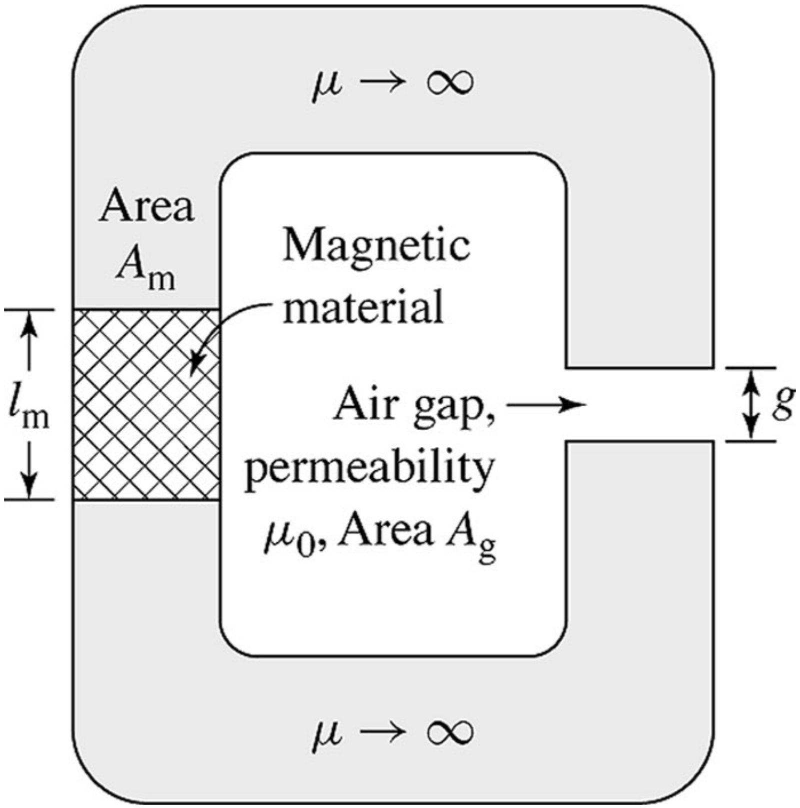


Figure 4.3 Typical cast Alnico magnet configurations.

Reference: R. Parker, *Advances in Permanent Magnetism*, John Wiley, 1990, pp. 65

Example: PM in Magnetic Circuits

- *Fitzgerald*, Example 1.9



g = 0.2 cm

l_m = 1.0 cm

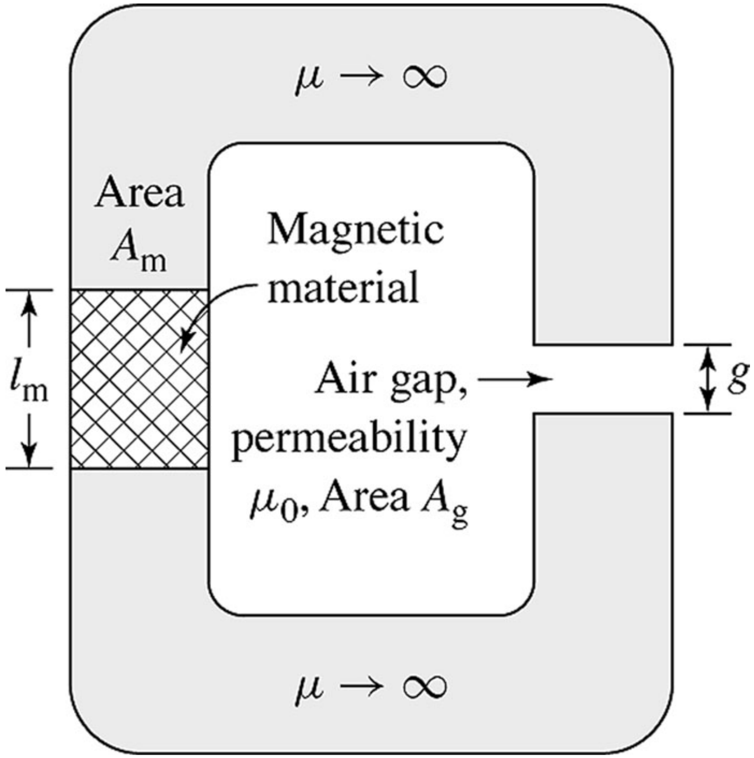
$A_m = A_g = 4\ cm^2$

Find flux in airgap B_g for magnetic materials

(a) Alnico 5

(b) M-5 steel

Example: Solution with Alnico PM



NI = 0, so by Ampere's law:

$$H_m l_m + H_g g = 0$$

Solve for H_g: $H_g = -H_m \left(\frac{l_m}{g} \right)$

Continuity of flux (Gauss' law):

$$A_g B_g = A_m l_m \Rightarrow B_g = B_m \left(\frac{A_m}{A_g} \right)$$

Next, solve for B_m as a function of H_m:

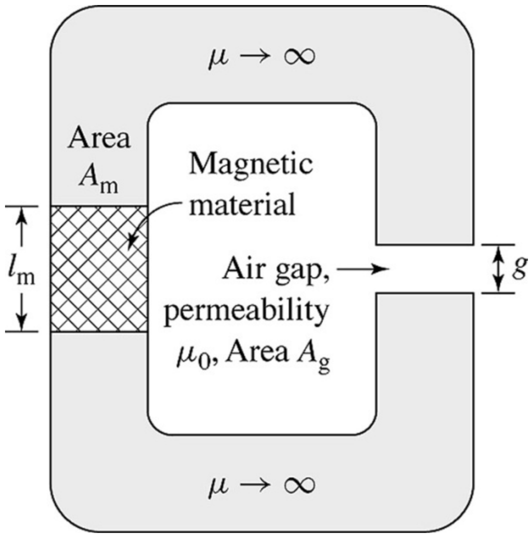
$$B_m = B_g \left(\frac{A_g}{A_m} \right) = \mu_o H_g \left(\frac{A_g}{A_m} \right)$$

$$\Rightarrow B_m = \mu_o \left(-H_m \frac{l_m}{g} \right) \left(\frac{A_g}{A_m} \right) = -6.28 \times 10^{-6} H_m$$

Plot this load line on Alnico BH curve

Example: Solution with Alnico

- Result: B_g = 0.3 Tesla



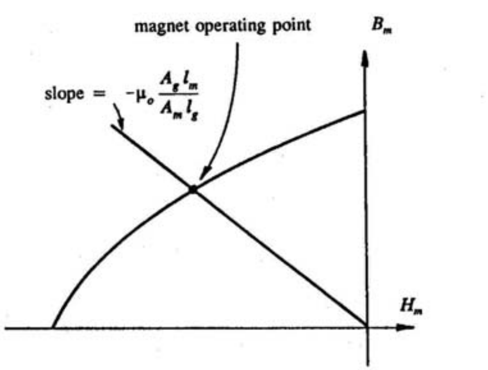
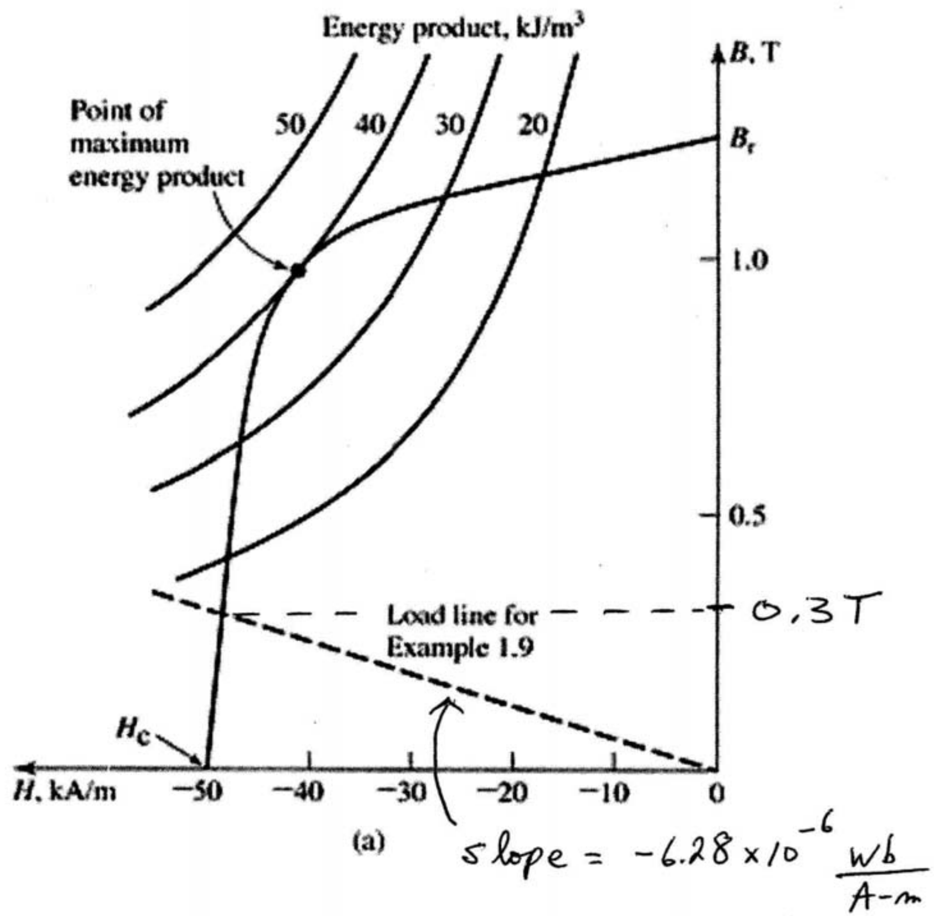


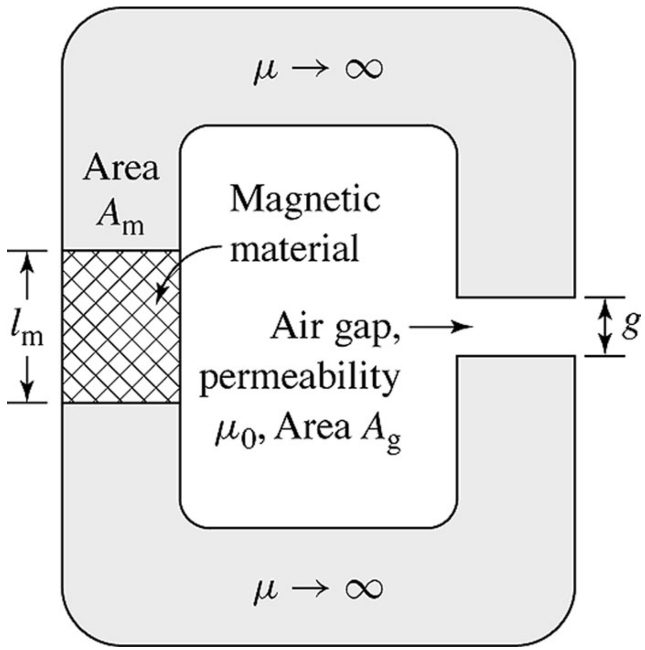
Figure 4.5. Demagnetization curve and load line for a magnet.

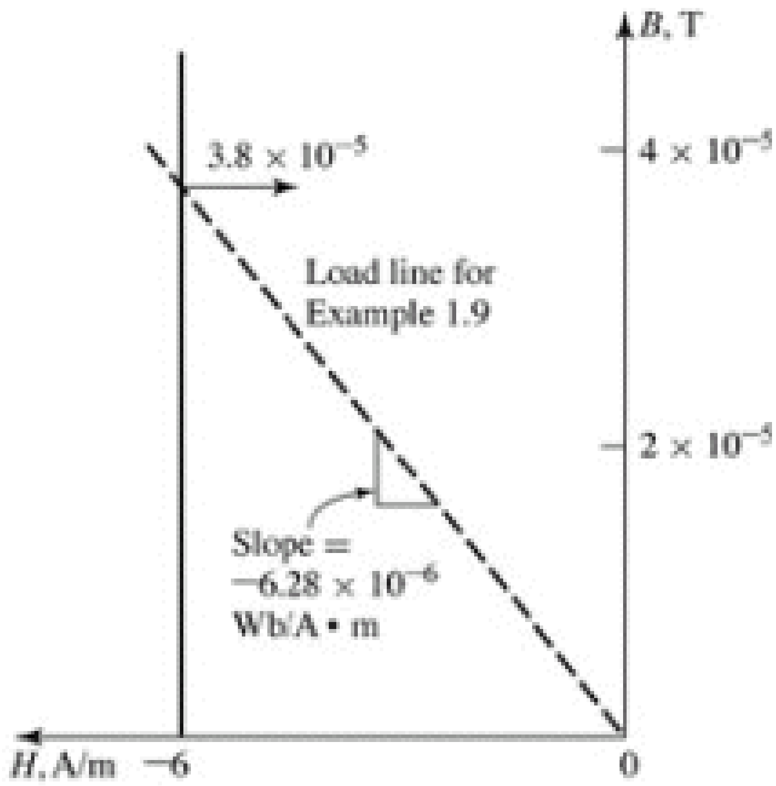


Reference: P. Campbell, *Permanent Magnet Materials and their Applications*, Cambridge University Press, 1994, pp. 89

Example: Load Line Solution with M-5 Steel

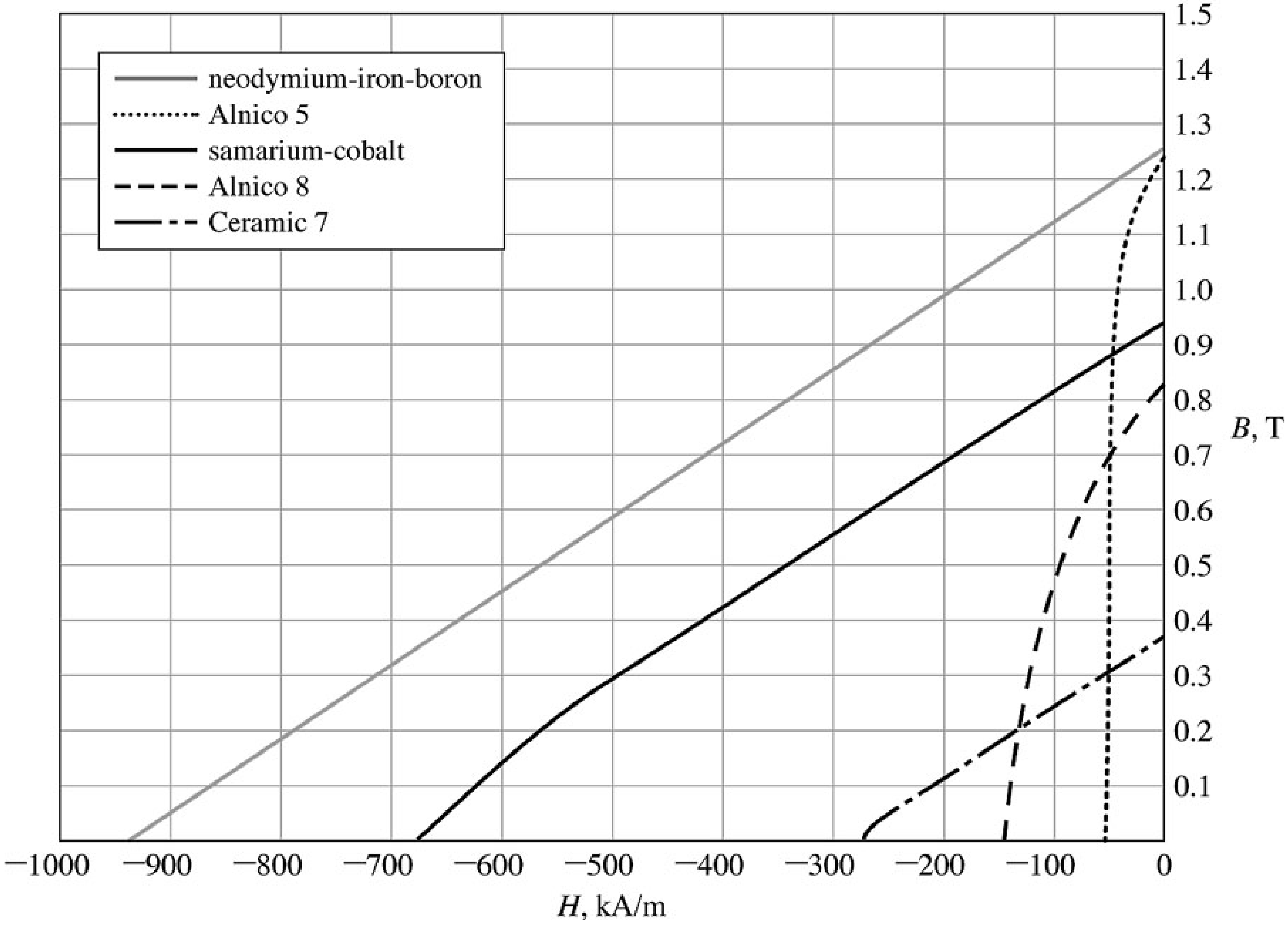
- Use same load line; B_g = 0.38 Gauss (much lower than with Alnico)
- Note: Earth's magnetic field ~ 0.5 Gauss





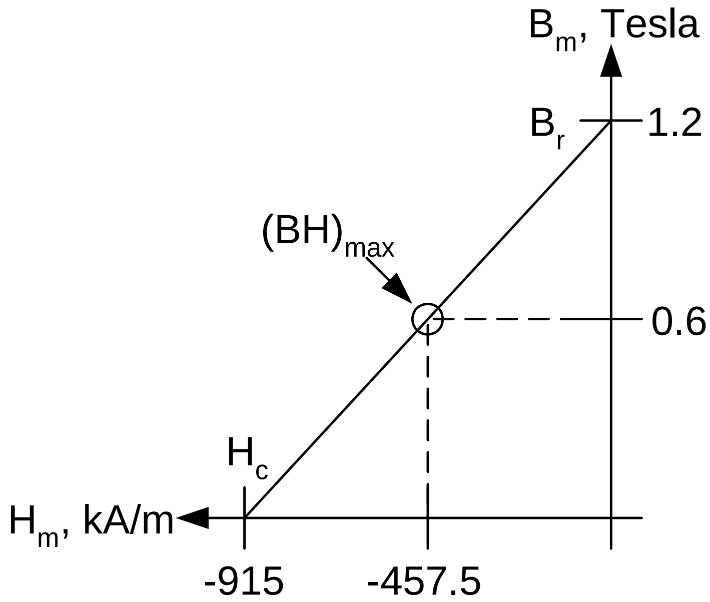
Some Common Permanent Magnet Materials

- Other tradeoffs not shown here include: mechanical strength, temperature effects, etc.

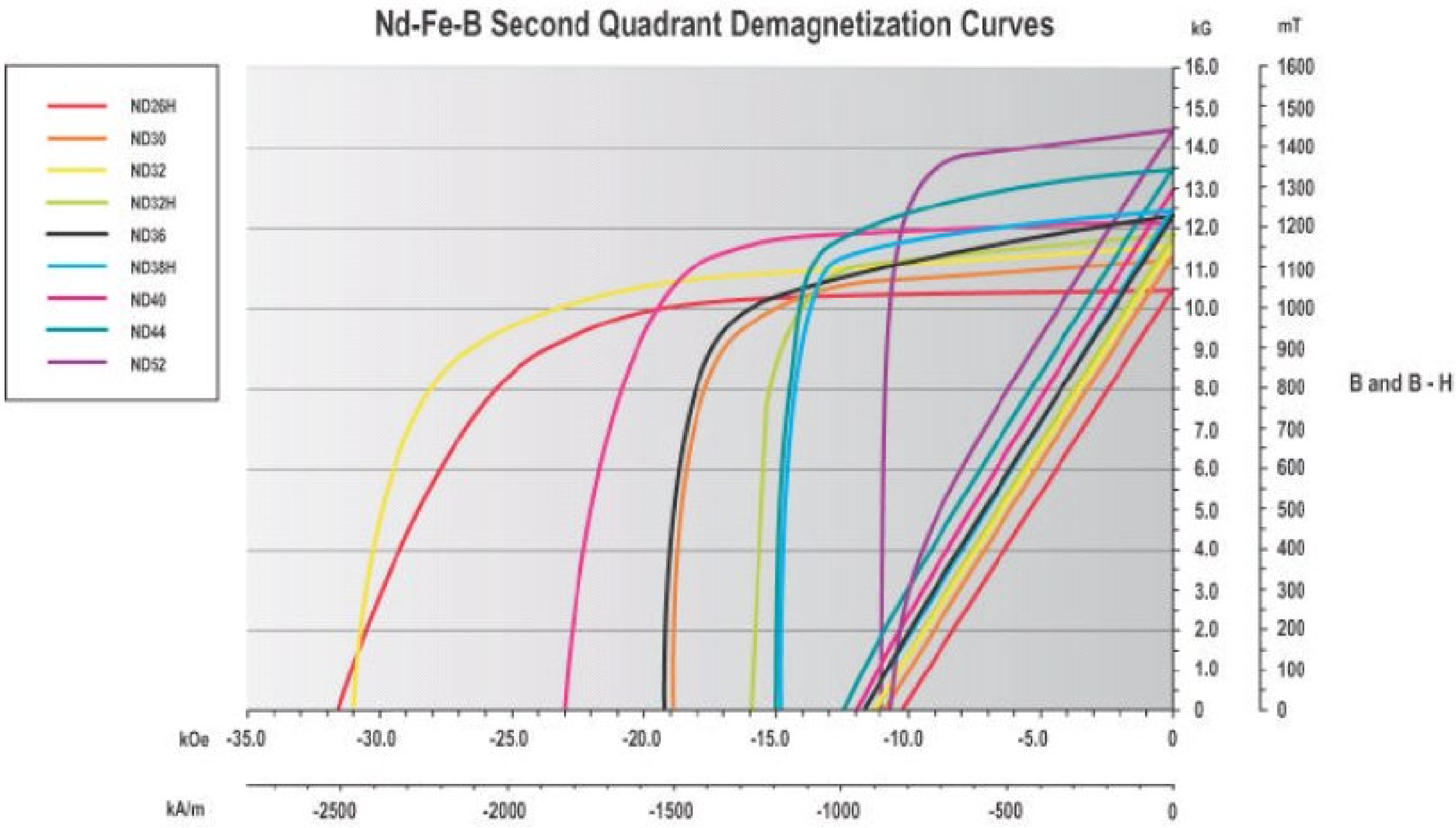


Typical NdFeB B-H Curve

- Neodymium-iron-boron (NdFeB) is the highest strength permanent magnet material in common use today
- Good material for applications with temperature less than approximately 80 - 150C
- Cost per pound has reduced greatly in the past few years
- B/H curve below for “grade 35” or 35 MGOe material



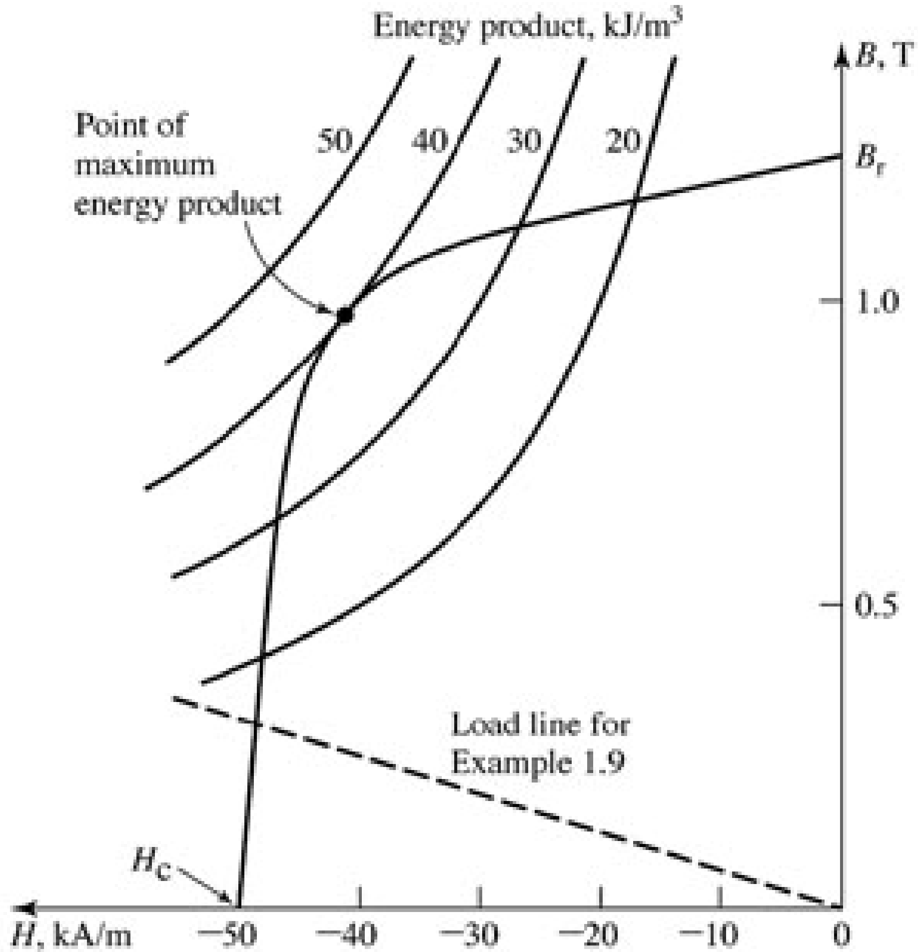
NdFeB B-H Curves for Different Grades



Reference: Dexter Magnetics, Inc. http://www.dextermag.com

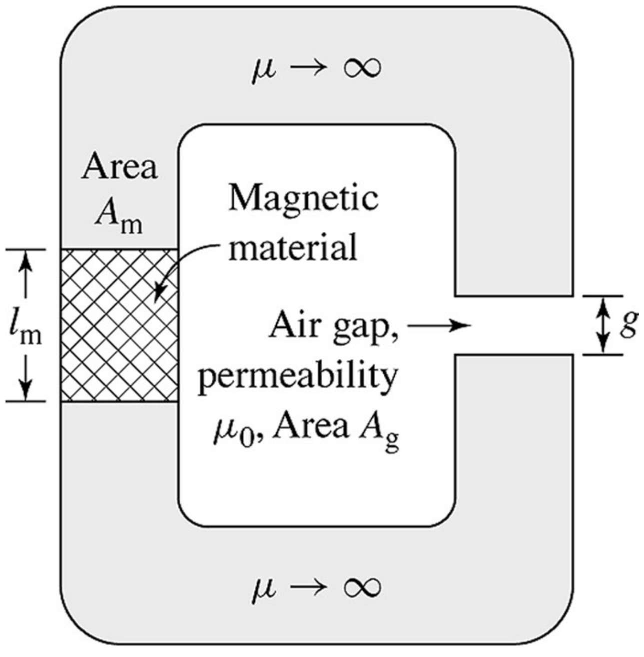
Maximum Energy Product

- BH has units of Joules per unit volume



(a)

Maximum Energy Product



Why is maximum energy product important?

(1) $B_g = B_m \left(\frac{A_m}{A_g} \right)$

(2) $\frac{H_m l_m}{H_g g} = -1$

Let's find B_g^2

$$B_g^2 = \left(B_m \frac{A_m}{A_g} \right) \times \left(\mu_o H_g \right)$$

$$= -\left(B_m \frac{A_m}{A_g} \right)\left(\mu_o \frac{H_m l_m}{g} \right)$$

$$= \mu_o \left(\frac{Vol_{mag}}{Vol_{gap}} \right)\left(-B_m H_m \right)$$

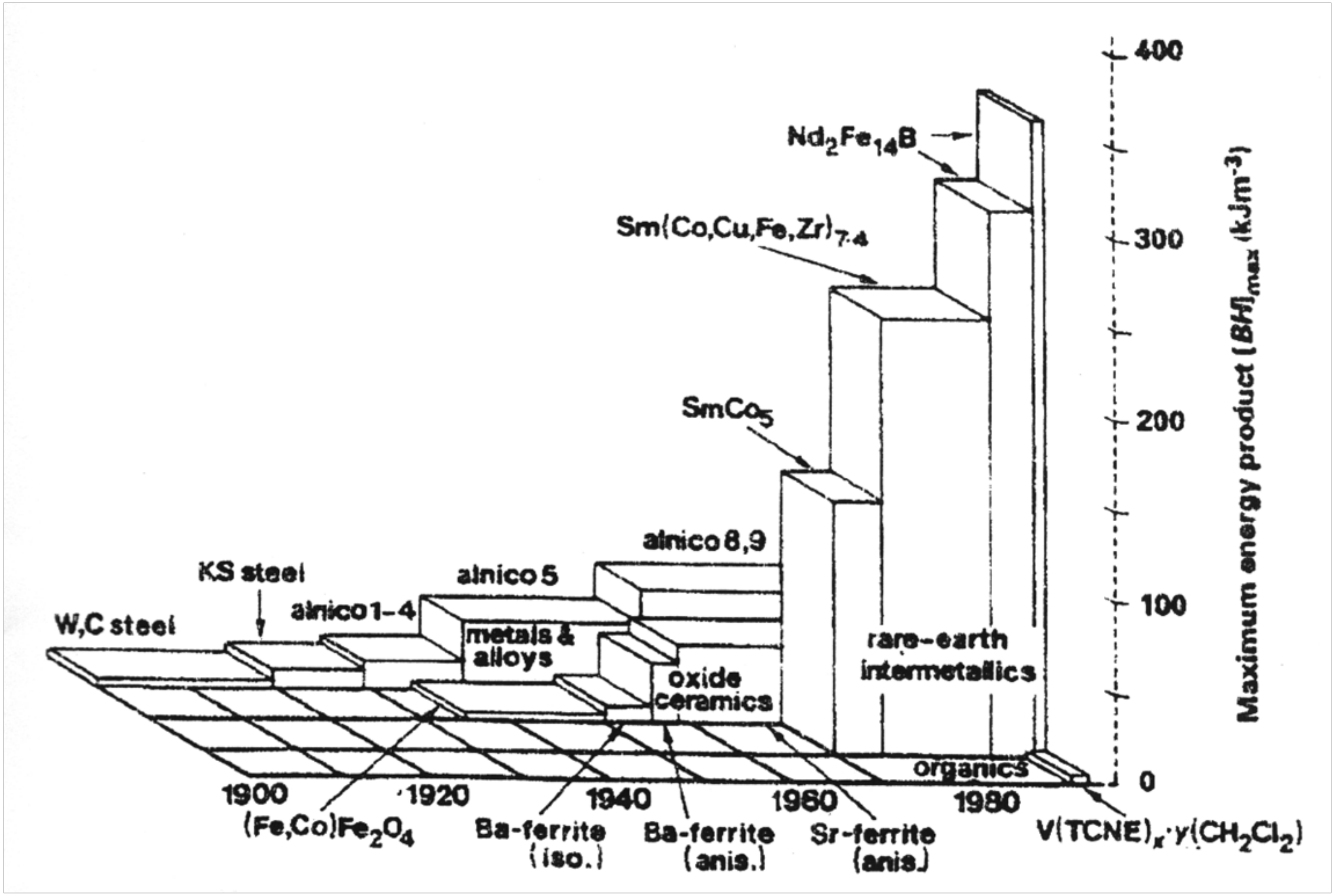
Solve for magnet volume Vol_{mag}

$$Vol_{mag} = B_g^2 \left(\frac{Vol_{gap}}{-B_m H_m} \right)$$

To use minimum volume of magnet for a given B_g, operate magnet at $(BH)_{max}$ point

Progress in PM Specs

- One figure of merit is $(BH)_{max}$ product



Reference: J. Evetts, *Concise Encyclopedia of Magnetic and Superconducting Materials,* Pergamon Press, Oxford, 1992

Progress in PM Specs

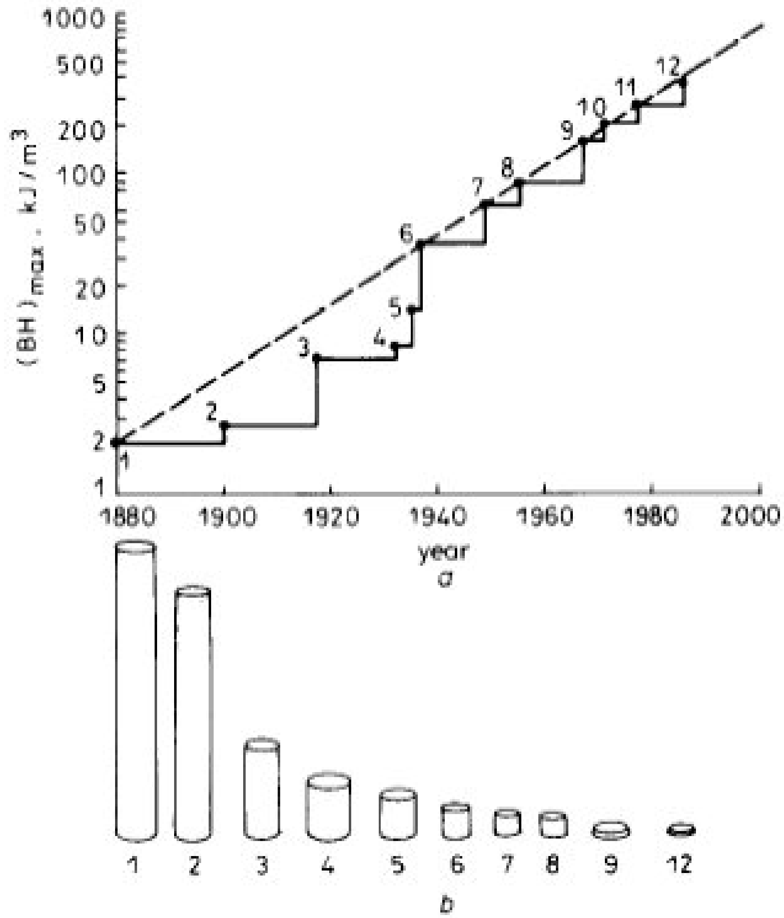


Fig. 7 *Development of permanent magnets*

a History of $(BH)_{max}$ values achieved since 1880
b Schematic representation of change in magnet size required for a specific application

Reference: K. Overshott, “Magnetism: it is permanent,” *IEE Proceedings-A*, vol. 138, no. 1, Jan. 1991, pp. 22-31

Applications for Permanent Magnets

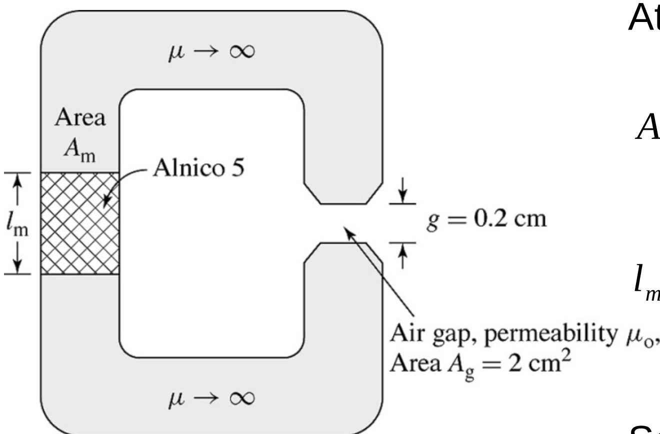
- Disk drives
- Speakers
- Motors
  - Rotary motors (Toyota Prius)
  - Linear motors (Maglev, people moving)
- Refrigerator magnets
- Proximity sensors and switches
- Compasses
- Magnetic bearings and magnetic suspensions (Maglev)
- Water filtration
- Plasma fusion research, NMR
- Eddy current brakes (ECBs)
- Etc.

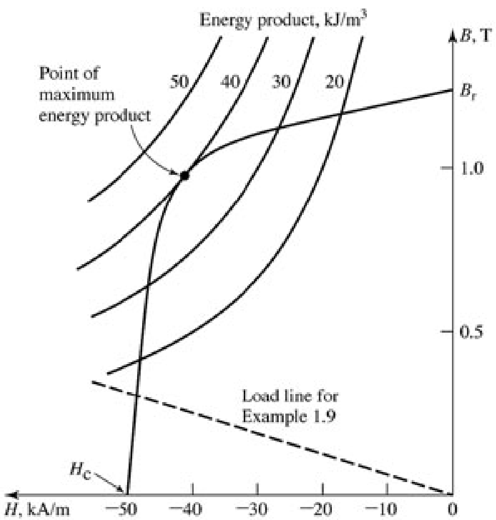
References:

1. R. Parker, *Advances in Permanent Magnetism*, John Wiley, 1990
2. P. Campbell, *Permanent Magnet Materials and their Applications*, Cambridge University Press, 1994

Example: Maximum Energy Product

- *Fitzgerald*, Example 1.10: Find magnet dimensions for desired airgap flux density B_g = 0.8 Tesla





(a)

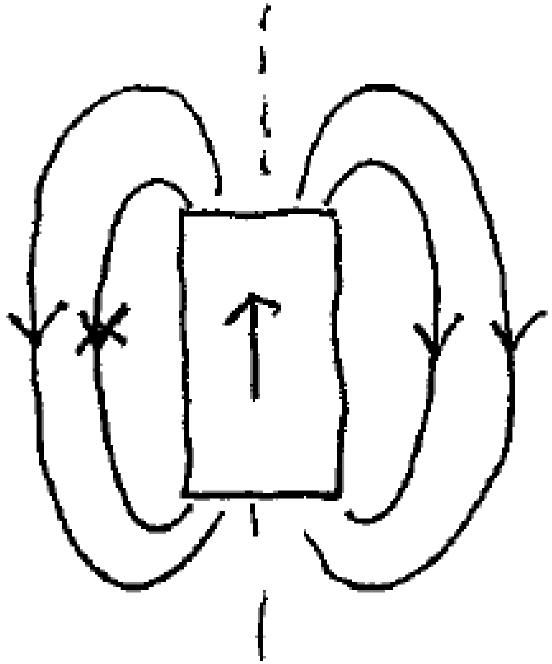
At maximum (BH), B_m=1.0 T and H_m = -40 kA/m

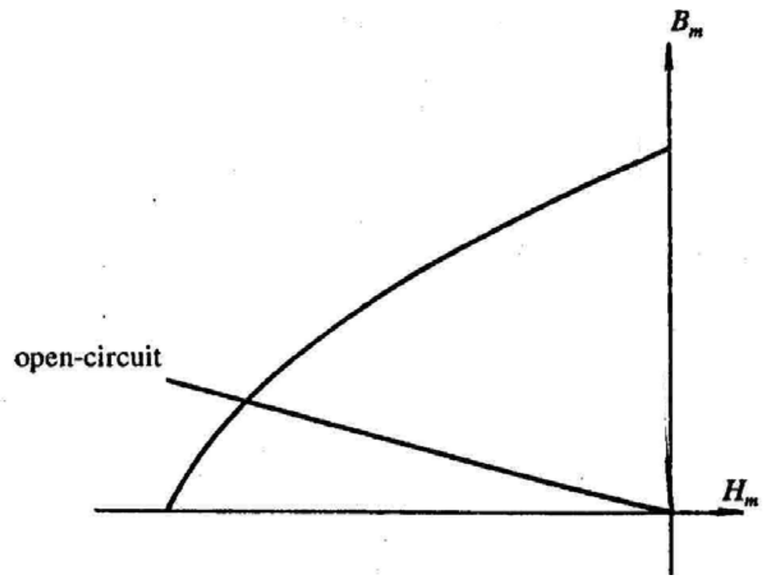
$$A_m = A_g\left(\frac{B_g}{B_m}\right) = (2cm^2)\left(\frac{0.8}{1.0}\right) = 1.6cm^2$$

$$l_m = -g\left(\frac{H_g}{H_m}\right) = -g\left(\frac{B_g}{\mu_o H_m}\right) = (-0.2cm)\left(\frac{0.8}{(4\pi\times10^{-7})(-40000)}\right) = 3.18cm$$

So, magnet is 3.18 cm long and 1.6 cm^2 in area

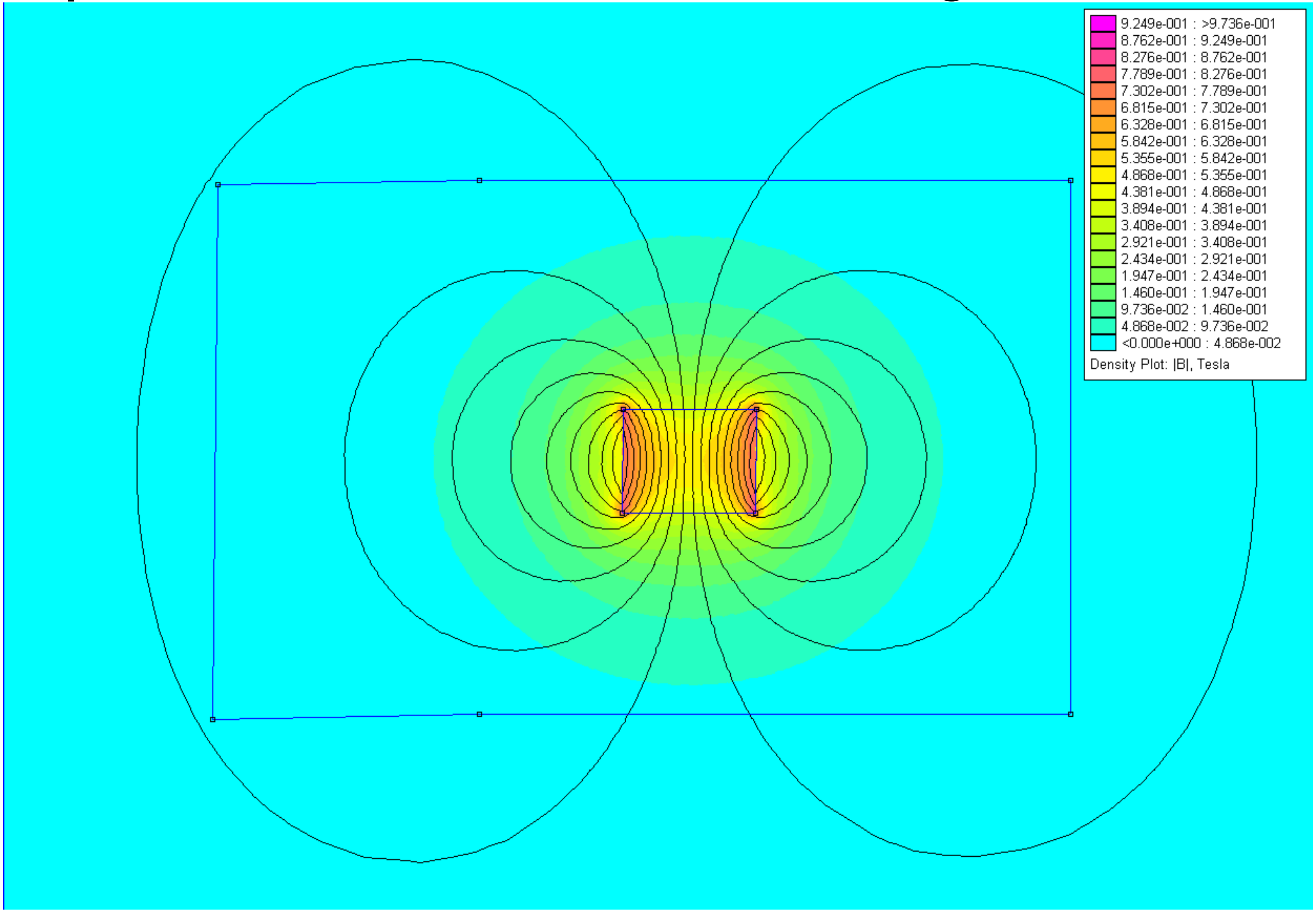
Open-Circuited Permanent Magnet





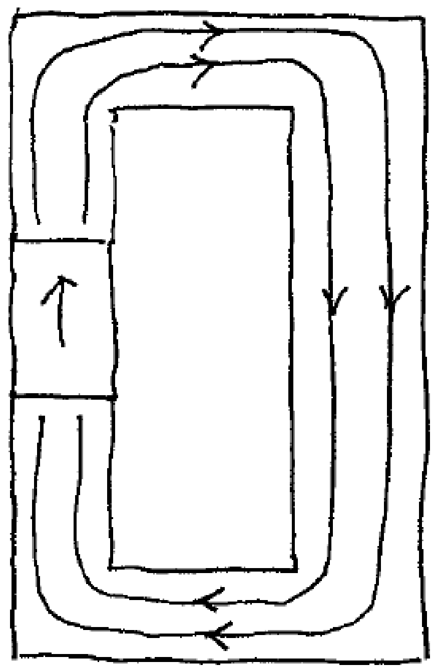
Reference: P. Campbell, *Permanent Magnet Materials and their Applications*, Cambridge University Press, 1994, pp. 89

Open Circuited Permanent Magnet --- FEA



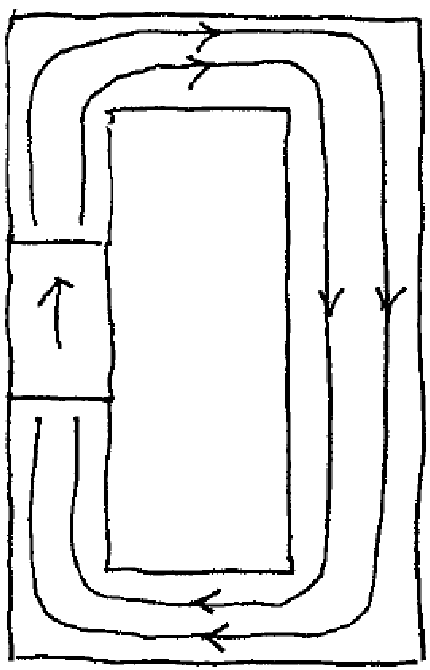
Short-Circuited Permanent Magnet

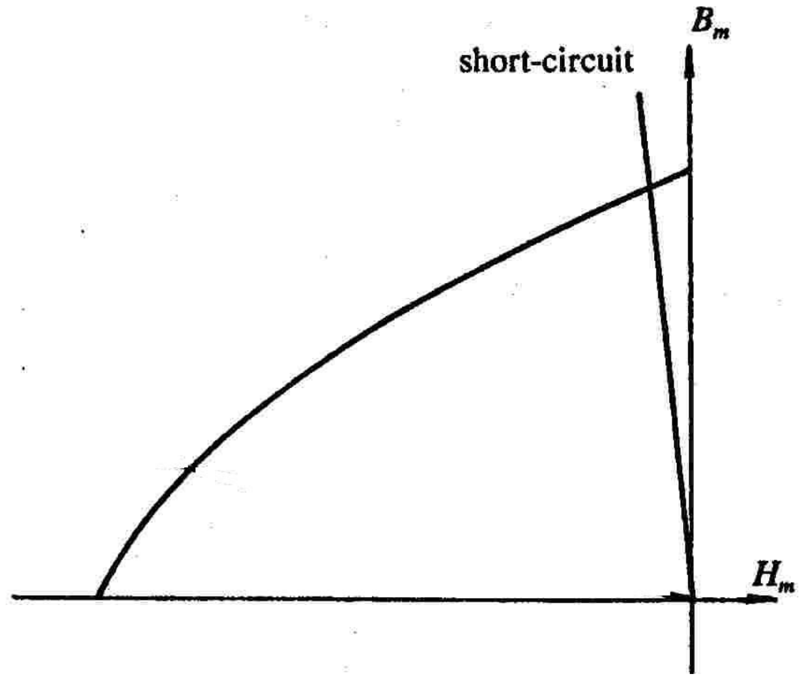
- Find B inside core, ignoring any leakage and assuming infinite permeability in core



Short Circuited Permanent Magnet

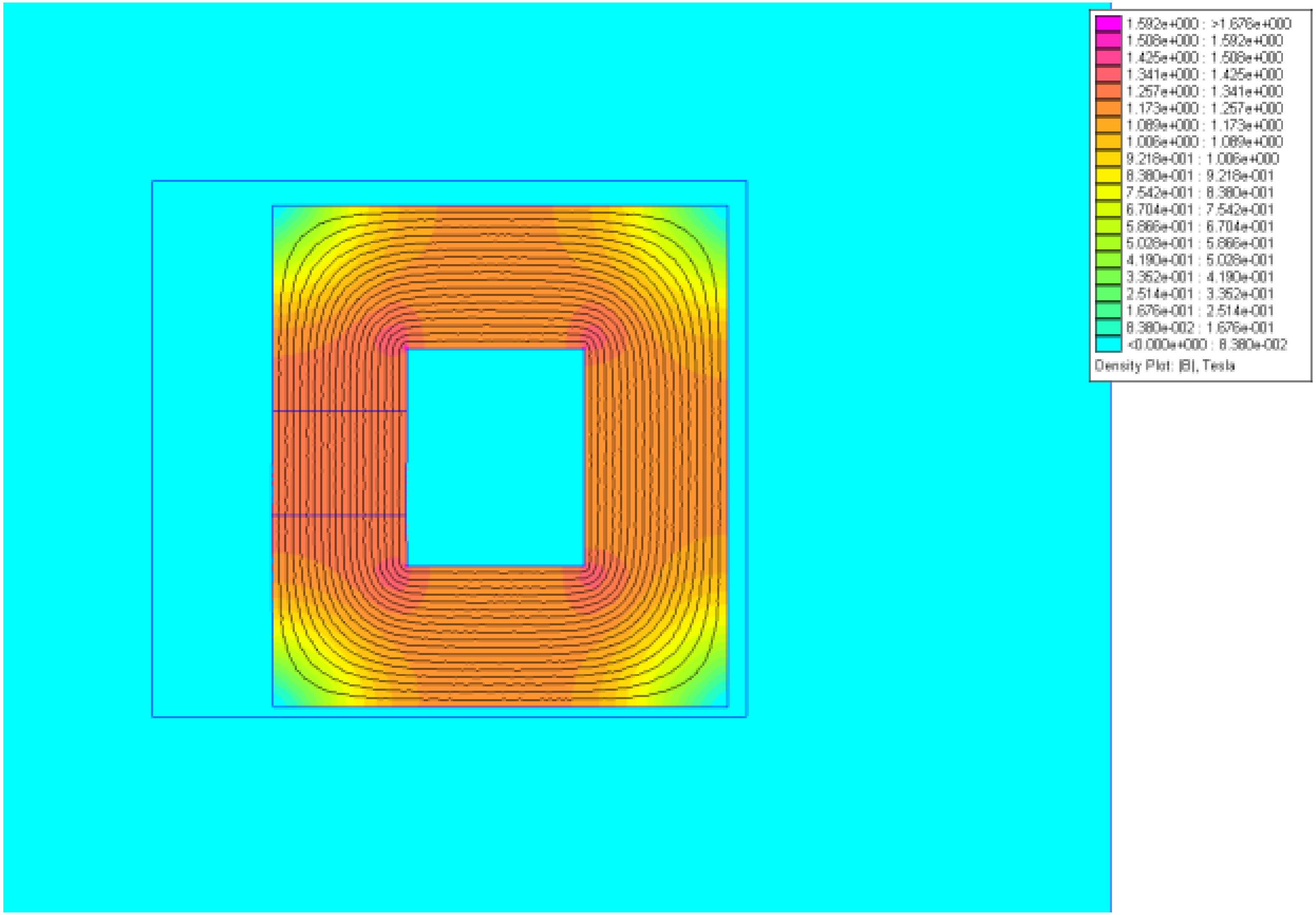
- For infinite permeability, load line is vertical
- Intersection of load lines occurs at $B \approx B_r$





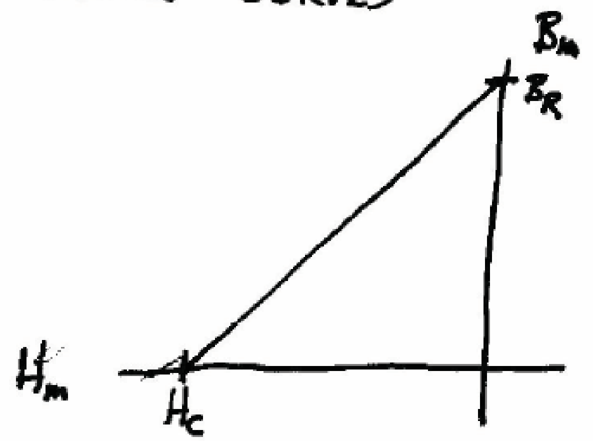
Reference: P. Campbell, *Permanent Magnet Materials and their Applications*, Cambridge University Press, 1994, pp. 89

Short Circuited Permanent Magnet --- FEA



Circuit Modeling of Permanent Magnets

SOME MAGNETS (I.E. ND-FE-B) HAVE LINEAR 2ND QUADRANT DEMAG CURVES



(1) $$B_m = B_R + \mu_m H_m$$

and

(2) $$\mu_m = \frac{B_R}{H_c}$$

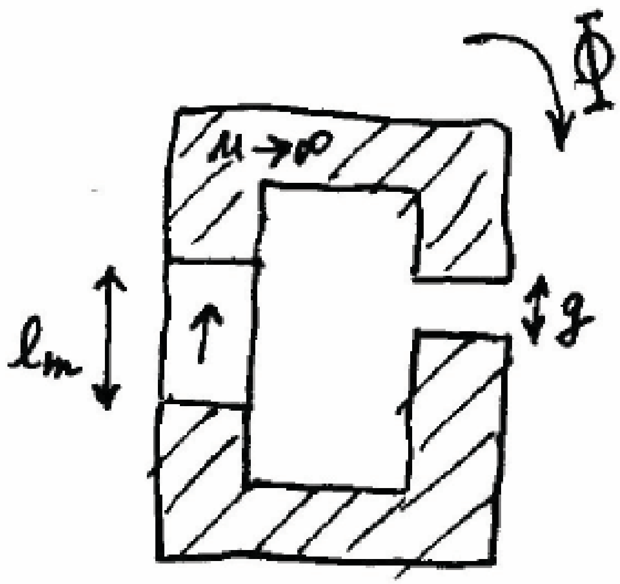
$$\Downarrow$$

(3) $$H_m = \frac{B_m}{\mu_m} - H_c$$

Reference: E. P. Furlani, *Permanent Magnet and Electromechanical Devices*, Academic Press, 2001

Circuit Modeling of Permanent Magnets

LET'S USE THIS IN A MAGNETIC CIRCUIT WITH AIRGAP:



AMPERE'S LAW

$$④ \quad H_m \ell_m + H_g g = 0$$

Circuit Modeling of Permanent Magnets

PUT (3) INTO (4)

$$\left(\frac{B_m}{\mu_m} - H_c\right) l_m + H_g g = 0$$

$$\Downarrow$$

(5) $H_g g + \dfrac{B_m}{\mu_m} l_m = H_c l_m$

OR $\dfrac{B_g g}{\mu_o} + \dfrac{B_m l_m}{\mu_o} = H_c l_m$

TOTAL FLUX:

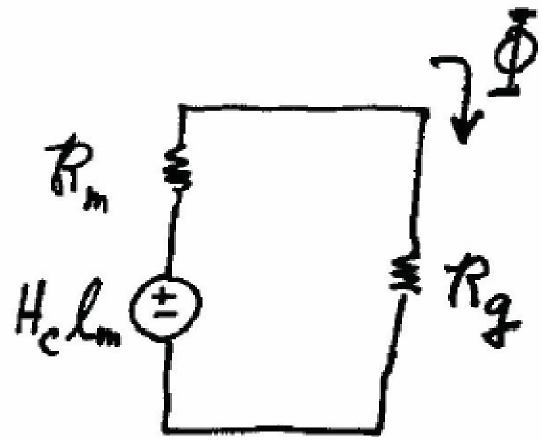
$$\Phi = B_g A_g = B_m A_m$$

SO, (5) REDUCES TO:

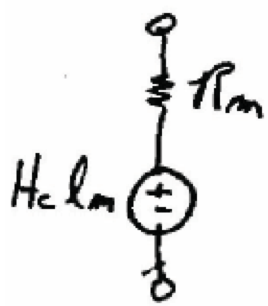
$$\Phi(\mathcal{R}_g + \mathcal{R}_m) = H_c l_m$$

Circuit Modeling of Permanent Magnets

WE CAN MODEL THIS AS FOLLOWS:



PM MODEL:



Example: Circuit Modeling of Permanent Magnets

<u>Example:</u>

Grade 37 NdFeB

$H_c = 950{,}000\ A/m$

$\mu_m = 1.048\,\mu_o$

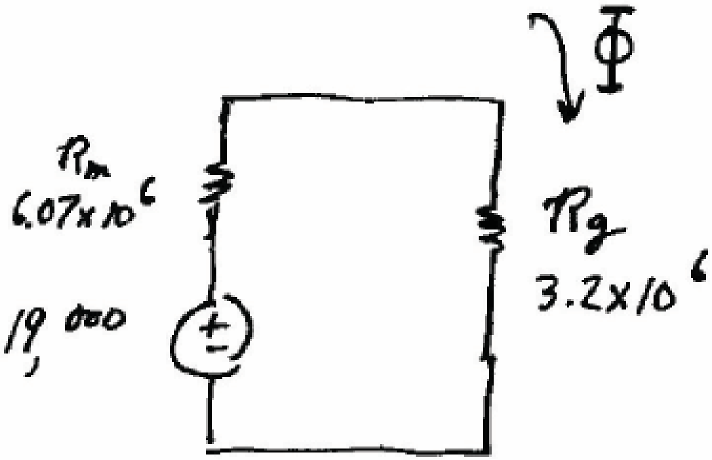
$l_m = 2cm$

$g = 1cm$

$A_m = A_g = 25cm^2$

Find B_g

Example: Circuit Modeling of Permanent Magnets



This example is particularly simple because $A_m = A_g$

Solution:

$$H_c l_m = (950,000)(0.02) = 19,000$$

$$R_m = \frac{l_m}{\mu_R A_m} = \frac{0.02}{(1.048)(4\pi\times10^{-7})(25\times10^{-4})}$$

$$= 6.07\times10^{6}$$

$$R_g = \frac{g}{\mu_0 A_g} = \frac{0.01}{(4\pi\times10^{-7})(25\times10^{-4})}$$

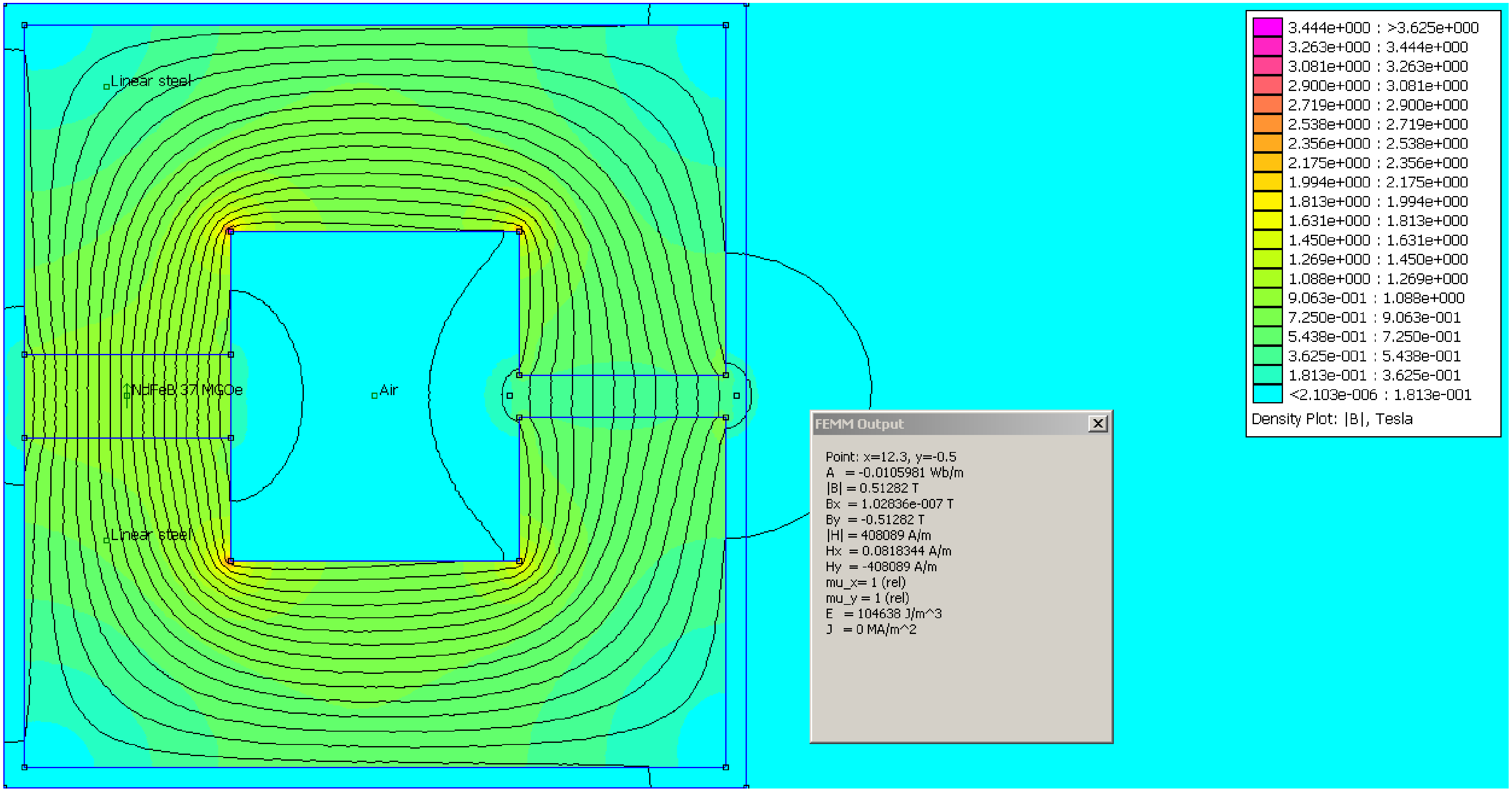
$$= 3.2\times10^{6}$$

$$\Phi = \frac{H_c l_m}{R_m + R_g} = 2.04\times10^{-3}\ Wb$$

$$B_g = \frac{\Phi}{A_g} = \frac{2.04\times10^{-3}}{25\times10^{-4}}$$

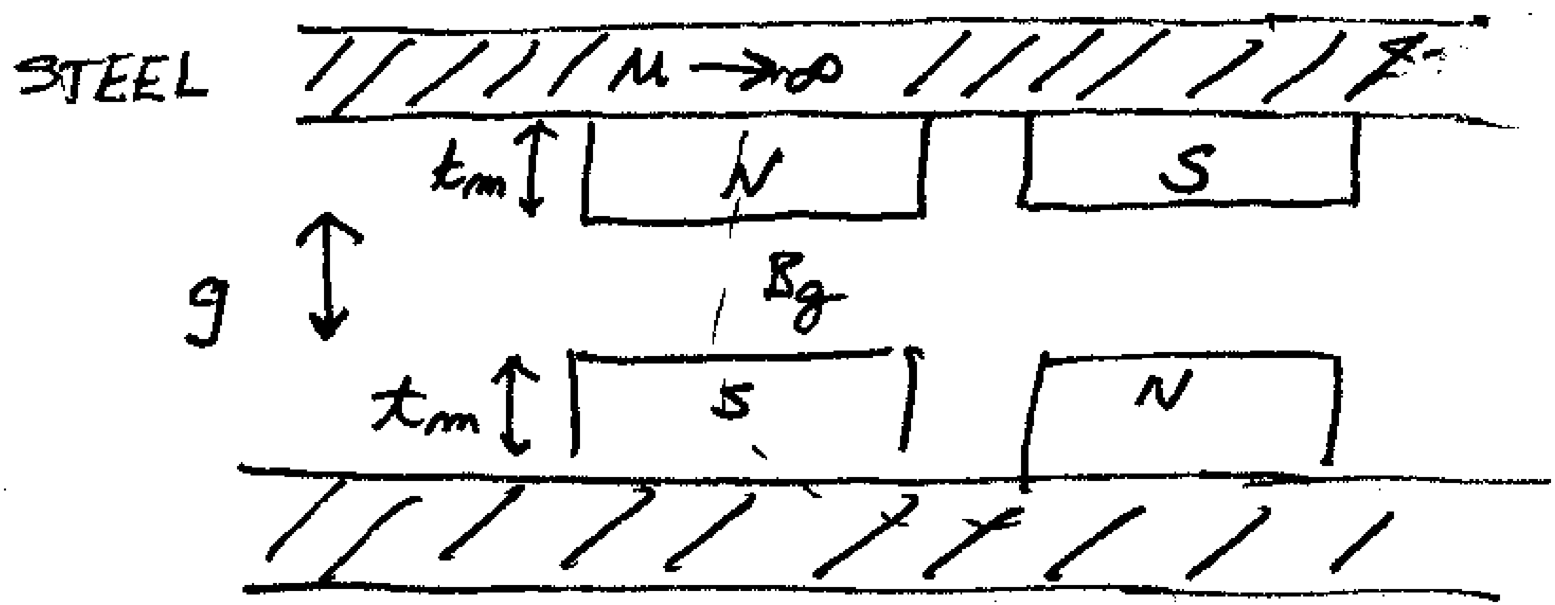
$$\boxed{\therefore B_g = 0.82\ Wb/m^2 \quad 0.82\ Tesla}$$

Example: Circuit Modeling of Permanent Magnets---FEA



Example: Magnetic Circuit With Steel

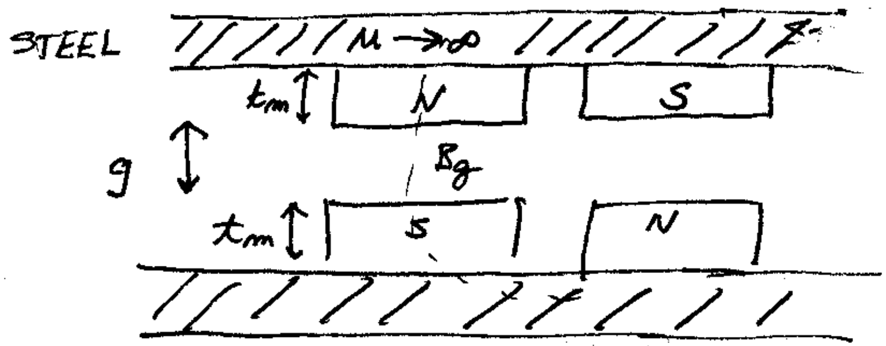
- Estimate airgap field B_g assuming grade 37 NdFeB
- Airgap g, magnet thickness t_m



Assume no saturation in steel

Example: Magnetic Circuit With Steel

- This analysis also ignores leakage



Assume no saturation in steel

① $2 H_m t_m + H_g g = 0$
(Ampere's law)

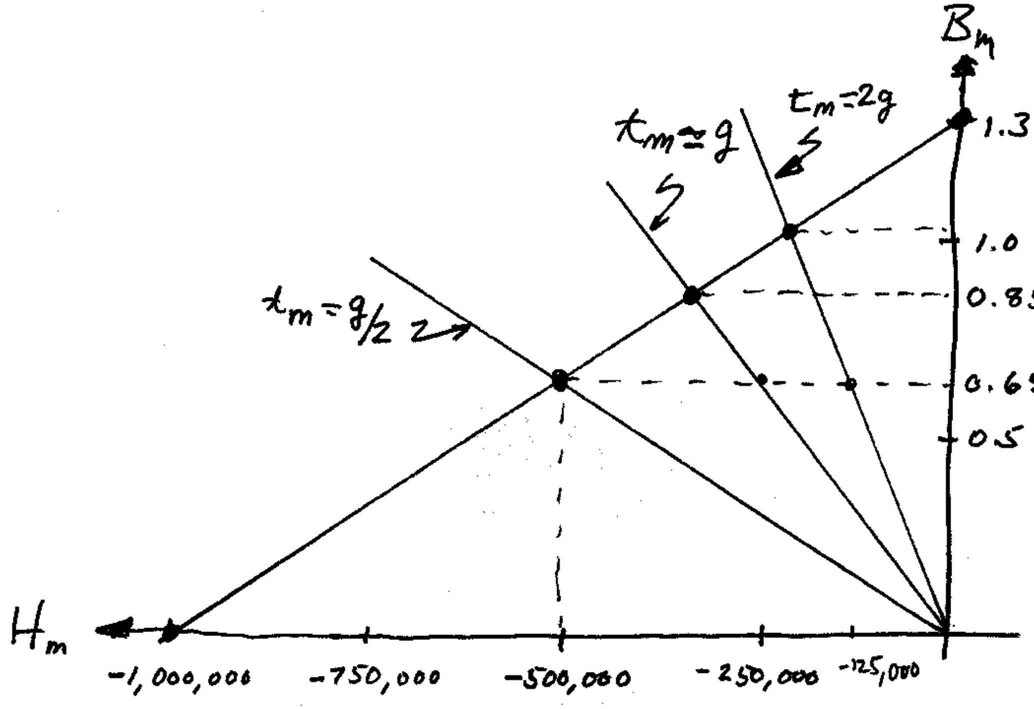
② $H_g = \dfrac{B_g}{\mu_0} = \dfrac{B_m}{\mu_0}$
(constitutive relation)

$$2 H_m t_m + \frac{B_m g}{\mu_0} = 0$$

$$\frac{B_m g}{\mu_0} = -2 H_m t_m$$

$$\boxed{\therefore \frac{B_m}{H_m} = \frac{-2 t_m \mu_0}{g}}$$

Example: Magnetic Circuit With Steel --- Operating Point vs. Magnet Thickness t_m

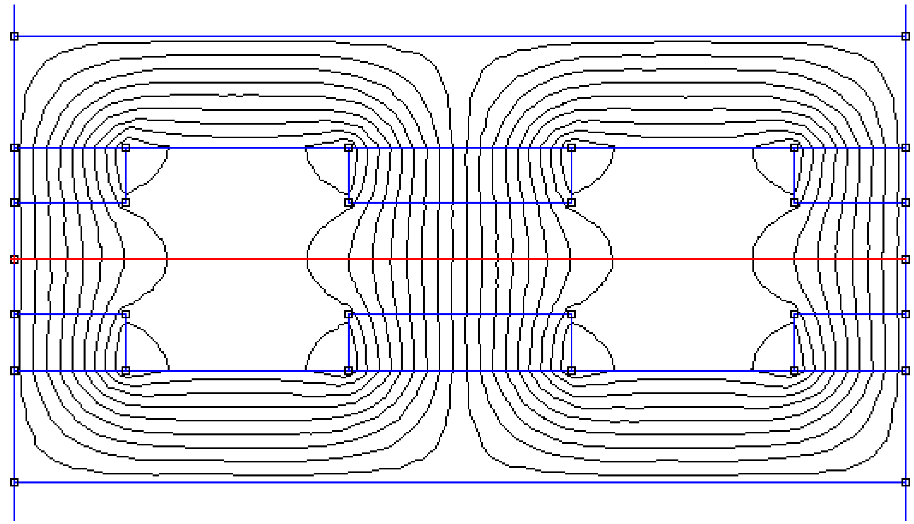


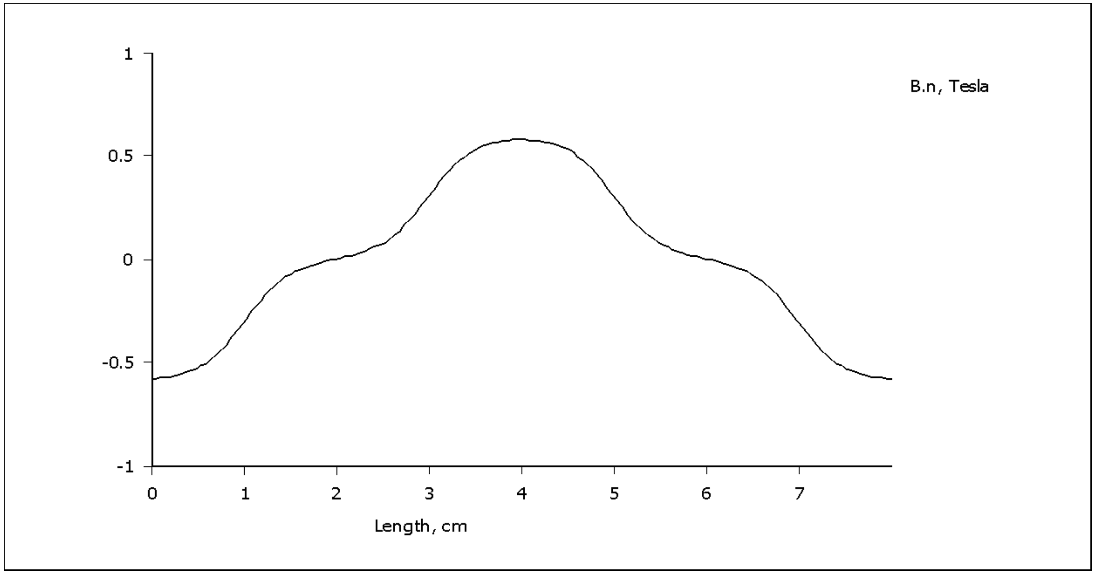
$t_m = g/2 \rightarrow B_g \sim 0.65\,T$

$t_m \simeq g \rightarrow B_g \sim 0.85\,T$

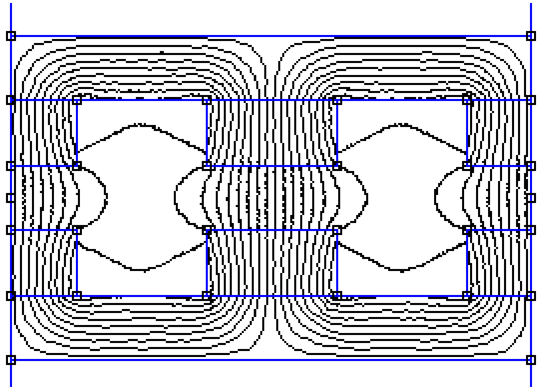
$t_m = 2g \rightarrow B_g \sim 1.05\,T$

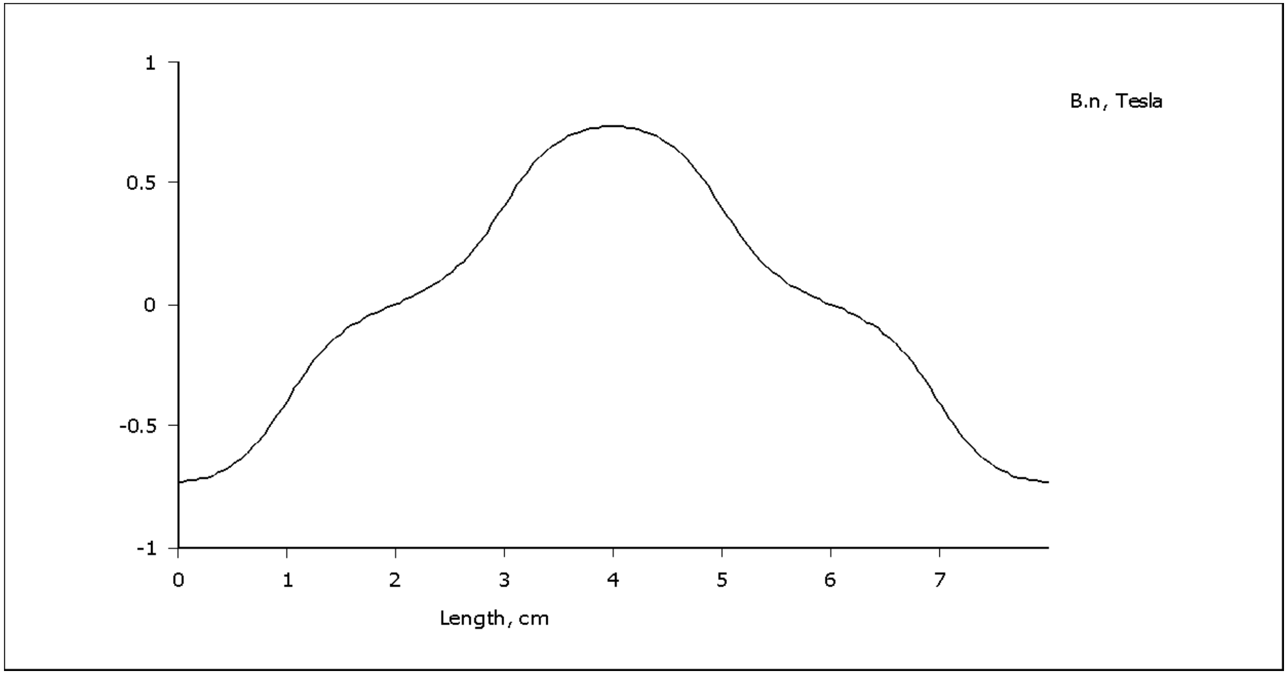
Example: FEA with Magnet Thickness $t_m = g/2$



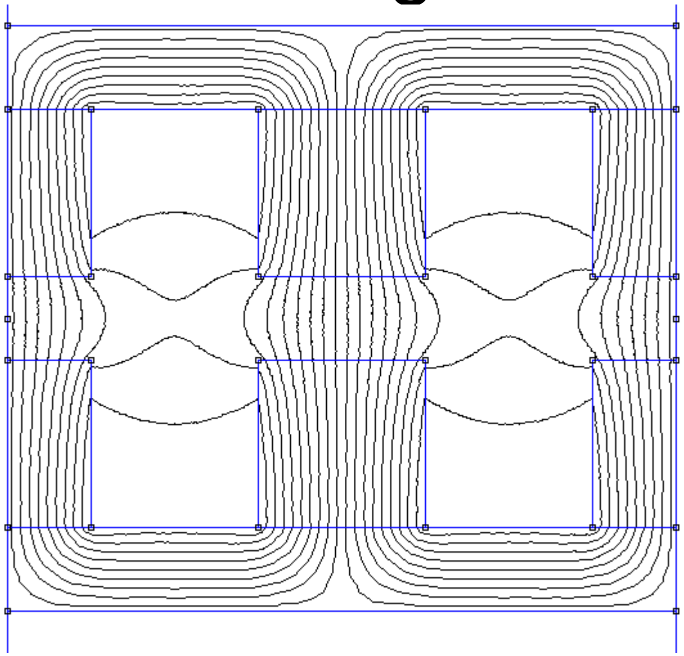


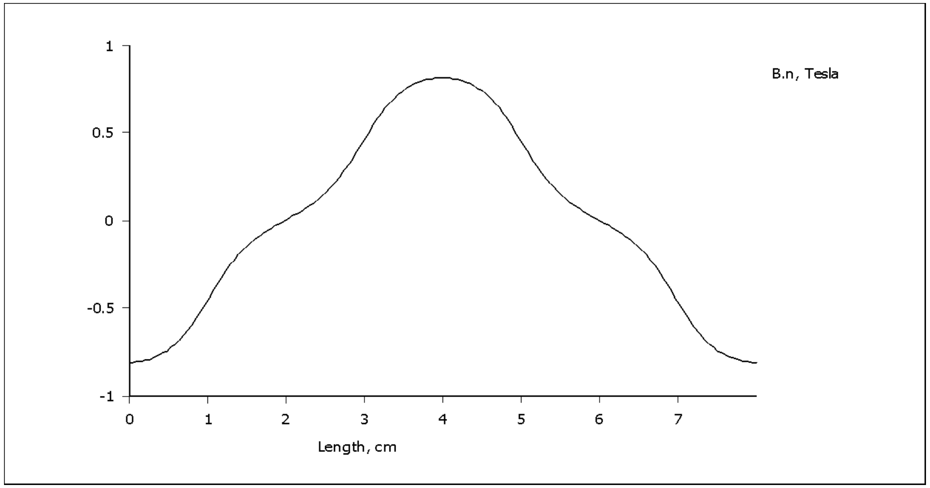
Example: FEA with Magnet Thickness $t_m = g$





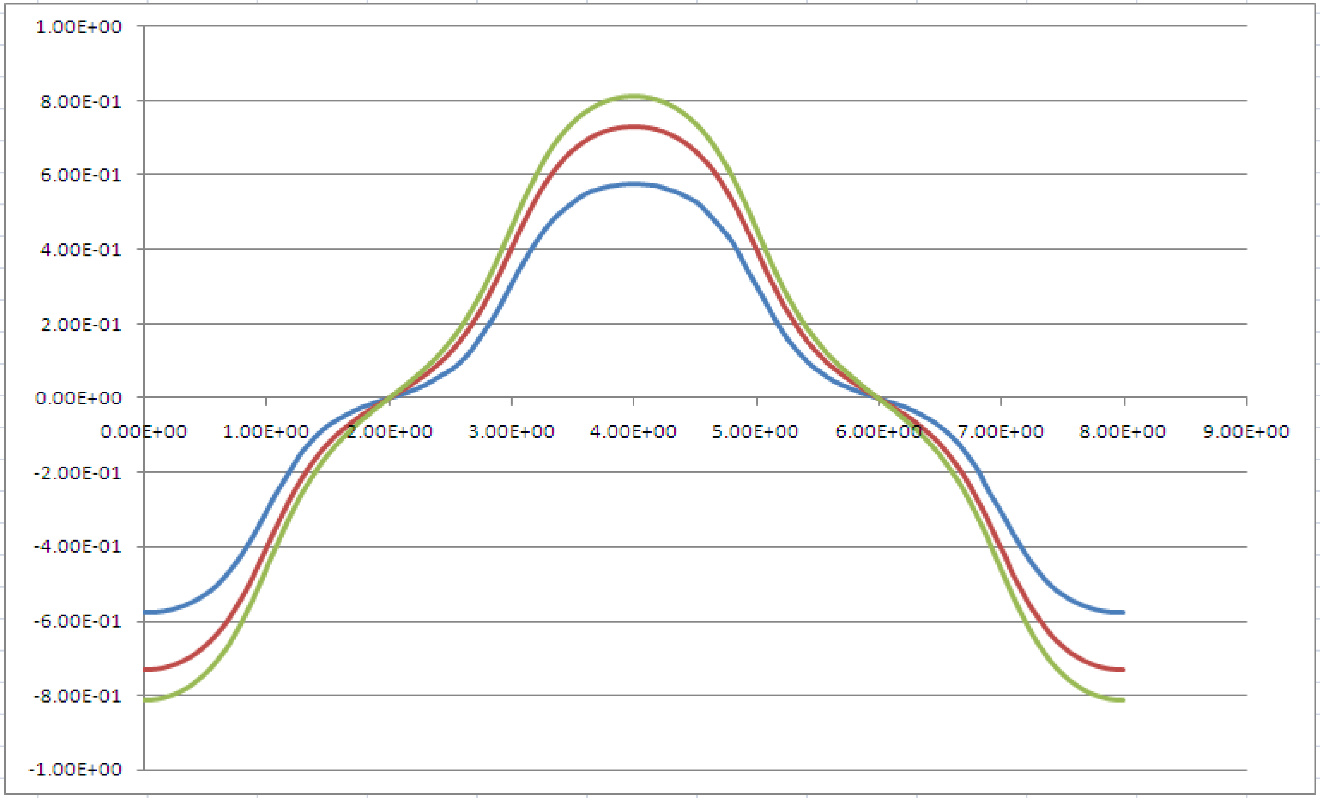
Example: FEA with Magnet Thickness $t_m = 2g$





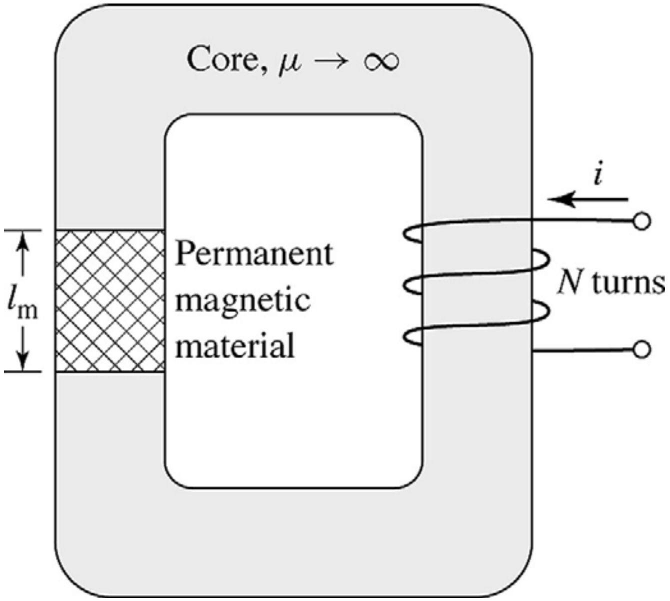
Example: Comparison of Different Magnet Thicknesses

	Bpk, 2D analytic	Bpk, 2D FEA
tm = g/2	0.65T	0.58T
tm = g	0.85T	0.73T
tm=2g	1.05T	0.82T



PM and a Winding

- Many motors have permanent magnets, steel and windings



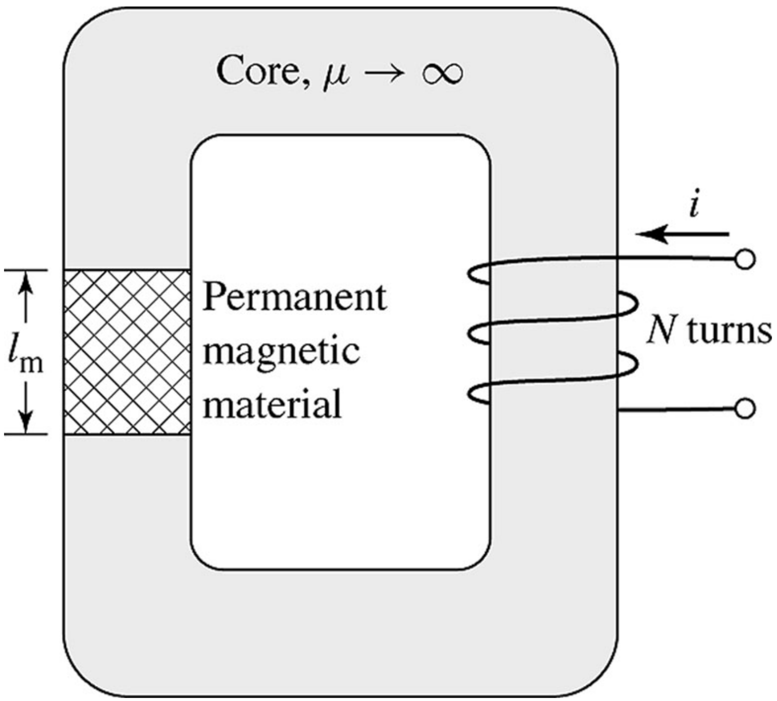
Analysis of PM in closed core, with excitation:

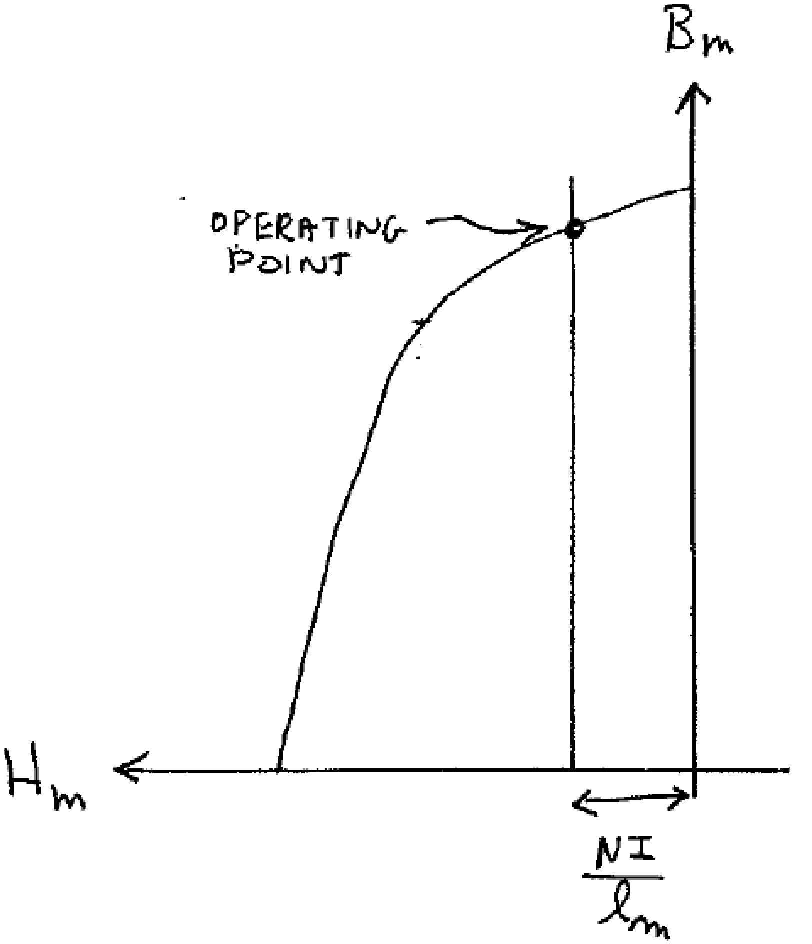
$$H_m l_m = NI$$

$$\therefore H_m = \frac{NI}{l_m}$$

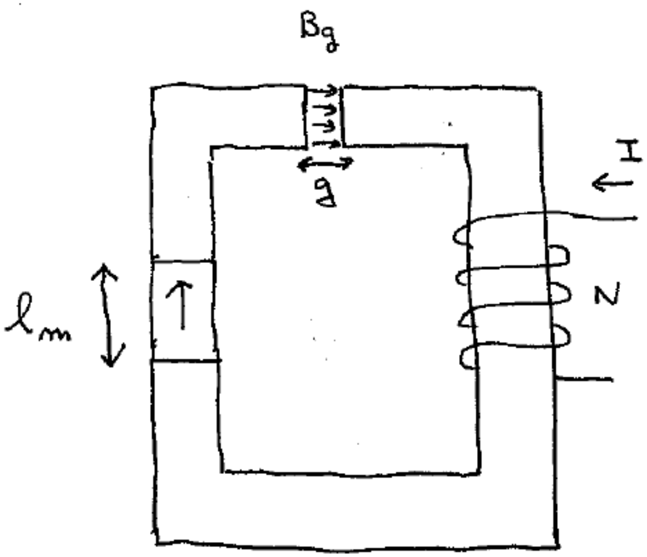
PM and a Winding --- Load Line

- Note that demagnetization can occur if current is sufficiently high





Another Example --- Excitation and Airgap



(1) Ampere's law:

$$H_m l_m + H_g g = NI$$

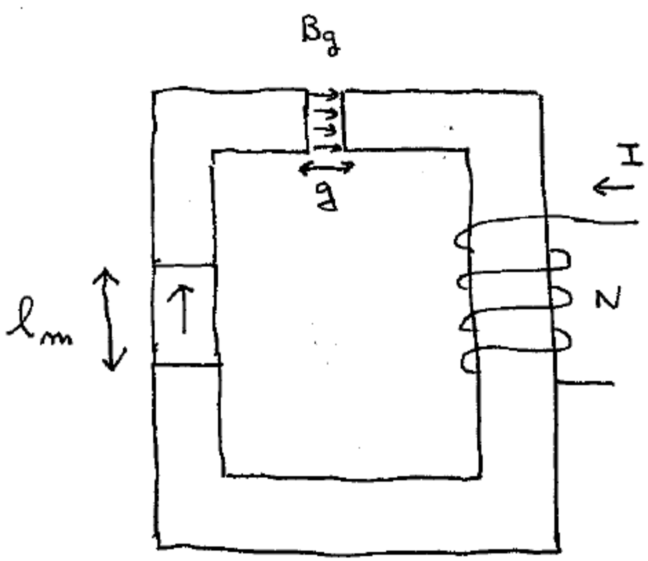
(2) Constitutive law: $B_g = \mu_o H_g$

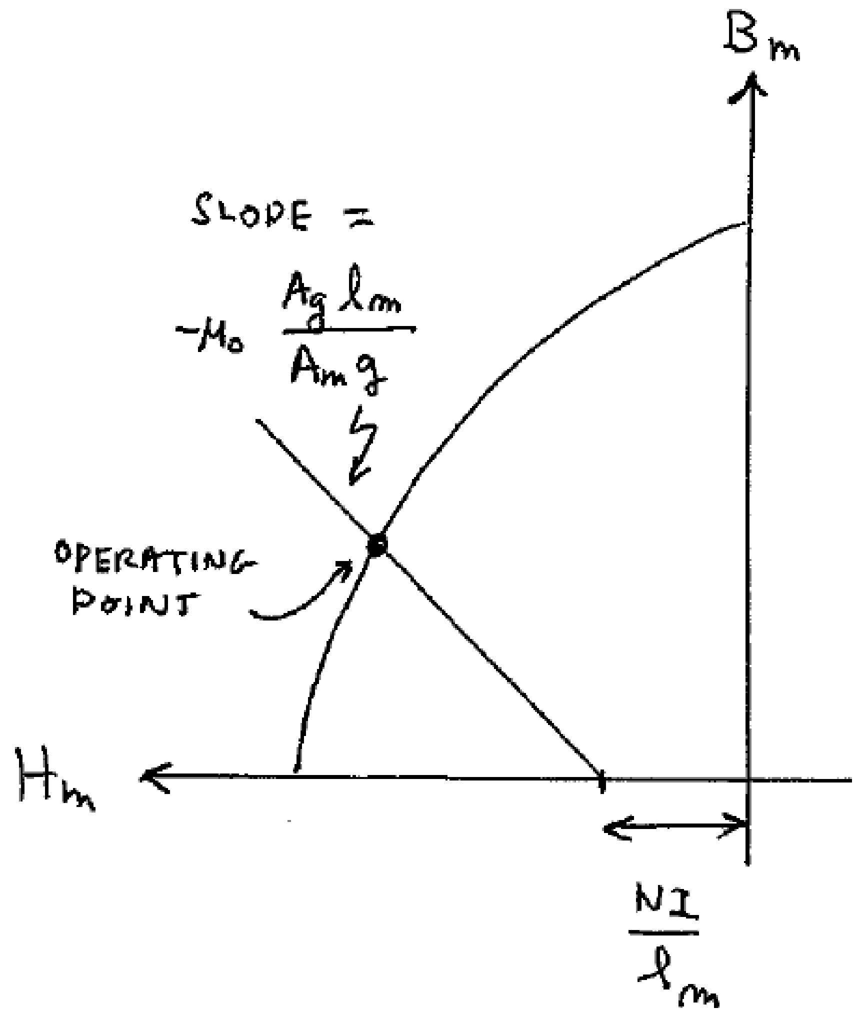
(3) Gauss' law: $B_m A_m = B_g A_g$

Solve for B_m-H_m load line:

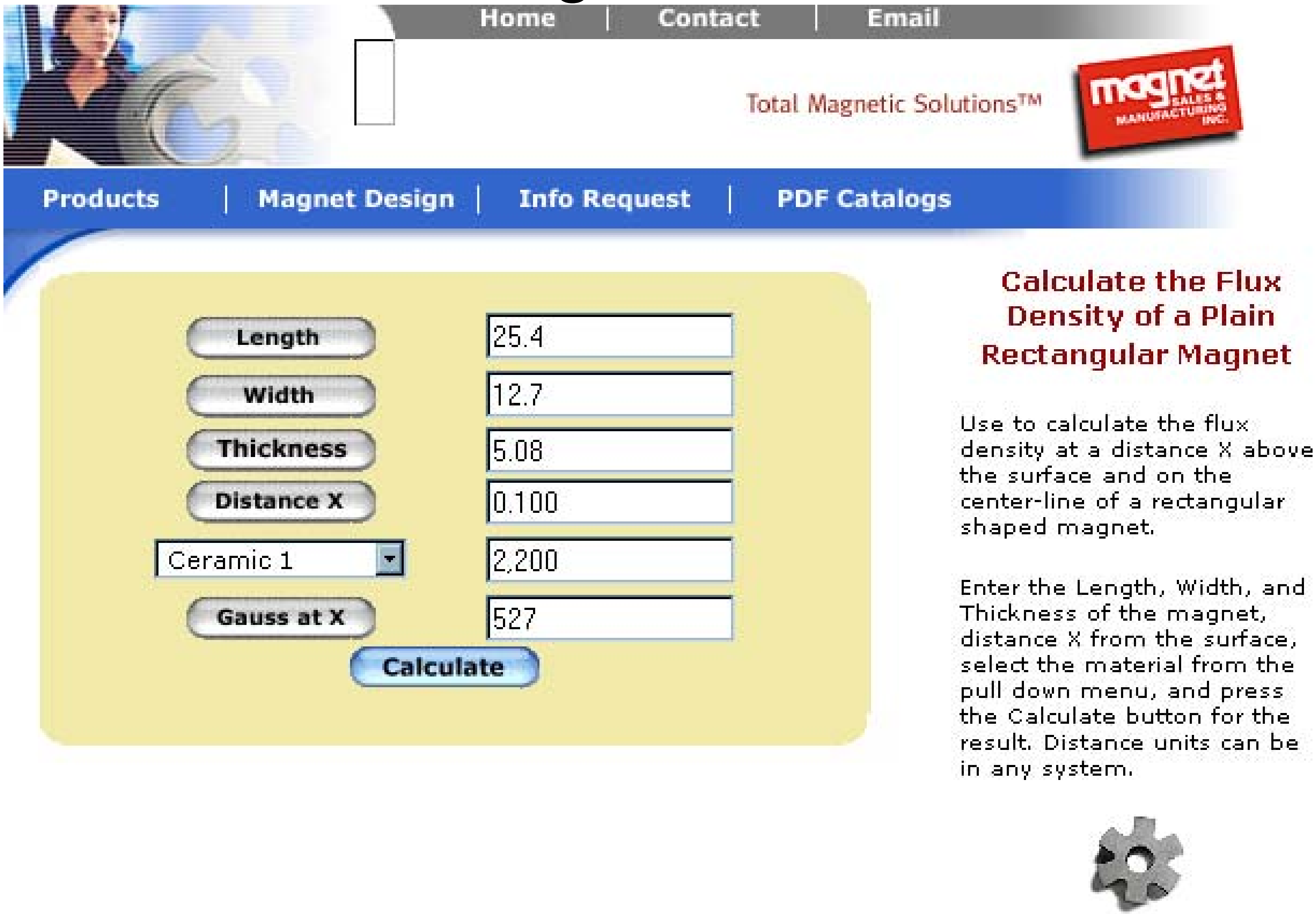
$$B_m = -\mu_o \left(\frac{A_m l_m}{A_m g} \right) \left(H_m - \frac{NI}{l_m} \right)$$

Another Example --- Excitation and Airgap --- Load Line





Interesting Calculation Tool

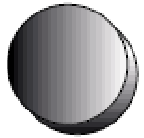


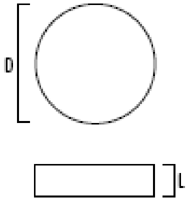
Reference: www.magnetsales.com

What Can You Buy?

NdFeB Rounds

Part Number	Material	Diameter	Length	Orientation
PN35C1250B-N	N36	0.123	0.250	L
PN45C0140B	N45	0.140	0.500	D
PN45C0189B	N45	0.188	0.057	L
94C5668B	94EB	0.250	0.100	L
CM40685	4014	0.250	0.125	L
PN36C0250B	N36	0.250	0.250	L
97C5513B	97CB	0.250	2.000	L
97C5613B	97CB	0.251	0.200	L
PN45C1200B	N45	0.328	1.200	D
PN36SHC0330B	N36SH	0.330	0.330	L
94C5536C	94EB	0.370	0.250	L
97C5446C	97CB	0.375	0.125	L
94C5562B	94EB	0.375	2.100	L
94C4822B	94EA	0.499	0.188	L
NA67A460	34B	0.500	0.190	L
CM41085-2630	2630	0.500	0.250	L
CM41086-3220	3520	0.500	0.250	L
CM40713-3714	4014	0.500	0.250	L
CM41087-3714	4014	0.625	0.250	L
CM41088-2630	2630	0.625	0.250	L
CM40884-3714	4014	0.866	0.393	L
CM41089-3714	4014	0.875	0.500	L
CM41090-2630	2630	0.875	0.500	L
CM41091-2630	2630	1.000	0.500	L
CM41092-3220	3520	1.000	0.500	L
CM41093-3714	4014	1.000	0.500	L



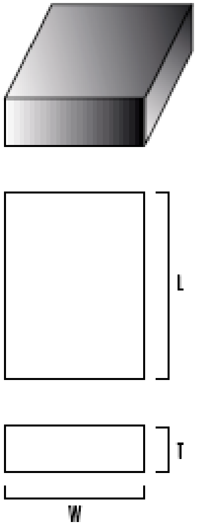


Reference: www.dextermag.com

What Can You Buy?

NdFeB Blocks

Part Number	Material	Thickness	Width	Length	Orientation
CM41030-3714	4014	0.250	0.325	0.500	T
CM41094-2630	2630	0.250	0.500	0.500	T
CM41095-3220	3520	0.250	0.500	0.500	T
CM41096-3714	4014	0.250	0.500	0.500	T
PN36HR2933B	N36H	0.293	0.560	1.044	T
PN38HR0990B	N38H	0.315	0.750	0.990	T
PN38HR0375B	N38H	0.375	0.866	2.362	T
CM40325-3714	4014	0.500	0.750	0.750	T
CM40729-3714	4014	0.500	1.000	1.000	T
CM41097-2630	2630	0.500	1.000	1.000	T
CM41098-3220	3520	0.500	1.000	1.000	T
CM40975-3714	4014	0.590	0.750	0.750	T
PN38HR0122B	N38H	1.000	2.000	2.000	T
CM41099-2630	2630	1.000	2.000	2.000	T
CM40680-3714	4014	1.000	2.000	2.000	T
PN48R0100B	N48	1.000	2.000	2.000	T



Reference: www.dextermag.com

What Can You Buy?

Rectangular Magnets - Finished Sizes sorted in ascending order of Length, Width, and then Thickness.

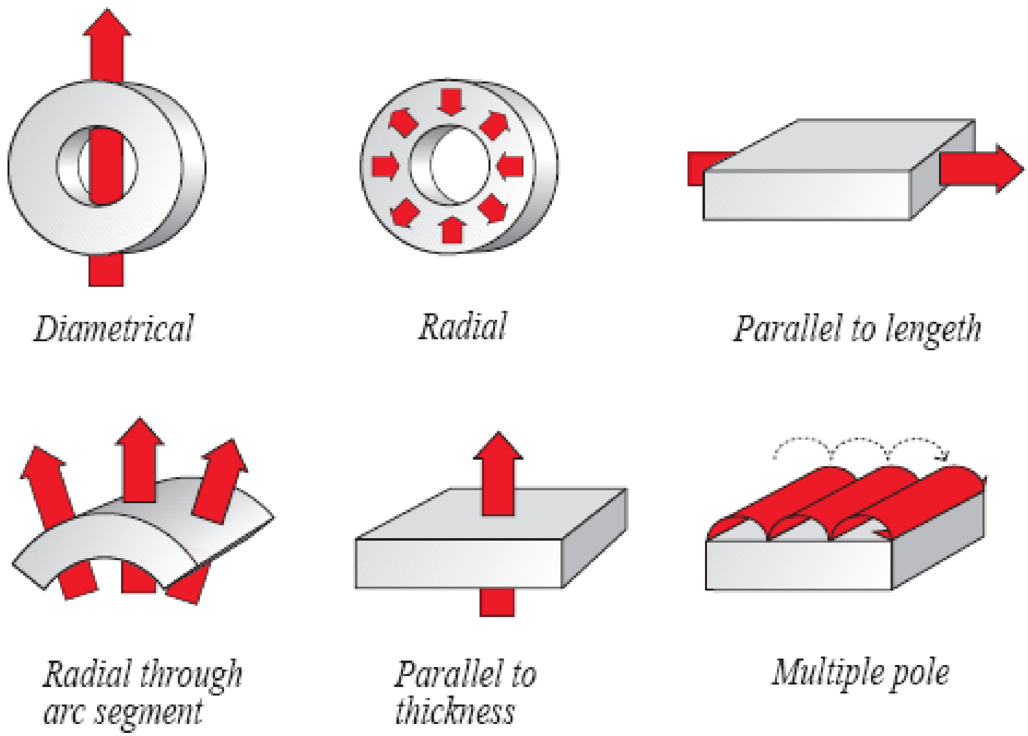
Item Number	Grade	L	W	T*	Condition
35NE111111	35	0.175	0.175	0.175	Die Pressed, Machined
35NE111111-NI	35	0.175	0.175	0.175	Die Pressed, Machined, Nickel Plated
30NE261916-NI	30	0.400	0.290	0.250	Die Pressed, Machined, Nickel Plated
35NE301412	35	0.475	0.230	0.190	Die Pressed, Machined
35NE323232-NI	35	0.500	0.500	0.500	Die Pressed, Machined, Nickel Plated
35NE501212	35	0.790	0.195	0.195	Die Pressed, Machined
35NE642424	35	1.000	0.375	0.375	Die Pressed, Machined
35NE646416	35	1.000	1.000	0.250	Die Pressed, Machined
35NE646432	35	1.000	1.000	0.500	Die Pressed, Machined
35NE646432-NI	35	1.000	1.000	0.500	Die Pressed, Machined, Nickel Plated
35NE209620	35	1.500	0.305	0.305	Die Pressed, Machined
35NE281107	35	2.000	0.170	0.107	Die Pressed, Machined
35NE2812832	35	2.000	2.000	0.500	Die Pressed, Machined
35NE2812832	35	2.000	2.000	0.500	Die Pressed, Machined
35NE2812864	35	2.000	2.000	1.000	Die Pressed, Machined

Tolerances on "machined" blocks are the greater of $\pm$1.5% of the dimension or $\pm$0.015" on cross sectional dimensions, and $\pm$0.005" on the orientation direction.

The T* dimension represents the orientation direction.

Reference: www.magnetsales.com

Magnetization Patterns



Different Magnetizing Patterns Give Different Results

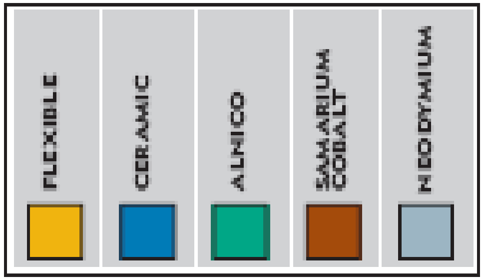
Except for the regular Flexible and Ceramic 1 materials, all magnet materials are "pre-oriented" and can only be magnetized in a particular direction.

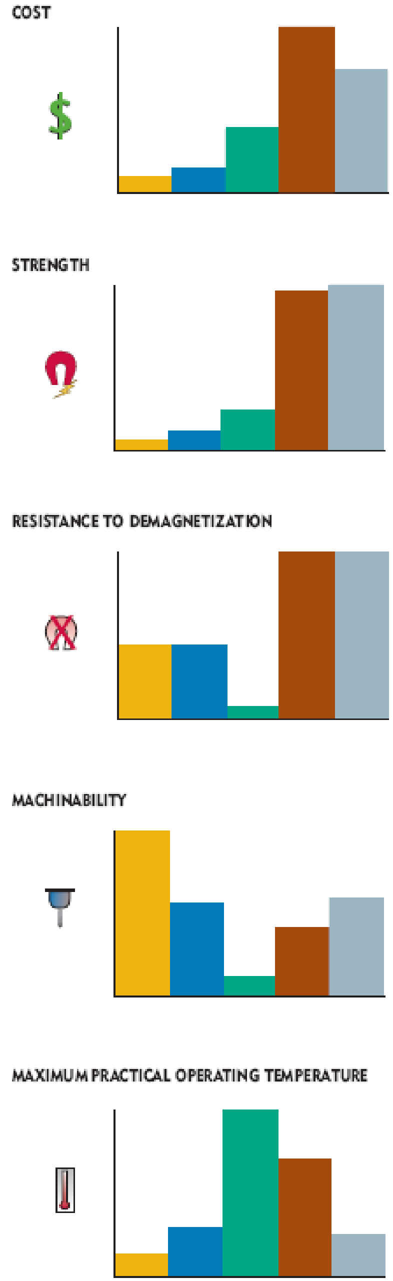
Standard (or "conventional") magnetization is straight through the orientation direction, and produces one North pole and one South pole. The Rare Earth magnets are extremely difficult to magnetize in non-standard ways. However, the Flexible and Ceramic types can be magnetized in many non-standard ways to give special results.

Reference: www.magnetsales.com

Comparison

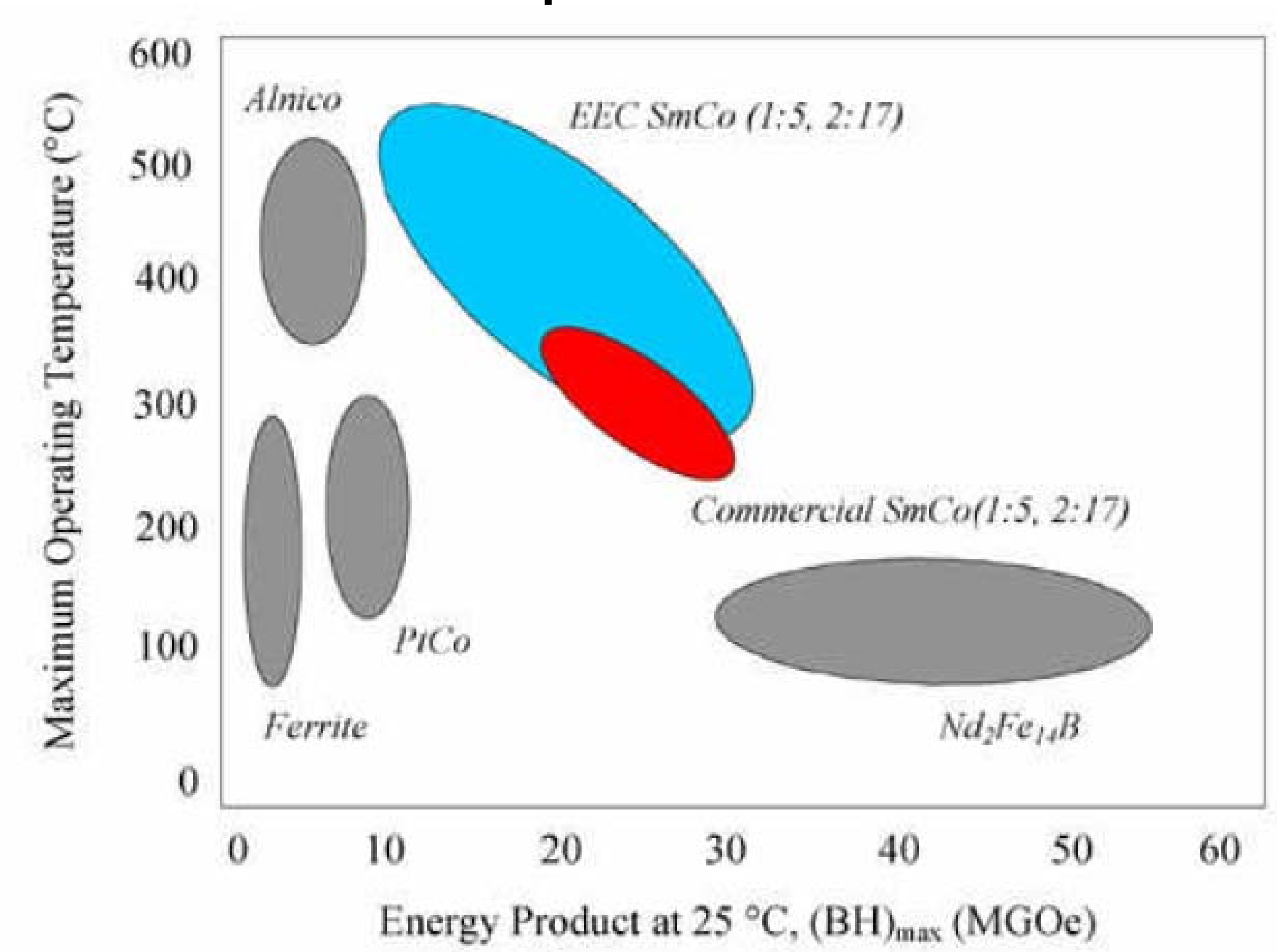
Magnet Materials Color Index





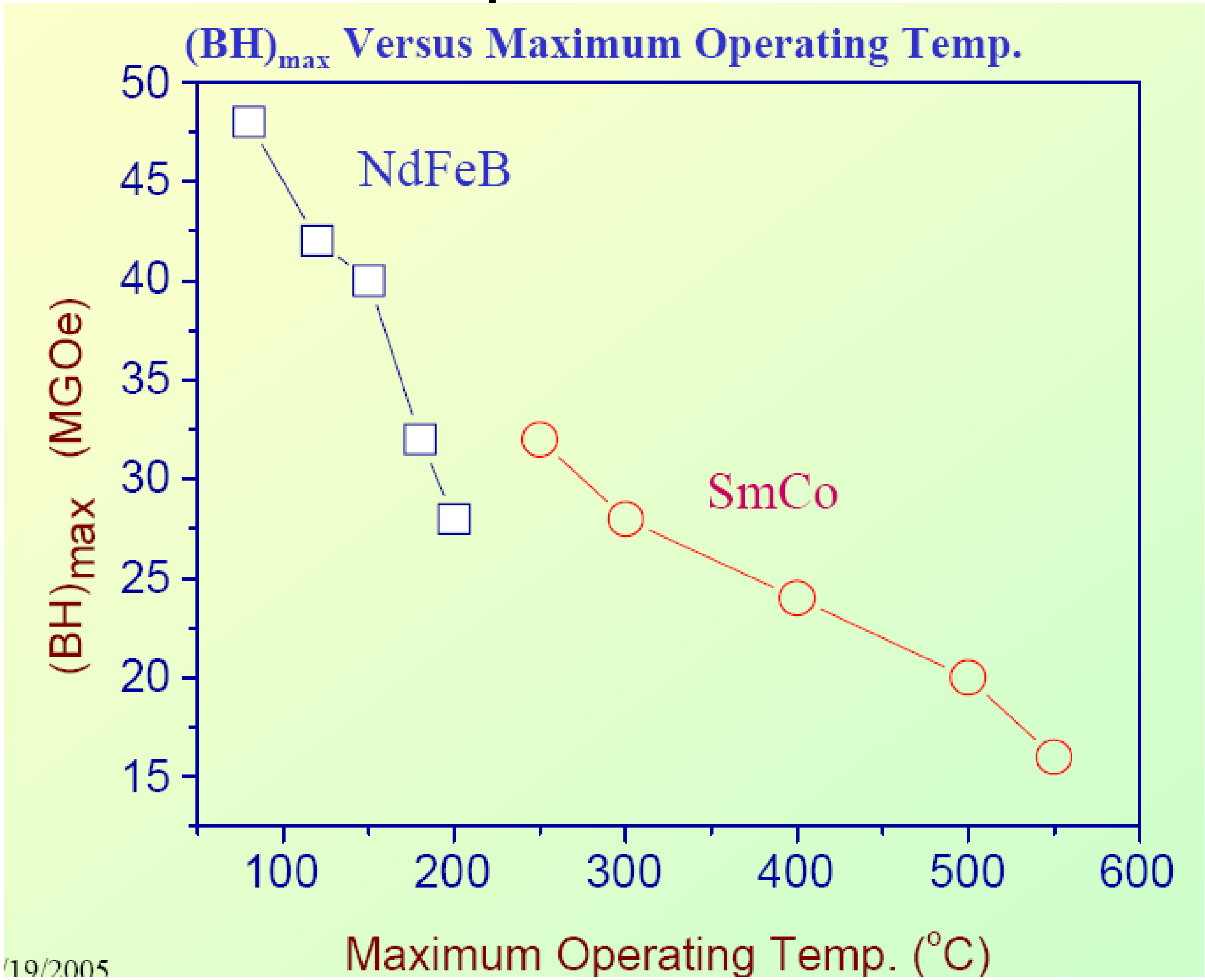
Reference: www.magnetsales.com

Comparison of Maximum Operating Temperatures



Reference: http://www.electronenergy.com/media/Magnetics%202005.pdf

Comparison of Maximum Operating Temperatures



Reference: http://www.electronenergy.com/media/Magnetics%202005.pdf

Magnet Comparisons

Material	Approx. Maximum Operating Temperatures	
	°C	°F
NdFeB	140	284
SmCo	300	572
Ferrite	300	572
Alnico	540	1,004
Flexible	100	212

Material	BHmax	Relative Cost ($ / pound)	Relative Cost ($ / BHmax)
Flexibible	1	$1.00	$0.60
Ceramic	3	$2.00	$0.50
Alnico	5	$20.00	$4.30
SmCo	20	$100.00	$6.00
NdFeB	40	$50.00	$1.40

Reference: www.magnetsales.com

PM Online Resources

- http://members.aol.com/marctt/
- www.dextermag.com
- www.magnetsales.com
- http://www.grouparnold.com/
- Magnetic Materials Producers Association (MMPA standard)

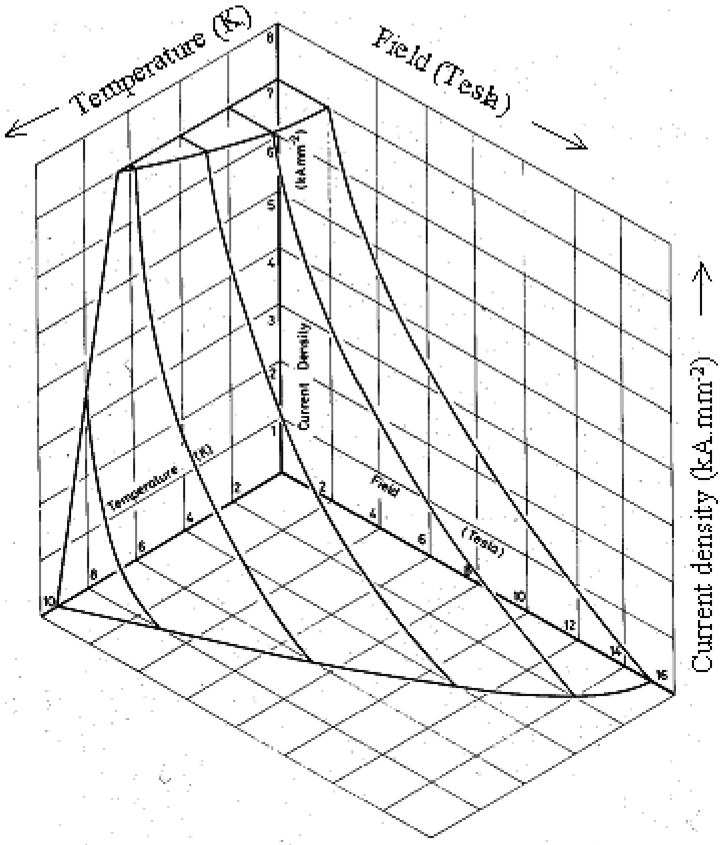
Some Very Brief Comments on Superconductors

- Superconductors have zero resistance if the temperature is low enough, the field acting on the superconductor is low enough, and the current through the superconductor is low enough
- Superconductors are classified as “low temperature” (NbTi, NbSn) or “high temperature” (YBCO, BSCCO)
- Low-Tc superconductors are usually chilled with liquid helium (4.2K)
- High-Tc superconductors are usually used in the 20K-77K range

Reference: Y. Iwasa “Case Studies in Superconducting Magnets,” Plenum Press, 1994

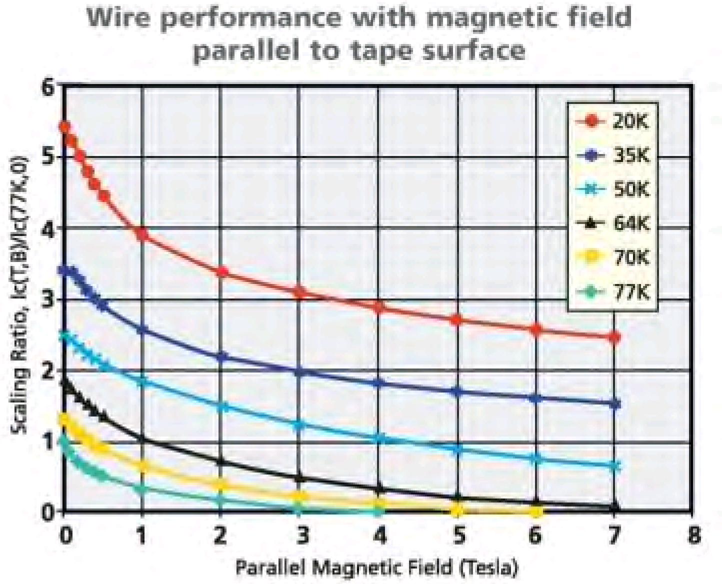
Some Data on Low-Tc Material

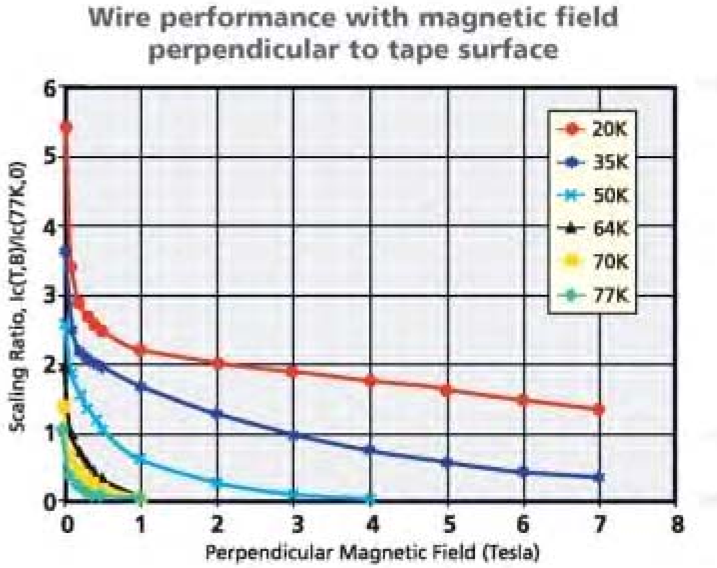
- Shown for niobium titanium
- This type of superconductor is used in the Japanese MLX 500 km/hr Maglev



Some Data on High-Tc Material

- Some superconductors are anisotropic; i.e. superconducting tapes





Reference: American Superconductor, www.amsuper.com

Quotes

It is well to observe the force and virtue and consequence of discoveries, and these are to be seen nowhere more conspicuously than in printing, gunpowder, and the magnet.
--- Sir Francis Bacon

The mystery of magnetism, explain that to me! No greater mystery, except love and hate.
---John Wolfgang von Goethe

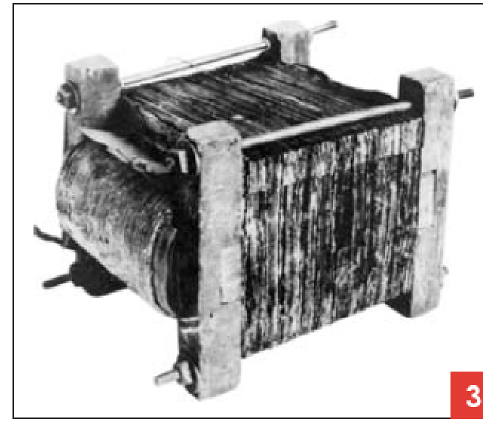
Transformers --- Overview

- Selected history
- Types of transformers
- Voltages and currents
- Equivalent circuits
- Voltage and current transformers
- Per-unit system

Selected History

- 1831 --- Transformer action demonstrated by Michael Faraday
- 1880s: modern transformer invented

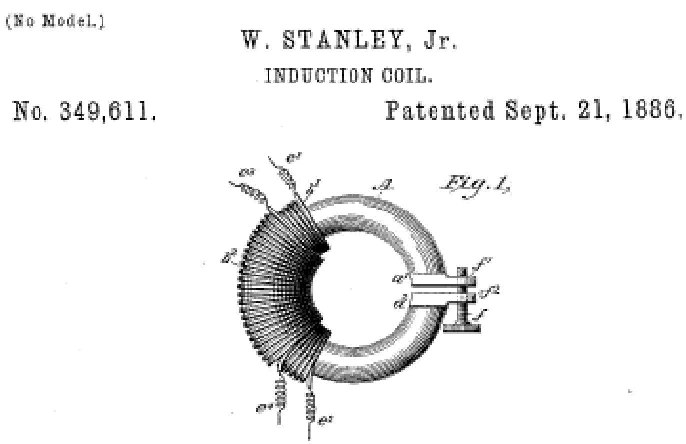
Photo of Faraday's original trans-
former (courtesy MIT Burndy Library).



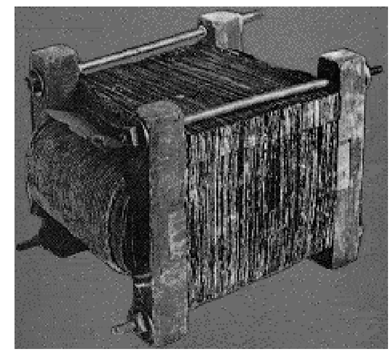
This Stanley transformer from the first ac power station in Great Barrington, Massachusetts, dates from 1885. The transformer is about a foot long; copper windings wrapped with cotton protrude between wood endpieces at the left. The middle arm of E-shaped iron laminations was slid into the prewound coil in alternating directions. The ends of the other two arms are visible as dense regions at the top and bottom of the laminations.

Reference: J. W. Coltman, "The Transformer (historical overview)," *IEEE Industry Applications Magazine*, vol. 8, no. 1, Jan.-Feb. 2002, pp. 8-15

Early Transformer (Stanley, c. 1880)



William Stanley

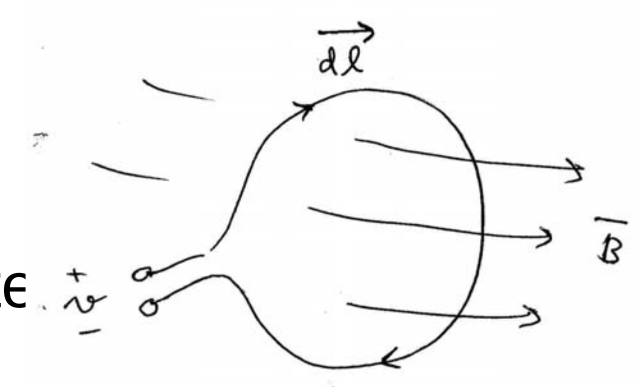


Faraday's Law

- A changing magnetic flux impinging on a conductor creates an electric field and hence a current (eddy current)

$$\oint_C E \cdot dl = -\frac{d}{dt}\int_S B \cdot dA$$

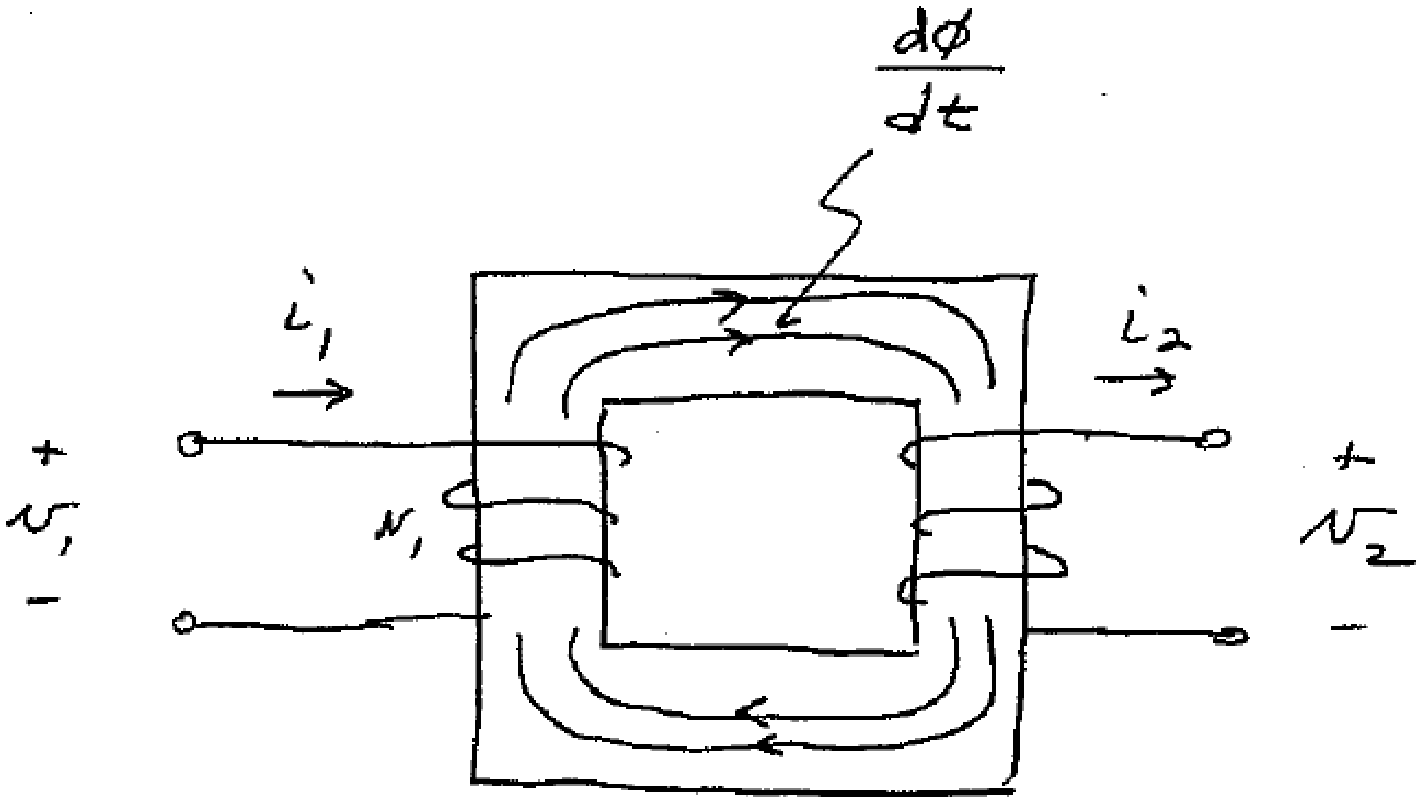
- The electric field integrated around a closed contour equals the net time-varying magnetic flux density flowing through the surface bound by the contour
- In a conductor, this electric field creates a current by: $J = \sigma E$



- Induction motors, brakes, etc.

Magnetic Circuit with Two Windings

- Note that flux is the sum of flux due to i_1 and that due to i_2

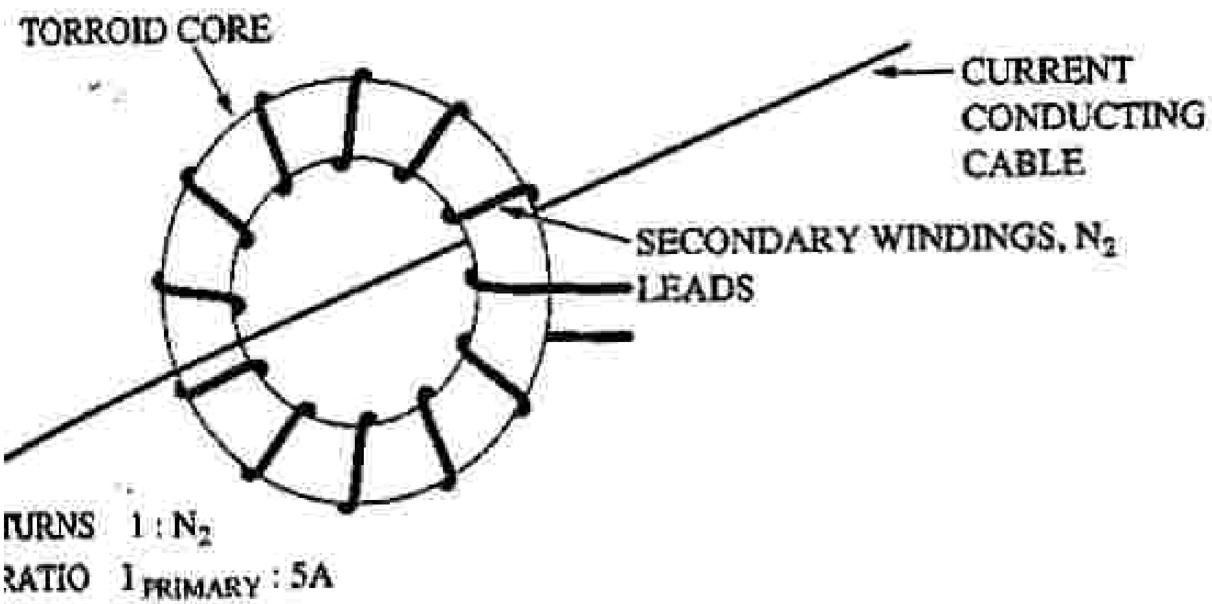


Types of Transformers

- There are many different types and power ratings of transformers: single phase and multi-phase, signal transformers, current transformers, etc.

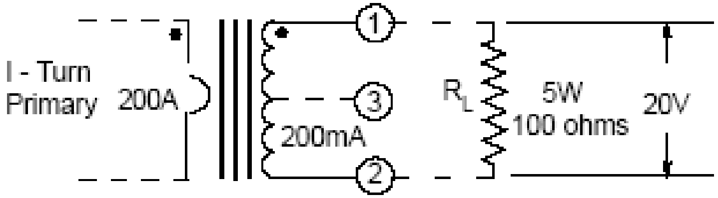
Instrument Transformer

- Instrument transformers (voltage and current) provide line current and line voltage information to protective relays and control systems
- Current transformer shown below



Reference: L. Faulkenberry and W. Coffer, <u>*Electrical Power Distribution and Transmission*</u>, Prentice Hall, 1996, pp. 131

200A Current Transformer (CT)



Applications

- Sensing Overload Current
- Ground fault detection
- Metering
- Analog to Digital Circuits

Electrical Specifications @ 20°C ambient

Electrical Specifications	
Primary Current	200A nom., 500A max.
Turns Ratio	1000:1 nominal
Volt per Amp Ratio at 200A for 100 ohm load	0.100 V/A
Volt per Amp Ratio at 20A for 100 ohm load	0.0991 V/A
DC Resistance at 20°C	11 ohms
Dielectric Withstanding Voltage (Hi-pot)	4KVrms
Mechanical Specifications	
Case	Polycarbonate
Encapsulant	Epoxy
Flammibility	Conforms to UL94-VO
Terminals	Pins Ø 1.0mm
Marking	TALEMA Date Code (W/Y) AC1200, Dot at start pin
Approximate Weight	150 grams
Tolerance	±0.2mm

Reference: http://rocky.digikey.com/WebLib/Amveco-Talema/Web%20Data/AC1200.pdf

Voltage Instrument Transformer

Voltage Transformers

Applications

- For single phase voltage measurement in AC power system

Features

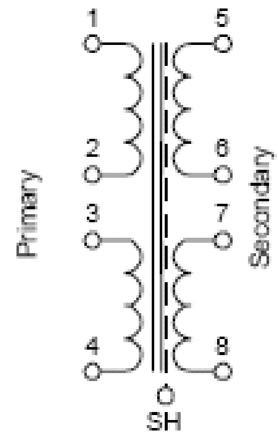
- Resin cast moulded plastic cases
- Integral fuses available on some models
- Designed to meet IEEE C57.13
- 50Hz design available
- UL Recognized and CSA Approved

Reference: http://www.geindustrial.com/products/brochures/ITI.pdf

Audio Transformer

P.C. BOARD MOUNT - EPOXY POTTED
BROADCAST QUALITY
AUDIO TRANSFORMERS

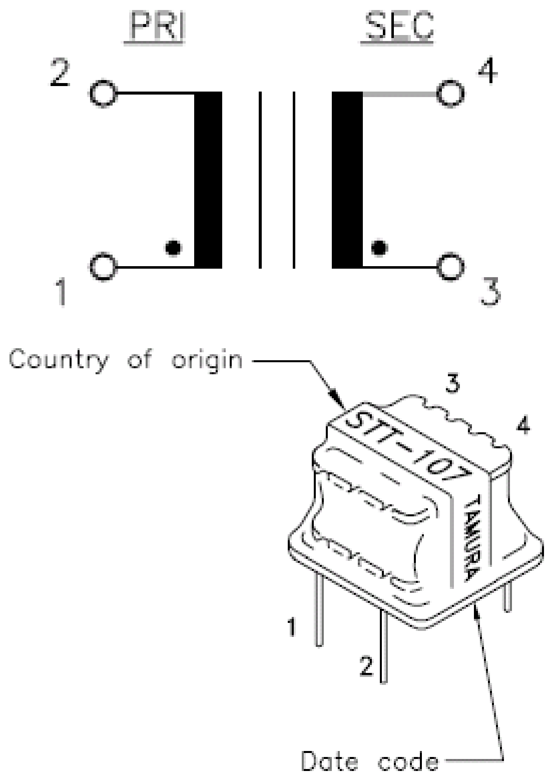
- Rugged black epoxy potted case with 9 pin connections (.025" square by 0.5" long).
- Frequency response @ 0 dbm +/- 1 db max. (+/- 0.5 db is typical) of 30 Hz. to 30 Khz., except 560Q which is 30 Hz. to 15 Khz.



Reference: http://rocky.digikey.com/WebLib/Hammond/Web%20Data/560,%20800-844,%20850%20Series.pdf

Pulse Transformer

- Used for triggering SCRs, etc. where isolation is needed



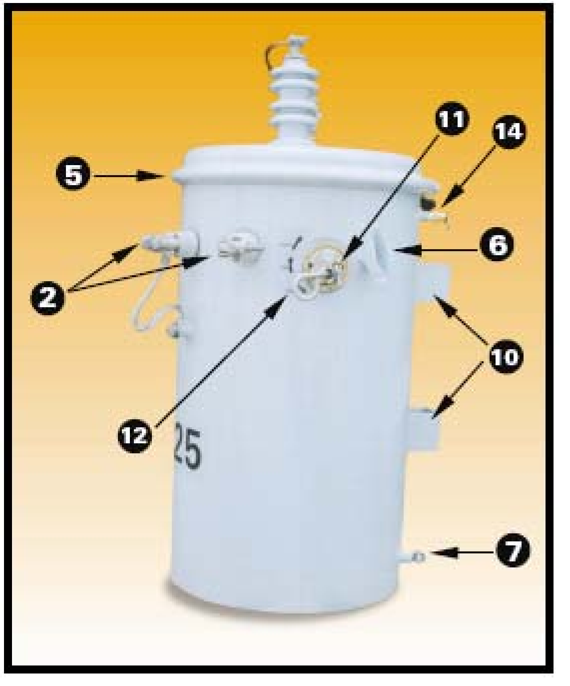
A. Electrical specifications (@ 25 ° C)

1. Leakage Inductance;
   20 μ H MAX
2. DC Resistance;
   Primary (1−2) 2.5 Ω MAX
   Secondary (3−4) 1.5 Ω MAX
3. Primary ET−constant;
   90.0V− μs MIN
4. Turns Ratio;
   (1−2) : (3−4) = 2 : 1.00 ±5%
5. Interwinding Capacitance;
   80.0 pF MAX
6. Primary Inductance;
   1.0 mH MIN
7. Dielectric Strength;
   AC 2600 Vrms 1 minute @ Pri to Sec

Reference: http://rocky.digikey.com/WebLib/Tamura-Microtran/Web%20Data/STT-107.pdf

Power Distribution Transformer

- Provides voltage for the customer
- Typical voltages are 2.3-34.5kV primary, and 480Y/277V or 208Y/120V 3-phase or 240/120V single phase
- Pole-top transformers typically 15-100 kVA



Reference: http://www.geindustrial.com/products/brochures/DEA-271-English.pdf

E Core Transformer

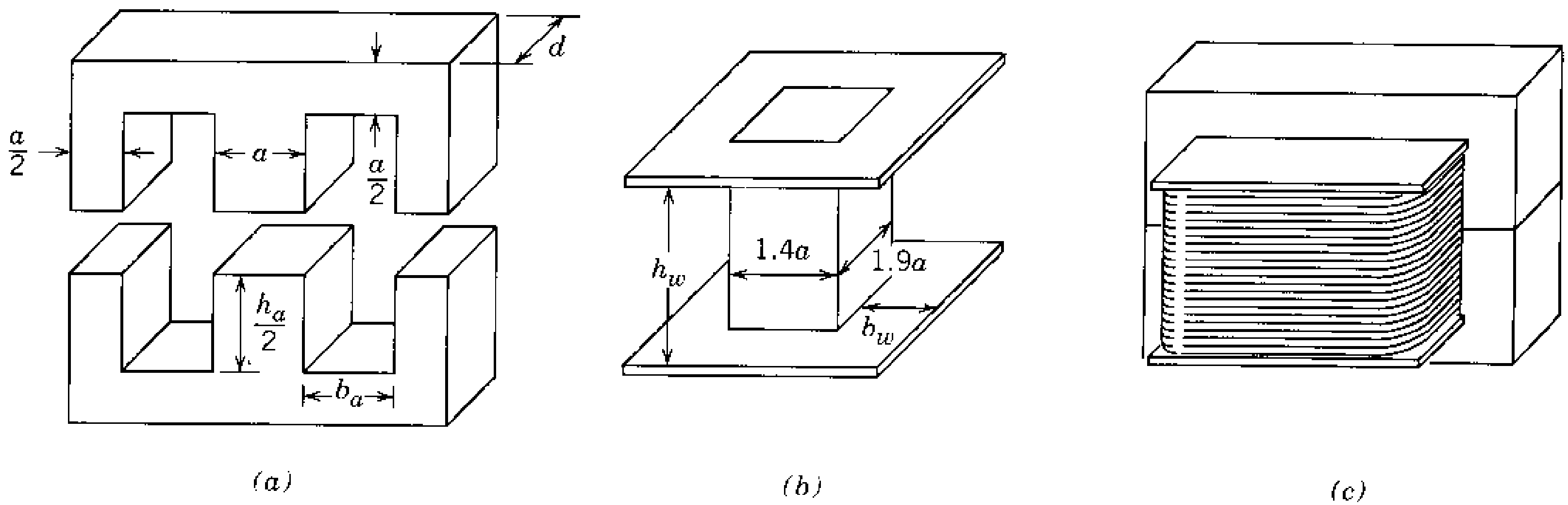


Figure 30-6 Dimensioned diagram of (a) a double-E core (b) bobbin, and (c) assembled core with winding.

Offline Flyback Power Supply

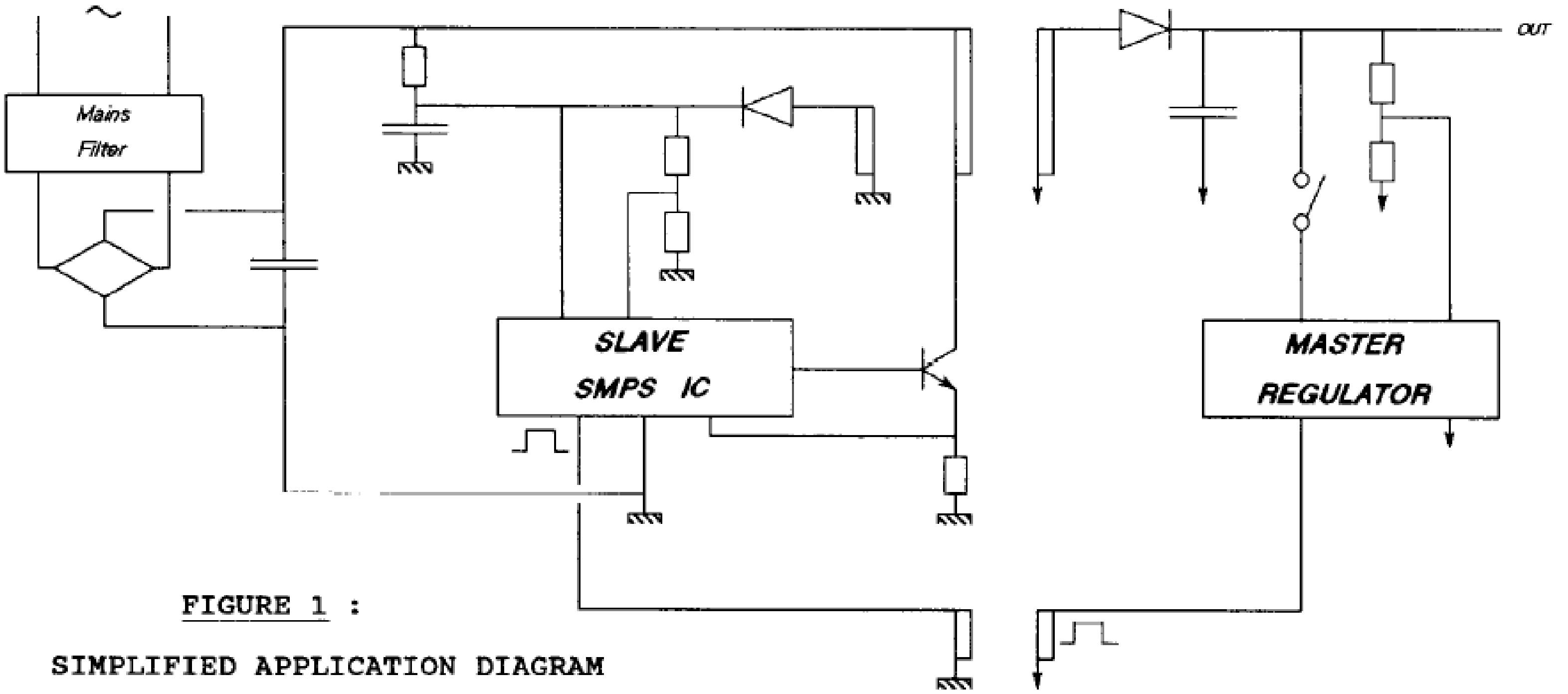


FIGURE 1 :

SIMPLIFIED APPLICATION DIAGRAM

P. Maige, “A universal power supply integrated circuit for TV and monitor applications,” *IEEE Transactions on Consumer Electronics*, vol. 36, no. 1, Feb. 1990, pp. 10-17

Transcutaneous Energy Transmission

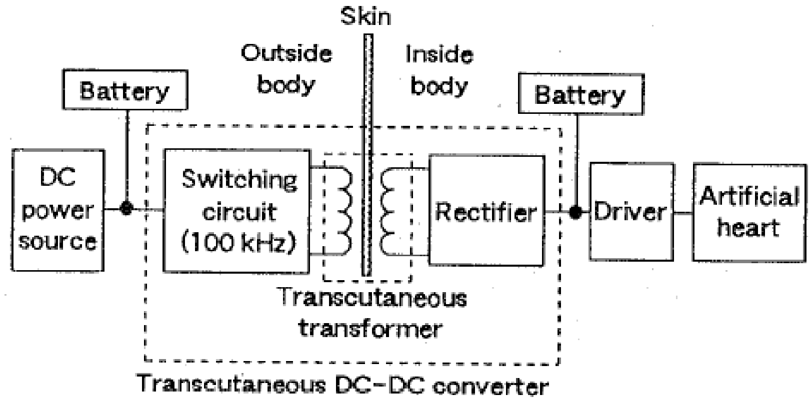


Fig. 1. Transcutaneous energy transmission system for an implantable artificial heart.

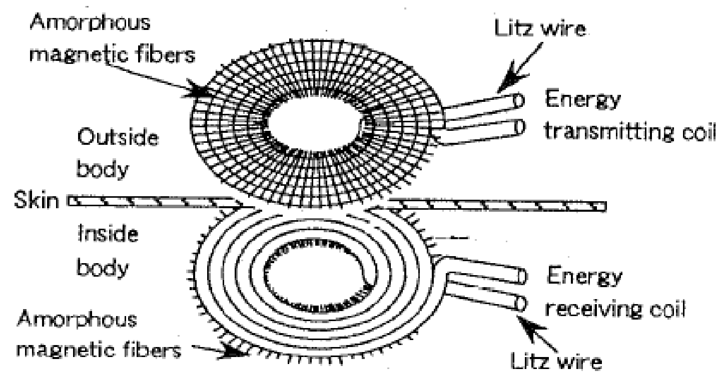
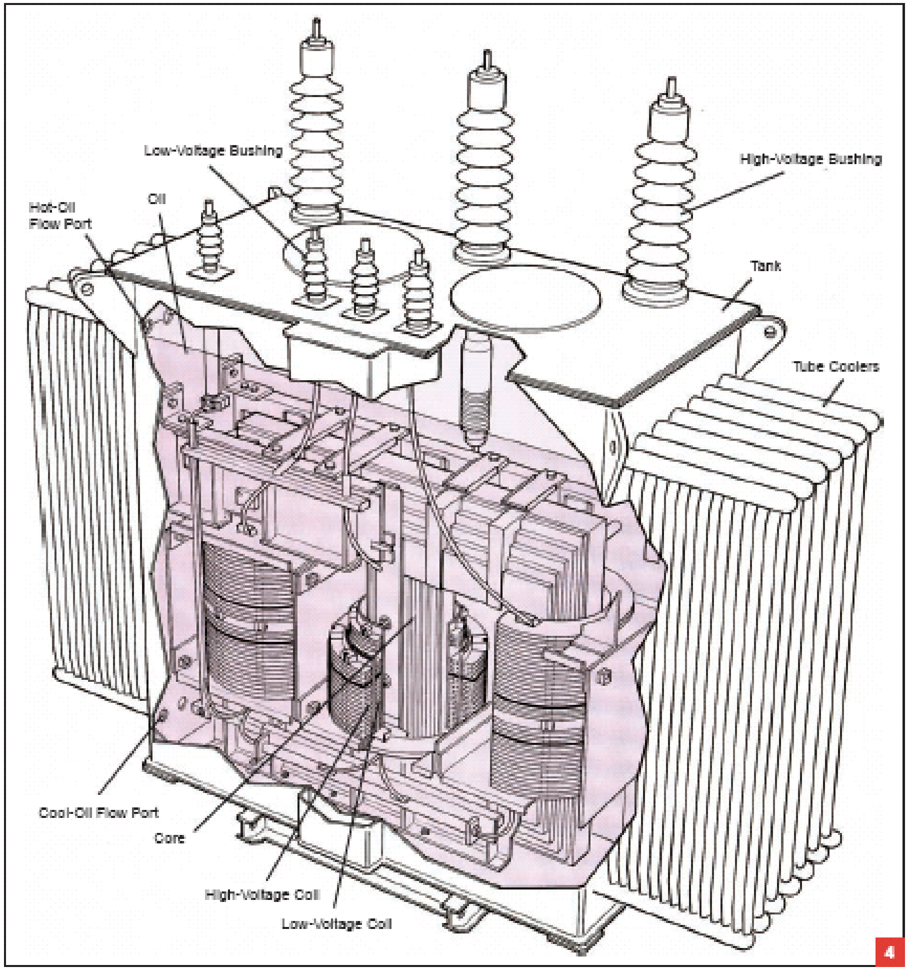


Fig. 2. Transcutaneous transformer.

H. Matsuki, Y. Yamakata, N. Chubachi, S.-I. Nitta and H. Hashimoto, "Transcutaneous DC-DC converter for totally implantable artificial heart using synchronous rectifier," *IEEE Transactions on Magnetics*, vol. 32 , no. 5 , Sept. 1996, pp. 5118 - 5120

Power Transformer



The typical modern transformer is submerged in oil for insulation and cooling and is sealed in an airtight tank. Low- and high- voltage power lines lead to and from the coils through ceramic bushings. Inside the transformer, coils and core are packed close together to minimize electrical losses and material costs. The oil coolant circulates by convection through external radiators. In large transformers cooling is expedited by attaching fans to the radiators and circulation the oil with pumps.

Reference: J. W. Coltman, "The Transformer (historical overview)," *IEEE Industry Applications Magazine*, vol. 8, no. 1, Jan.-Feb. 2002, pp. 8-15

Superconducting Transformer

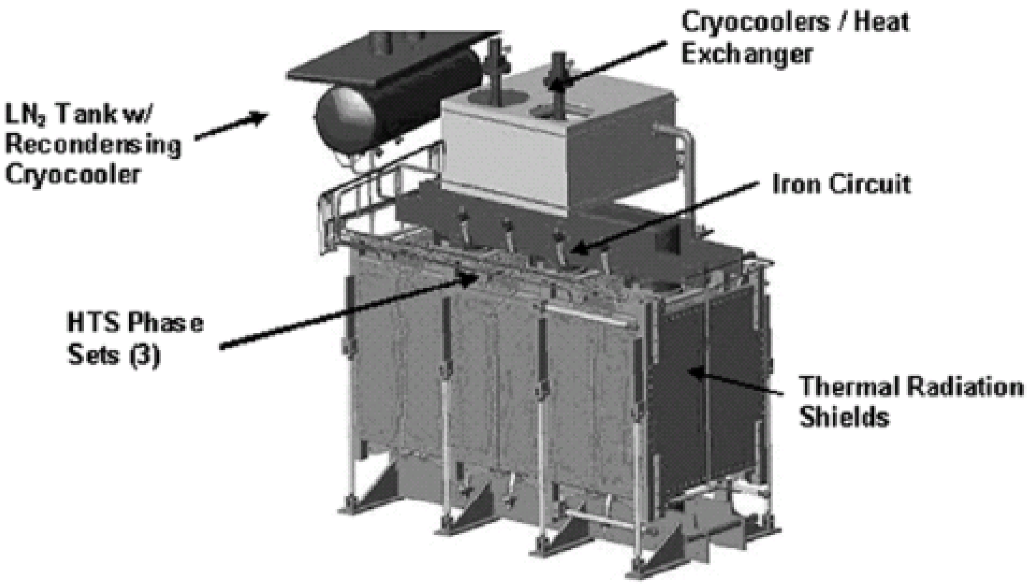


Fig. 13. Schematic of the major system components of a 5/10 MVA HTS transformer.

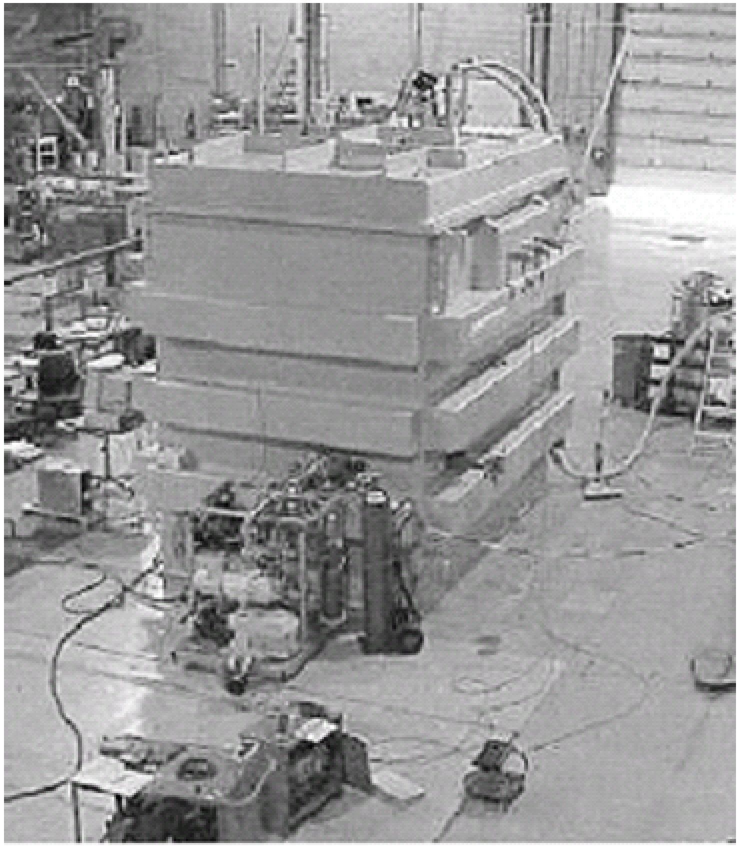
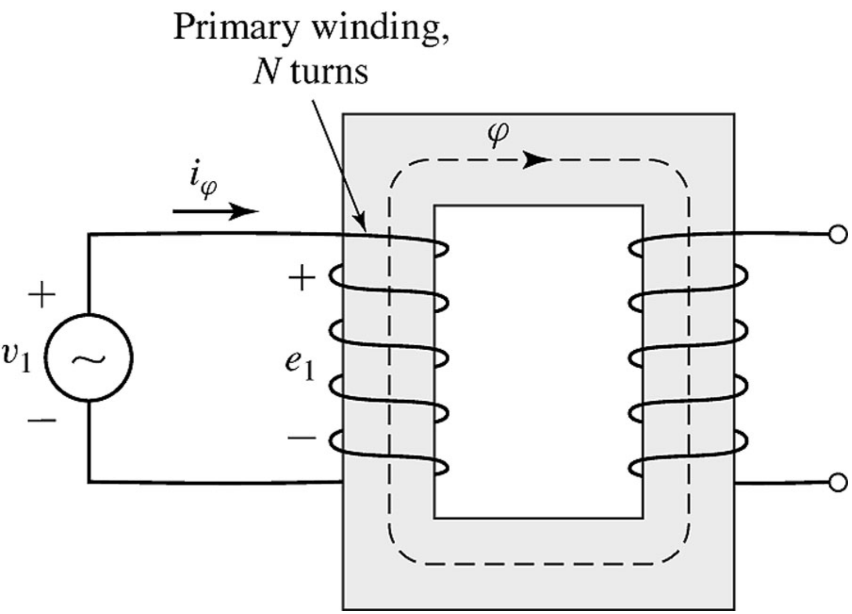


Fig. 15. Completed 5/10 MVA WES/SP/ORNL HTS transformer under test at the Waukesha Electric test facility.

Reference: W. Hassenzahl et. al., "Electric Power Applications of Superconductivity," *Proceedings of the IEEE*, vol. 92, no. 10, October 2004, pp. 1655-1674

Transformer Under No-Load Condition



By Faraday's law:

$$e_1 = \frac{d\lambda_1}{dt} = N\frac{d\Phi}{dt}$$

$$= \omega N\Phi_{\max}\cos(\omega t)$$

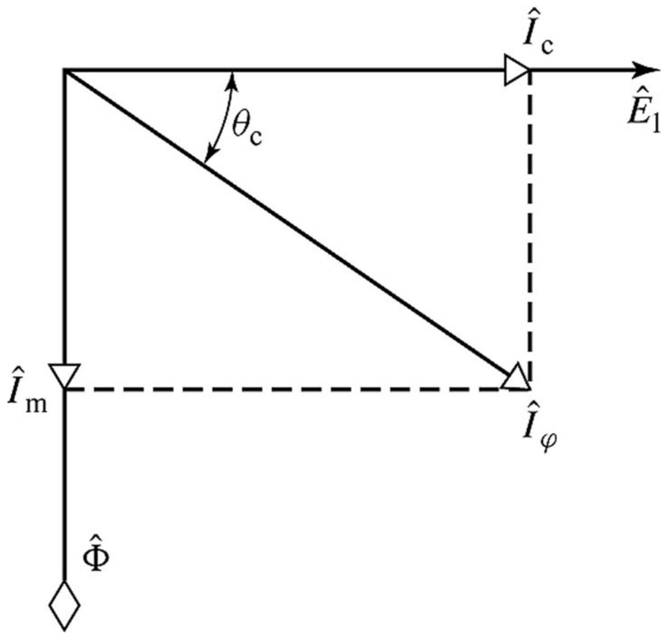
RMS value of e_1:

$$E_{1,rms} = \frac{2\pi f N\Phi_{\max}}{\sqrt{2}} = \sqrt{2}\pi f N\Phi_{\max}$$

If resistive drop in winding is negligible:

$$\Phi_{\max} = \frac{E_{1,rms}}{\sqrt{2}\pi f N}$$

No-Load Phasor Diagram



- Winding current has harmonics, and fundamental is generally out of phase with respect to flux
- In-phase component is from core losses

$$P_c = E_1 I_\varphi \cos\theta_c$$

- Magnetizing current is 90 degrees out of phase

Example: Transformer Calculations

- *Fitzgerald*, Example 2.1. In Example 1.8, the core loss and VA at Bmax = 1.5T and 60 Hz were found to be: P_c = 16W and VI = 20 VA with induced voltage 194V. Find power factor, core loss current I_c and magnetizing current I_m.

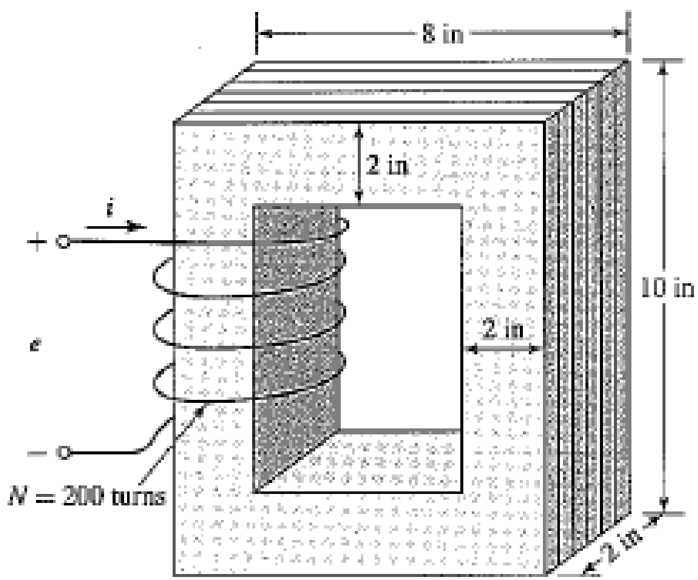
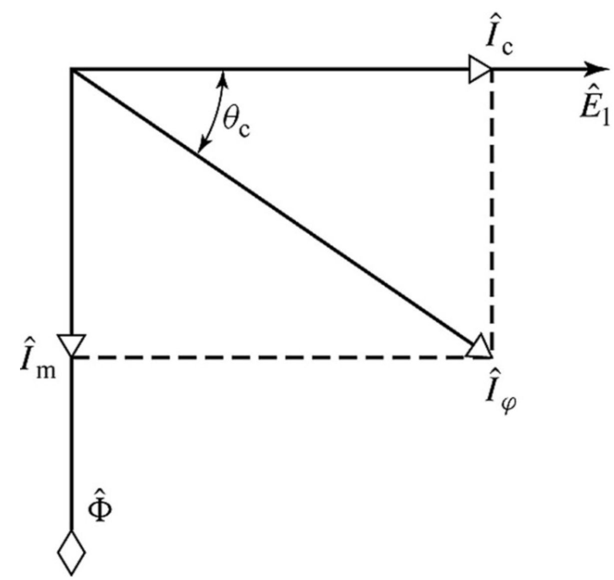


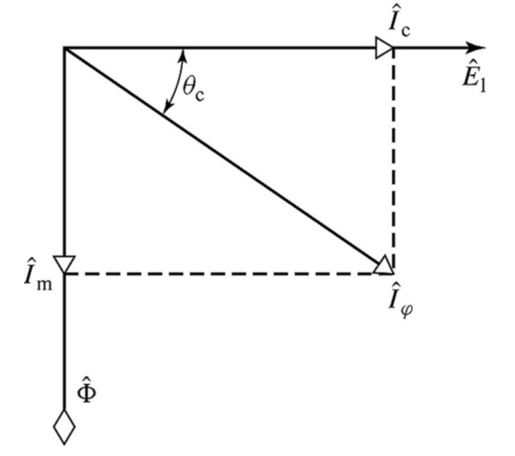
Figure 1.15 Laminated steel core with winding for Example 1.8.



Example: Transformer Calculations --- Solution

Power factor:

$$PF = \cos(\theta_c) = \frac{16}{20} = 0.8$$

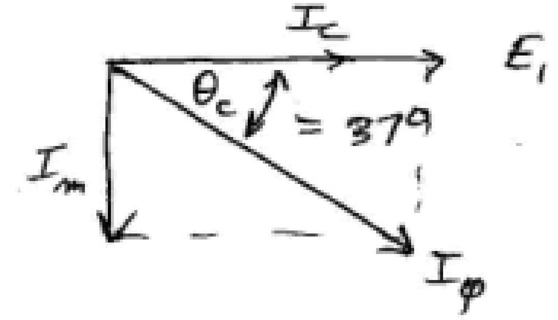


Exciting current

$$I_\varphi = \frac{VA}{V} = \frac{20}{194} = 0.1A$$

Core loss component

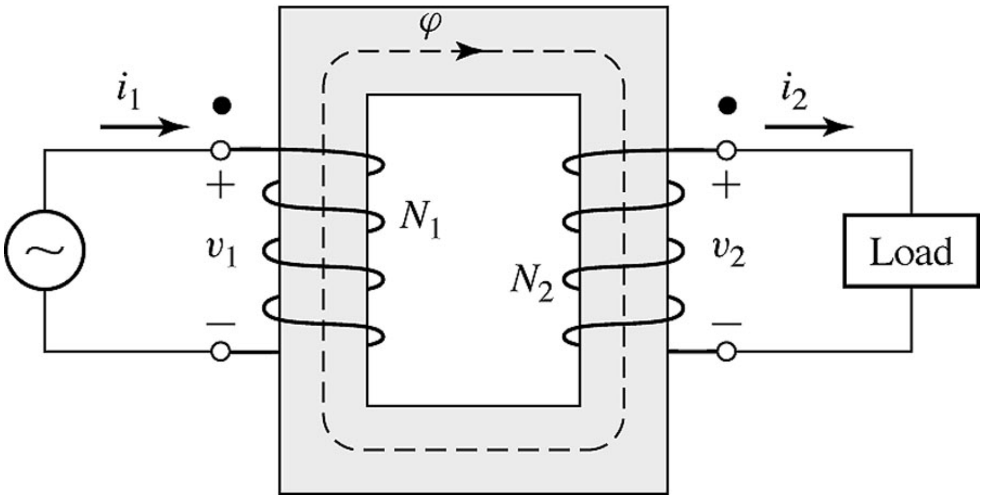
$$I_c = \frac{16}{194} = 0.082A$$



Magnetizing component

$$I_m = I_\varphi \left|\sin\theta_c\right| = 0.060\ A$$

Ideal Transformer with Load



Ideal transformer:

$$v_1 = N_1 \frac{d\Phi}{dt}$$

$$v_2 = N_2 \frac{d\Phi}{dt}$$

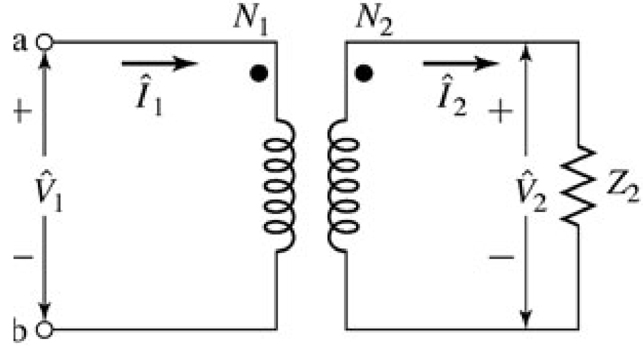
$$\frac{v_2}{v_1} = \frac{N_2}{N_1}$$

By Ampere's law:

$$N_1 i_1 - N_2 i_2 = 0 \Rightarrow \frac{i_1}{i_2} = \frac{N_2}{N_1}$$

(Also, by power balance, $v_1 i_1 = v_2 i_2$)

Impedance Transformation



(a)

$$\hat{V}_1 = \frac{N_1}{N_2}\hat{V}_2$$

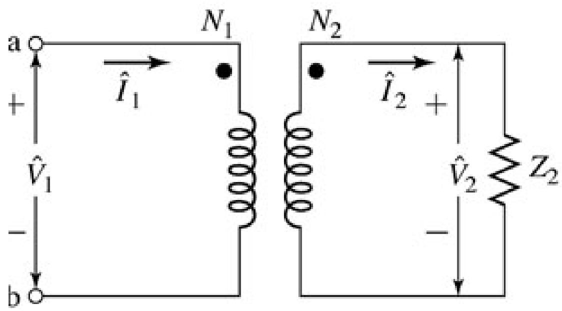
$$\hat{I}_1 = \frac{N_2}{N_1}\hat{I}_2 = \frac{N_2}{N_1}\frac{\hat{V}_2}{Z_2} = \left(\frac{N_2}{N_1}\right)\left(\frac{N_2}{N_1}\right)\hat{V}_1\left(\frac{1}{Z_2}\right)$$

Therefore, impedance at input terminals is:

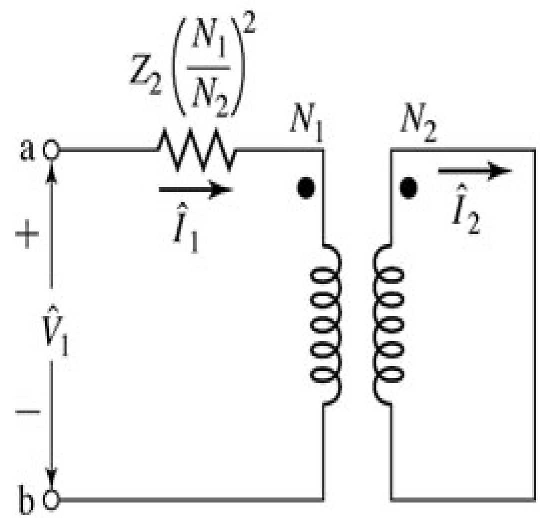
$$\frac{\hat{V}_1}{\hat{I}_1} = Z_2\left(\frac{N_1}{N_2}\right)^2$$

Equivalent Circuits

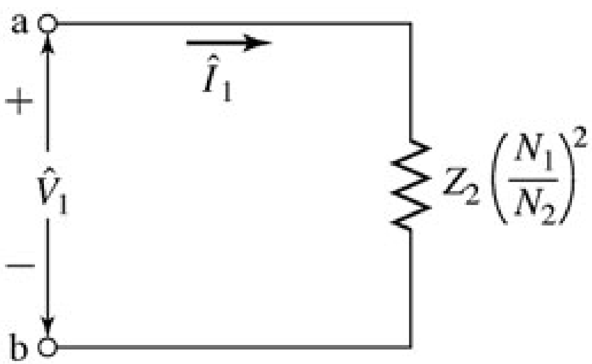
- These 3 circuits have the same impedance as seen from the a-b terminals



(a)



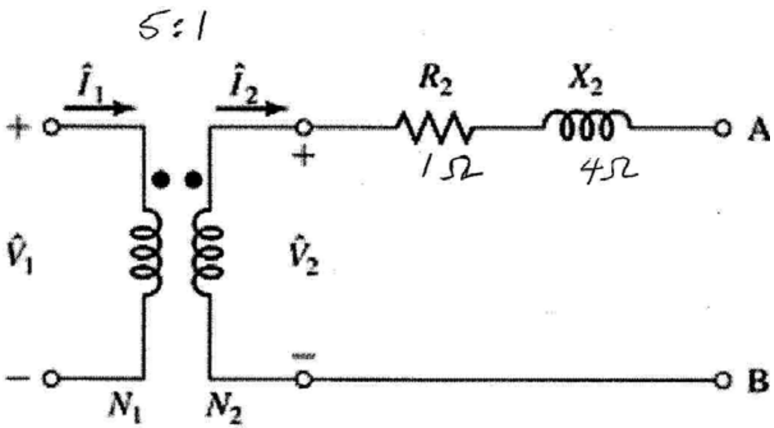
(b)

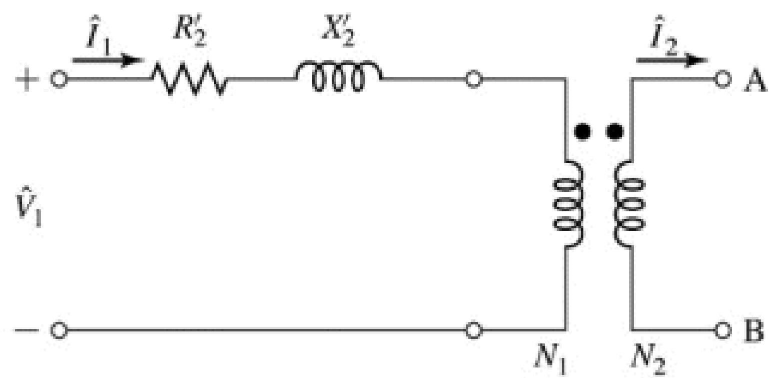


(c)

Example: Use of Equivalent Circuits

- *Fitzgerald*, Example 2.2
- (a) Draw equivalent circuit with series impedance referred to primary
- (b) For a primary voltage of 120VAC and a short at the output, find the primary current and the short circuit current at the output

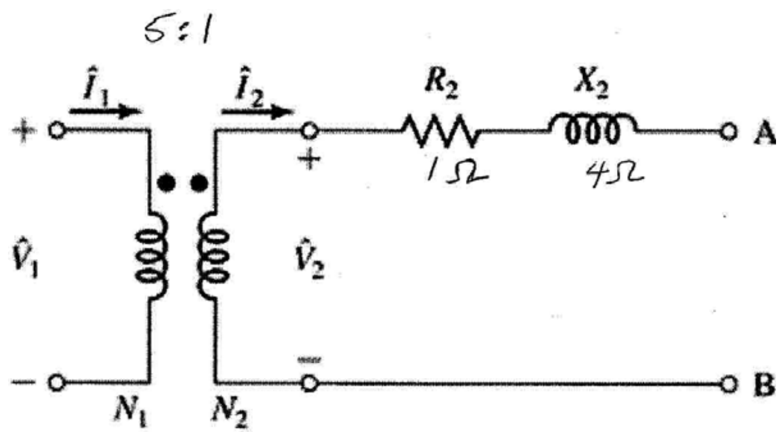


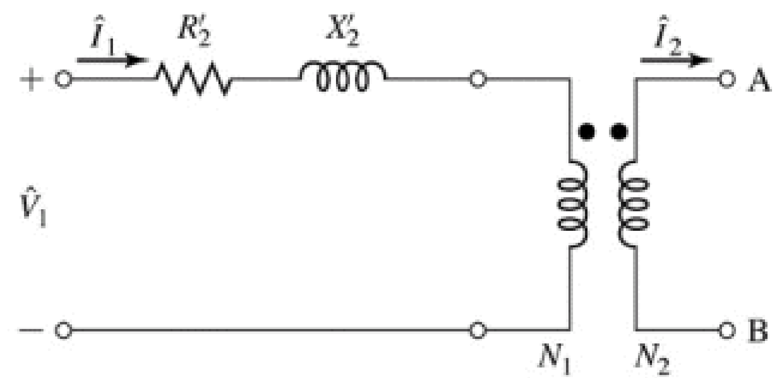


Example: Impedance Transformation

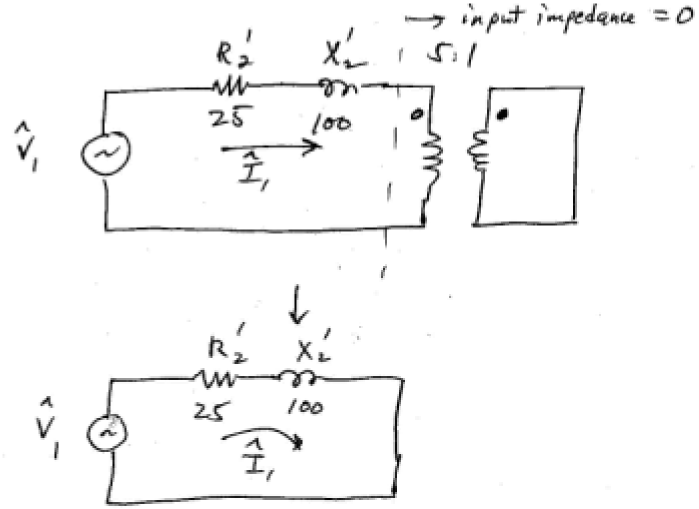
$$R_2^{'} + jX_2^{'} = \left(\frac{N_1}{N_2}\right)^2 \left(R_2 + jX_2\right)$$

$$= 25 + j100$$





Example: Input Current with Output Short Circuit



$$\hat{I}_1 = \frac{\hat{V}_1}{R_2^{'} + jX_2^{'}} = \frac{120}{25 + j100}\left(\frac{25 - j100}{25 - j100}\right)$$

$$\hat{I}_1 = 0.28 - j(1.13)$$

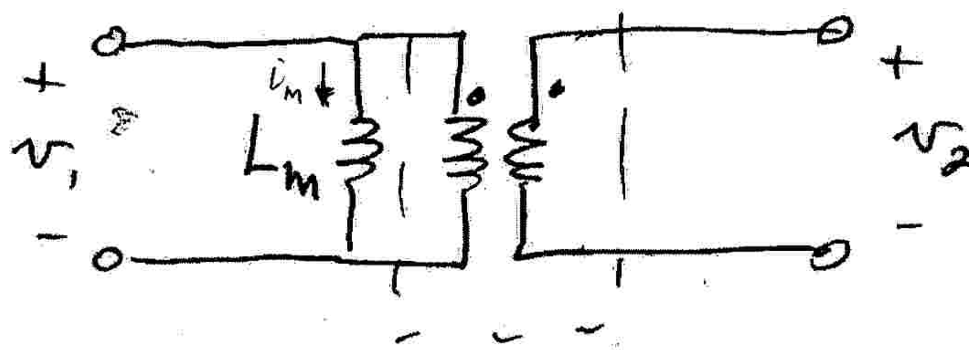
$$I_{1,rms} = \sqrt{0.28^2 + 1.13^2} = 1.16A$$

$$\hat{I}_2 = 5\hat{I}_1 = 1.4 - j(5.65)$$

$$I_{2,rms} = \sqrt{1.4^2 + 5.65^2} = 5.8A$$

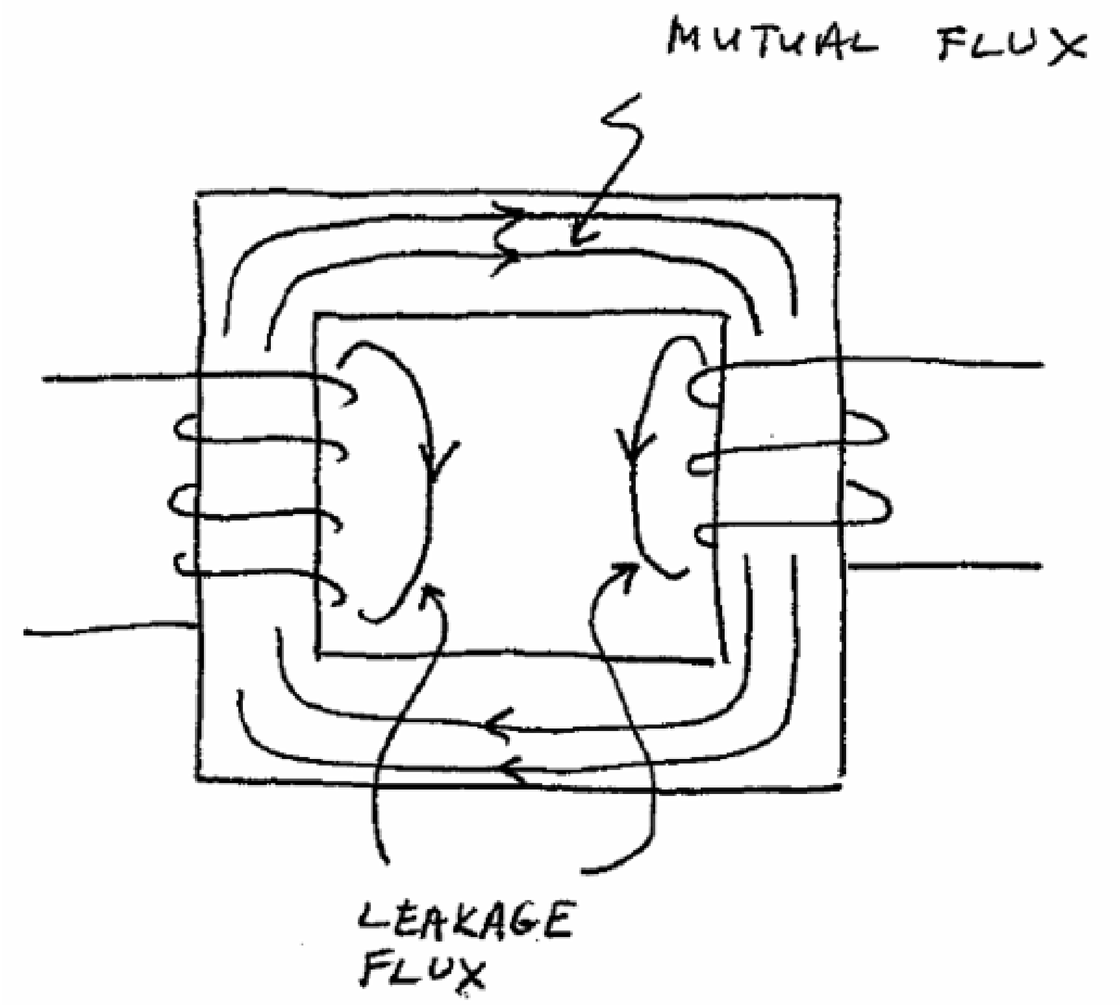
Non-Ideal Effects in Transformers --- Magnetizing Inductance

- A real-world transformer doesn't pass DC
- From either set of terminals, the impedance looks like an inductor if the other set of terminals is open-circuited
- Can model this as an ideal transformer with a <u>magnetizing inductance</u> added.
- The magnetizing current i_m produces the mutual flux which couples to the secondary

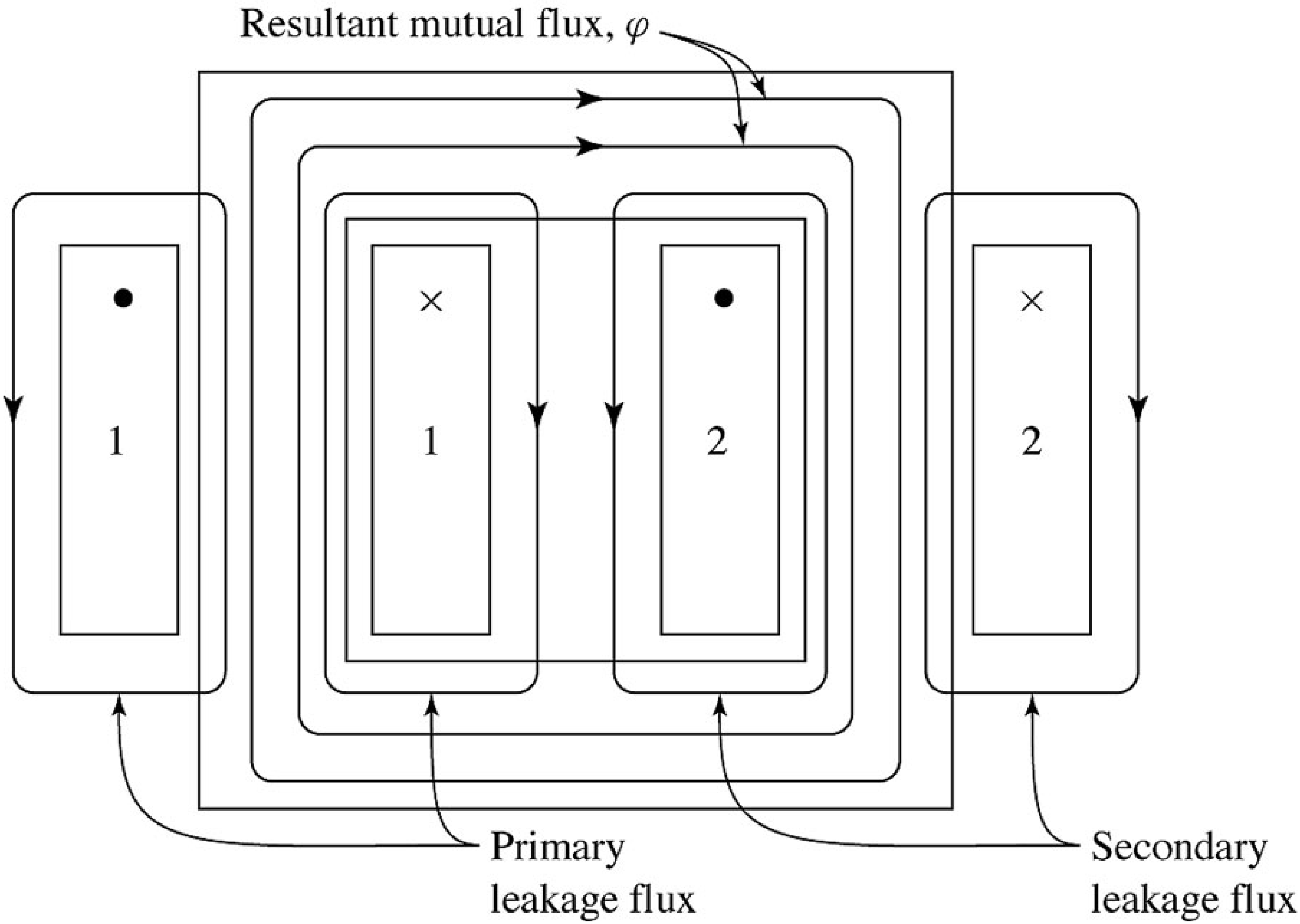


Mutual and Leakage Flux

- Not all of the flux created by winding #1 links with winding #2.

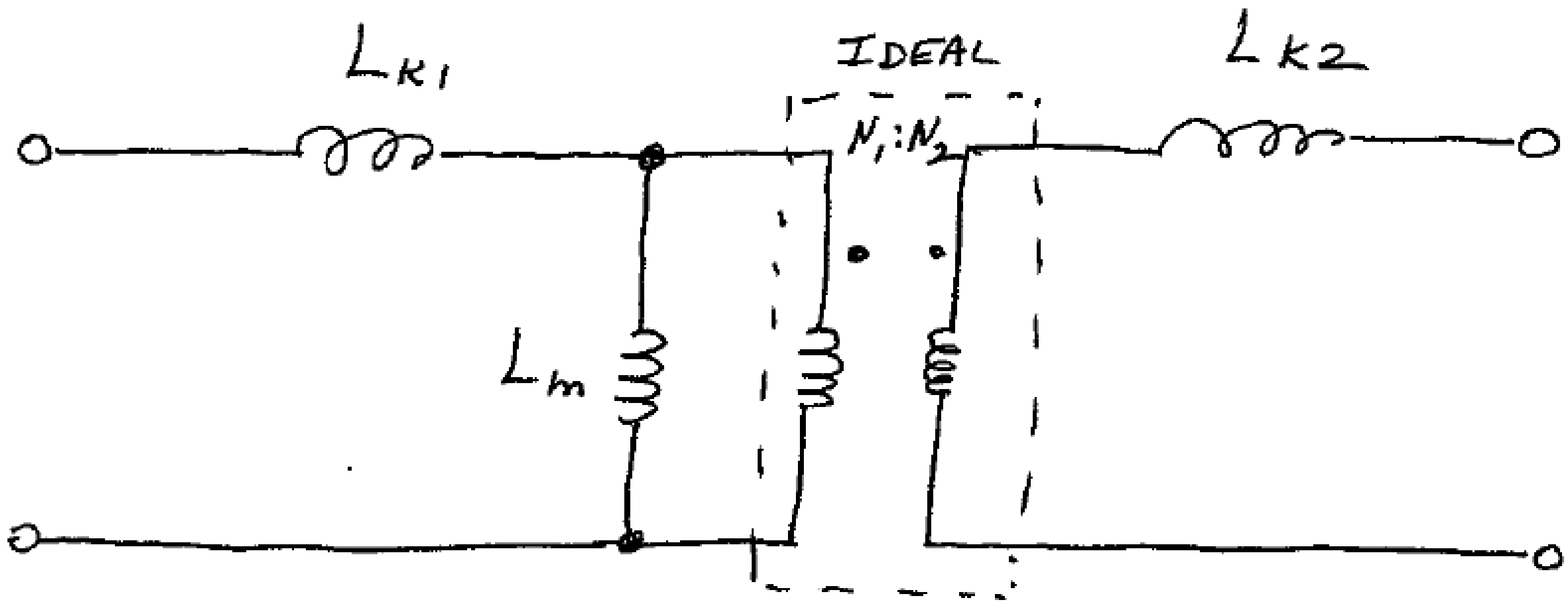


Mutual and Leakage Flux

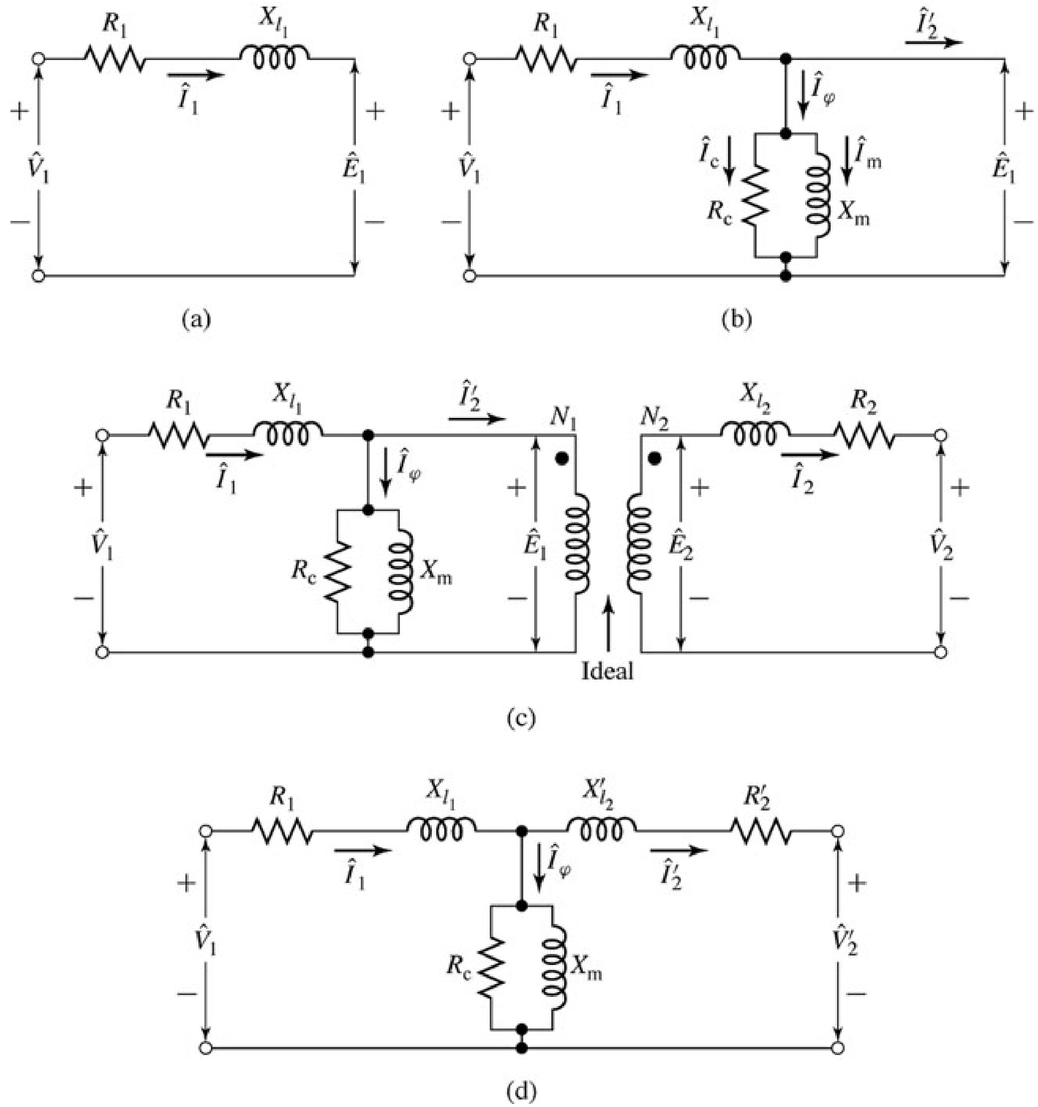


Non-Ideal Effects in Transformers --- Leakage

- Not all of the flux created by winding #1 links with winding #2.
- Therefore, real-world voltage transformation is not exactly equal to the turns ratio, due to the voltage drops on L_{k1} and L_{k2}



Transformer Equivalent Circuits



Example: Use of Equivalent Circuits

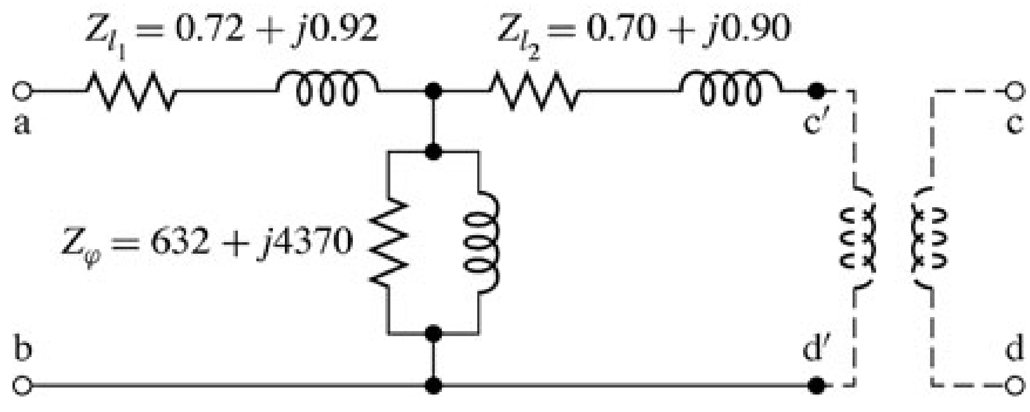
- Fitzgerald, Example 2.3: A 50-kVA 2400:240V 60 Hz distribution transformer has a leakage impedance of 0.72+j0.92Ω in the high voltage winding and 0.0070 + j0.0090 Ω in the low-voltage winding. The impedance Zφ of the shunt branch (equal to R_c +jX_m in parallel) is 6.32 + j43.7 Ω when viewed from the low voltage side.

Draw the equivalent circuits referred to the high voltage side and the low voltage sides.

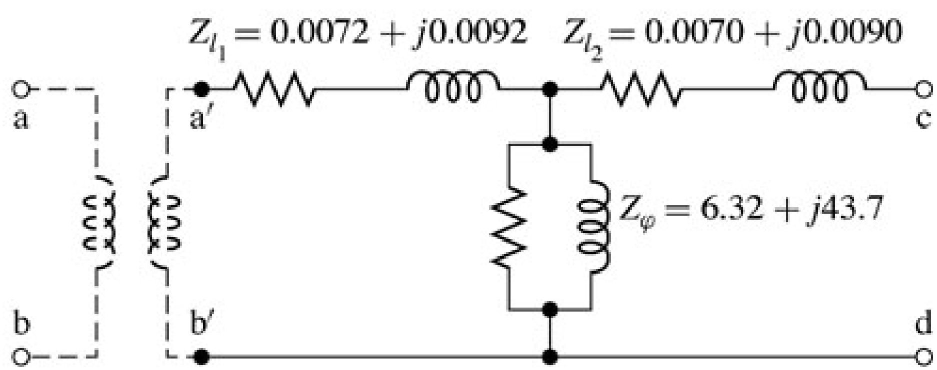
<u>SOLUTION:</u>
Note that N1:N2 is 1:10, so impedances step up and down by 100

Example: Solution

--- Leakage impedance of 0.72+j0.92Ω in the high voltage winding and 0.0070 + j0.0090 Ω in the low-voltage winding.
--- The impedance Zφ of the shunt branch (equal to R_c +jX_m in parallel) is 6.32 + j43.7 Ω when viewed from the low voltage side.



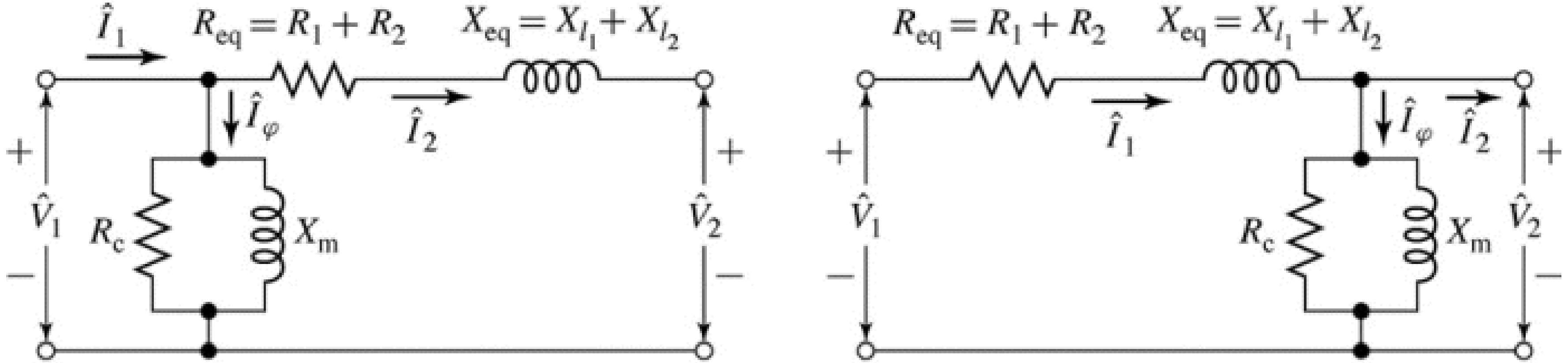
(a)



(b)

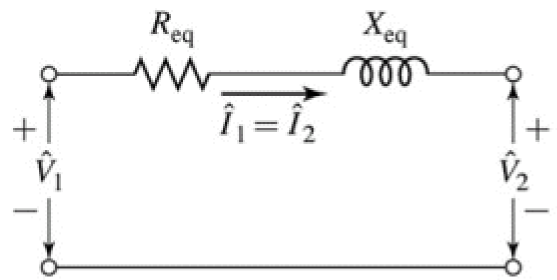
Approximate Transformer Equivalent Circuits

- “Cantilever circuits”
- Ignoring voltage drop in primary or secondary leakage impedances

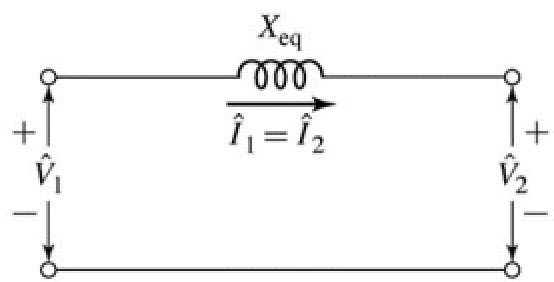


Approximate Transformer Equivalent Circuits

- Circuit if we ignore the magnetizing inductance and core resistance

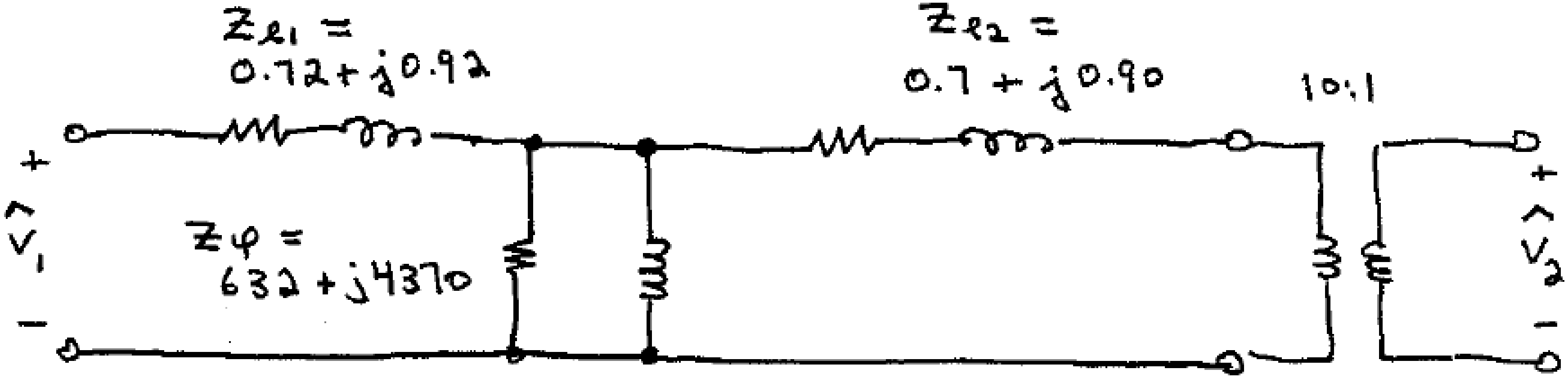


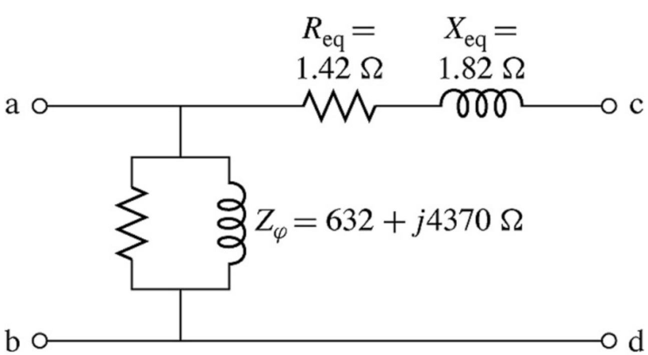
- Circuit if we further ignore the winding resistance



Example: Use of Cantilever Circuit

- *Fitzgerald*, Example 2.4





Using T-model:

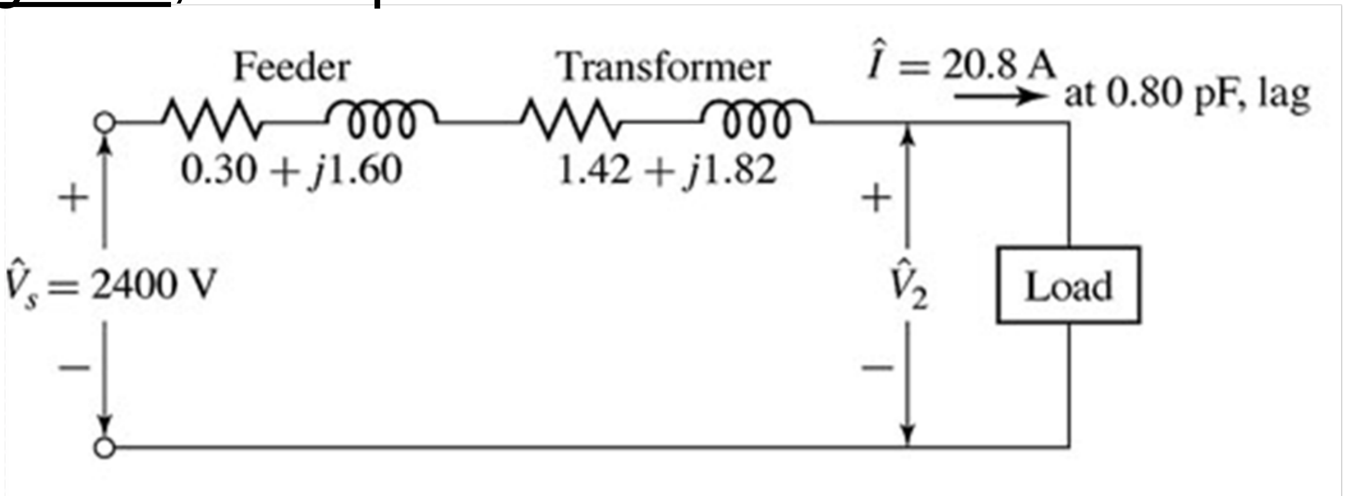
$$\hat{V}_2 = (2400)\left(\frac{Z_\varphi}{Z_{l1} + Z_\varphi}\right)\left(\frac{1}{10}\right) = 239.9 + j0.0315$$

Using cantilever model:

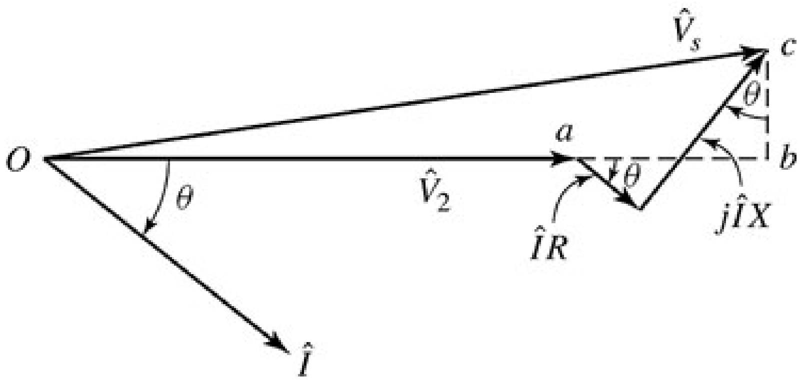
$$\hat{V}_2 = (2400)\left(\frac{1}{10}\right) = 240$$

Example: Find V_2

- *Fitzgerald*, Example 2.5

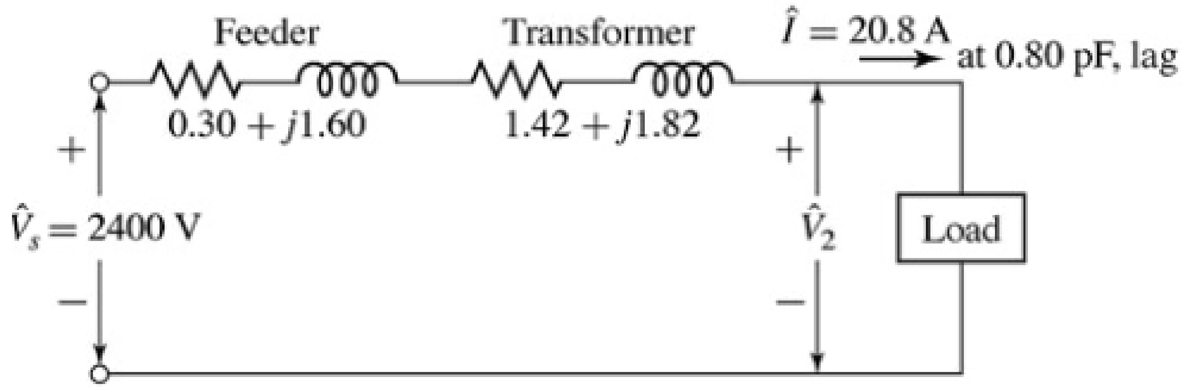


(a)



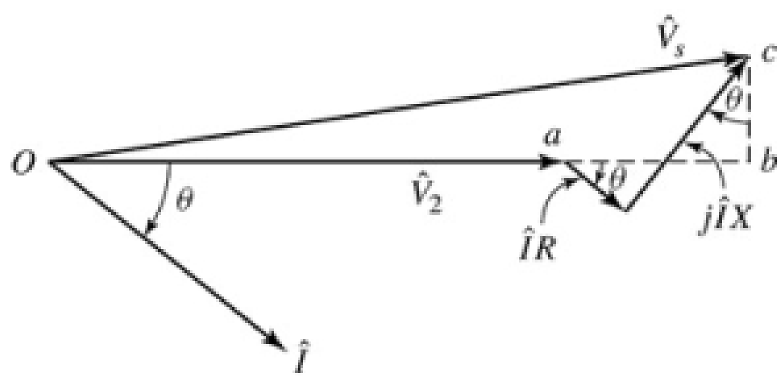
(b)

Example: Find V_2

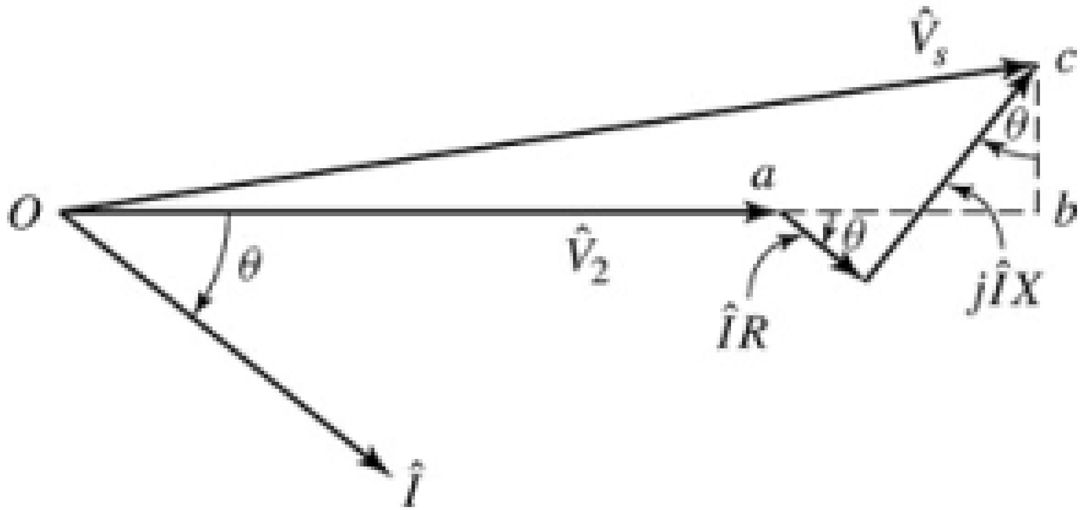


From node equations:

$$\hat{V}_s = \hat{V}_2 + \hat{I}_L R + jX\hat{I}_L$$



Example: Find V_2



-We need length of vector Oa (which is V_2)
-We know length of vector Oc (which is 2400V)

$$ab = IR\cos\theta + IX\sin\theta = (20.8)(1.72)(0.8) + (20.8)(3.42)(0.6) = 71.4$$

$$bc = IX\cos\theta - IR\sin\theta = 35.5$$

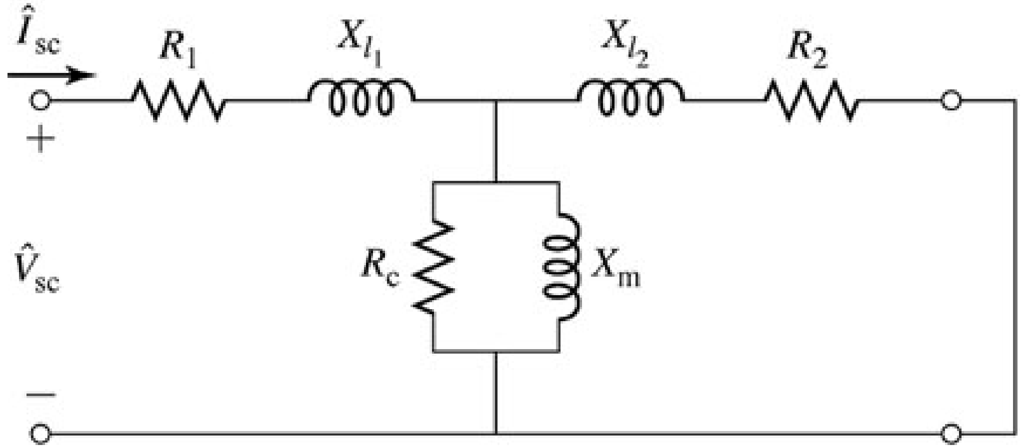
Solve for V_2:

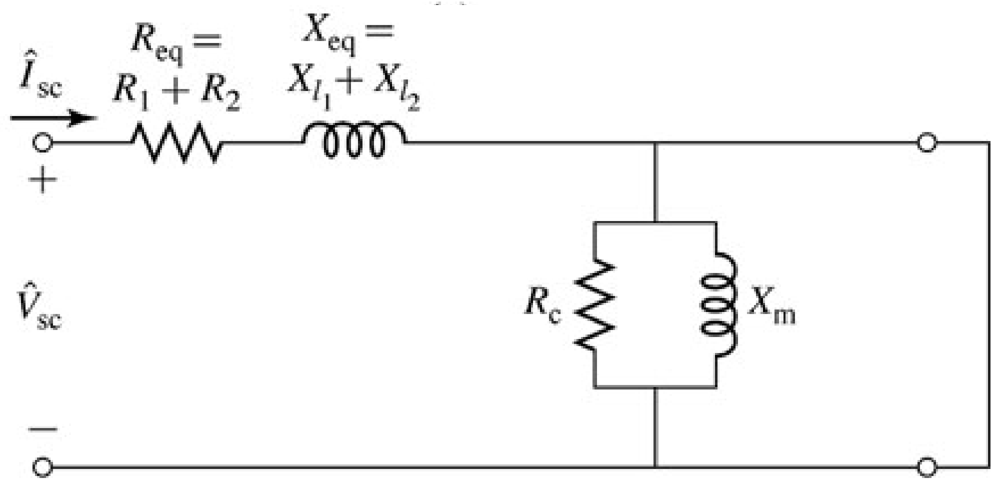
$$(V_2 + ab)^2 + (bc)^2 = V_s^2 \Rightarrow V_2 = 233V$$

Transformer Testing to Determine Parameters

- By doing various tests on a transformer, we can determine the equivalent circuit parameters
- Testing includes open-circuit and short-circuit testing

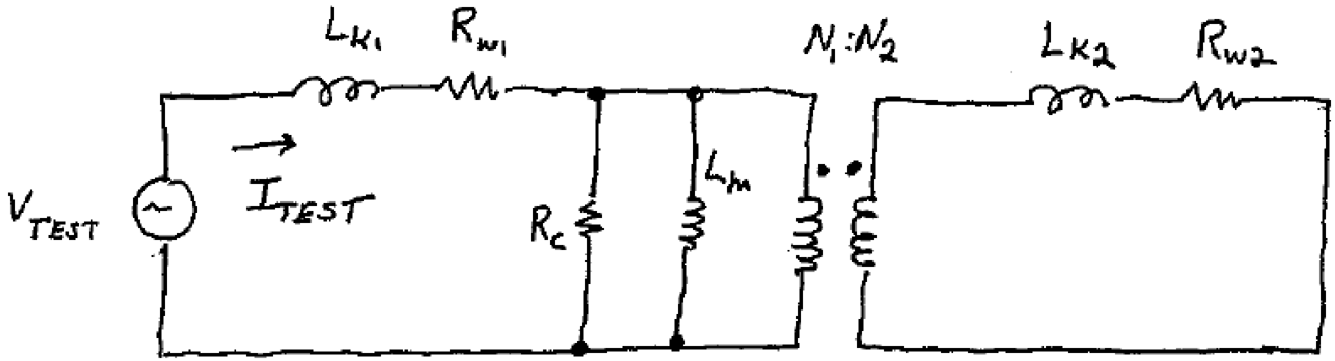
Short-Circuit Test





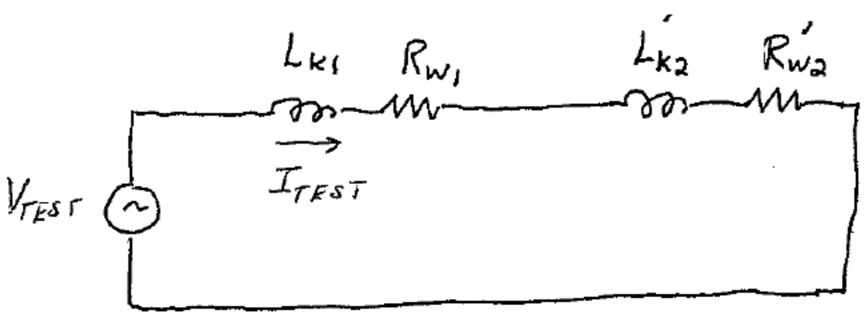
(b)

Short-Circuit Testing



Normally:

$L_m >> L_{k1}, L_{k2}$ and $R_c >> R_{w1}, R_{w2}$



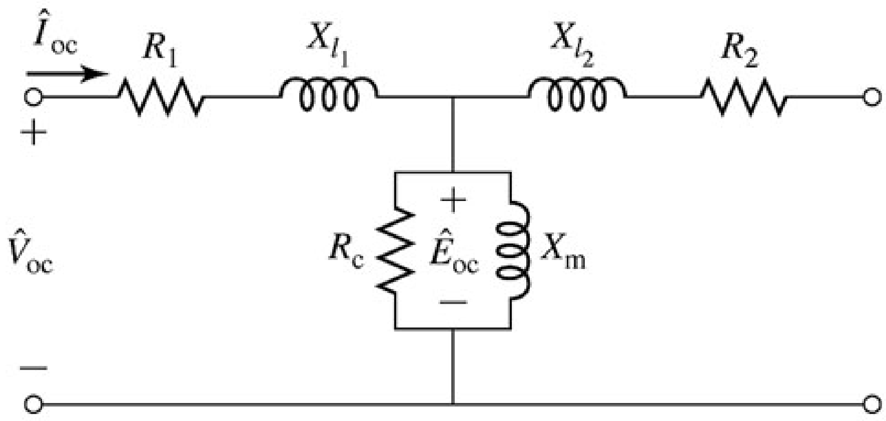
Using the simplified circuit, we can approximate:

$$\left|Z_{eq}\right| \approx \frac{V_{TEST}}{I_{TEST}}$$

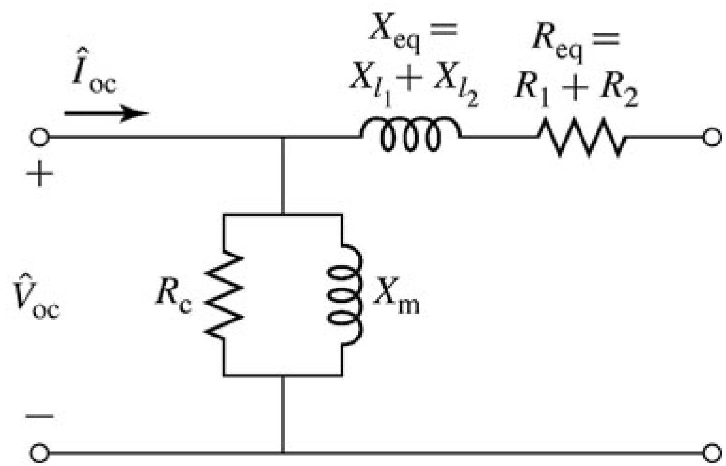
$$R_{SC} = R_{W1} + R_{W2}^{'} = \frac{P_{SC}}{I_{TEST}^2}$$

$$X_{SC} \approx \sqrt{\left|Z_{eq}\right|^2 - R_{SC}^2}$$

Open-Circuit Testing



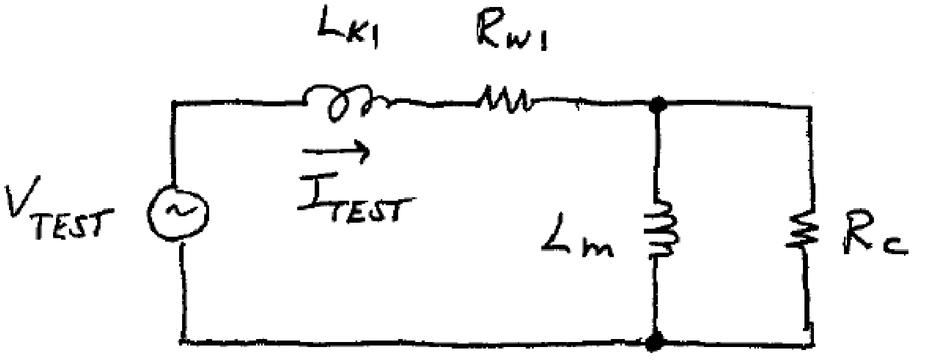
(a)



(b)

Open-Circuit Testing

- There is no secondary current

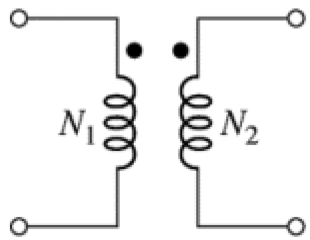


$$R_c \approx \frac{V_{TEST}^2}{P_{OC}}$$

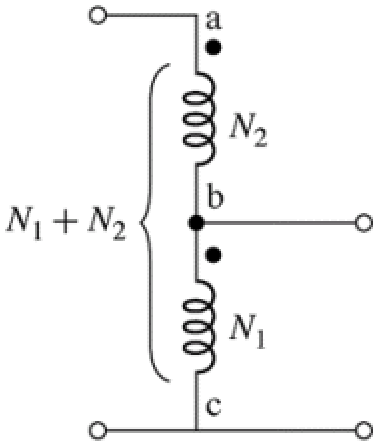
$$\left|X_m\right| \approx \frac{V_{TEST}}{I_{TEST}}$$

Autotransformer

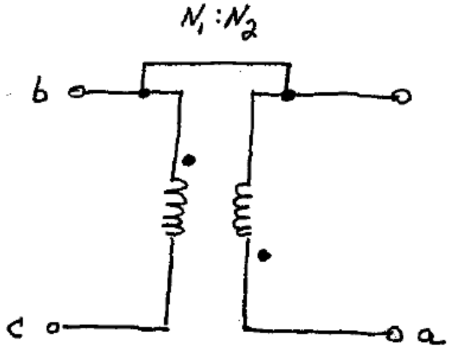
Conventional transformer



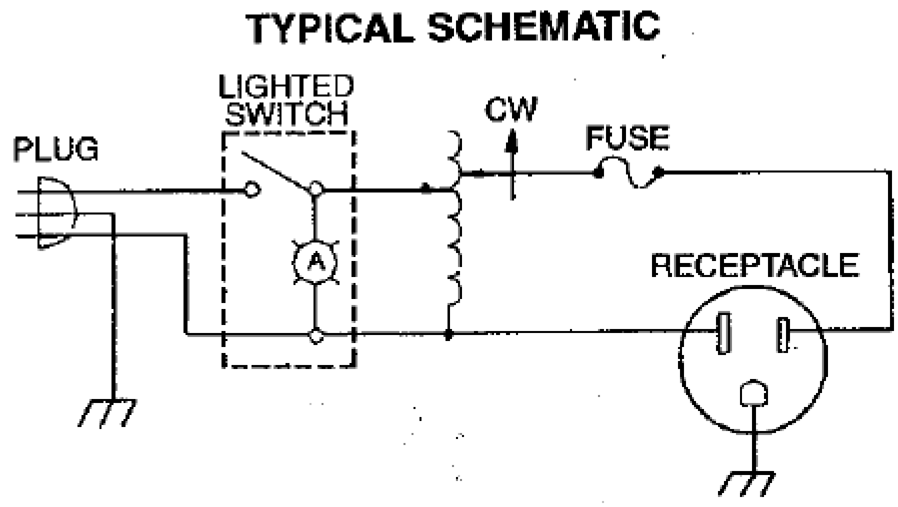
Autotransformer redrawn



Connection as autotransformer

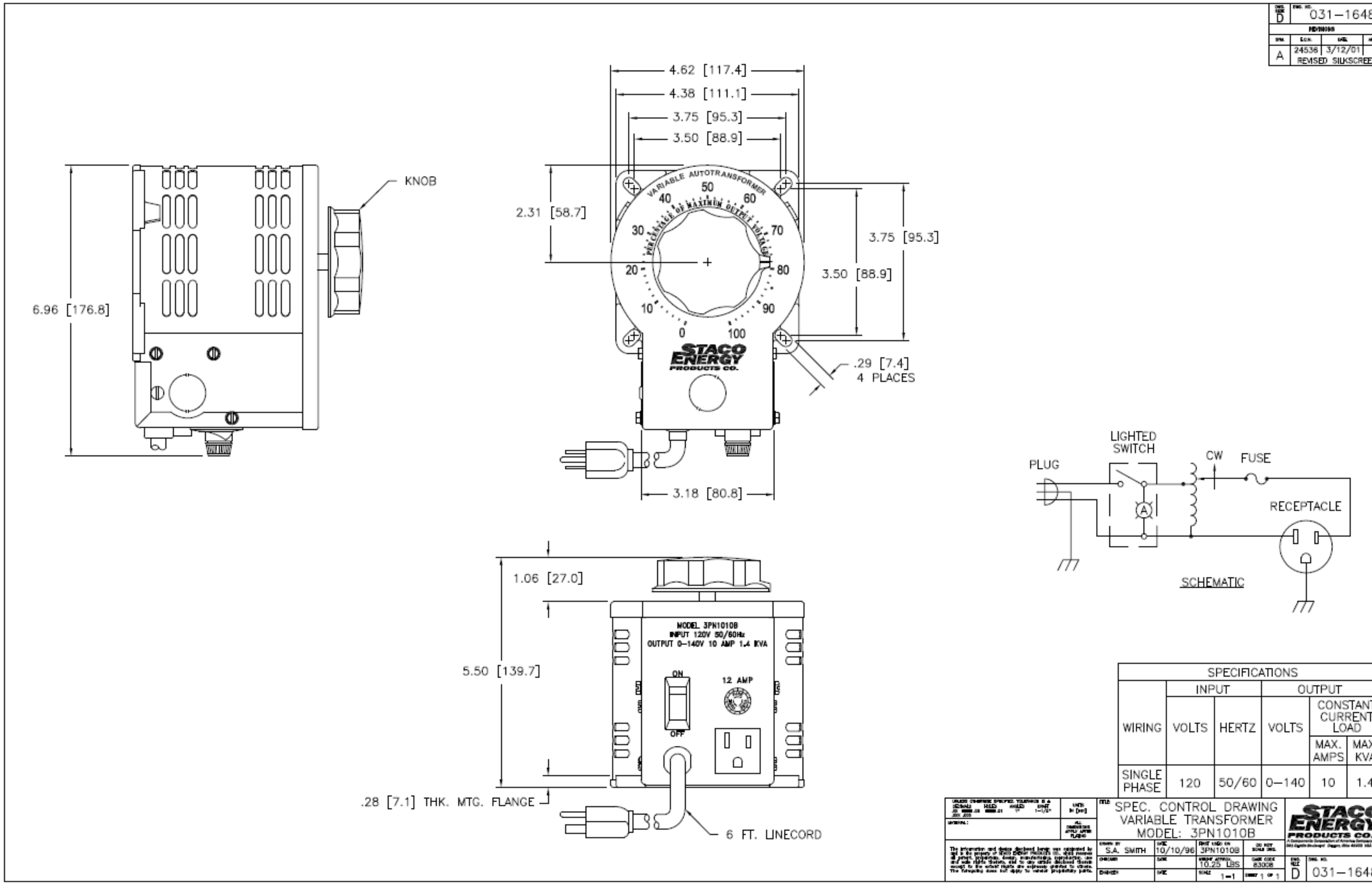


Staco Autotransformer



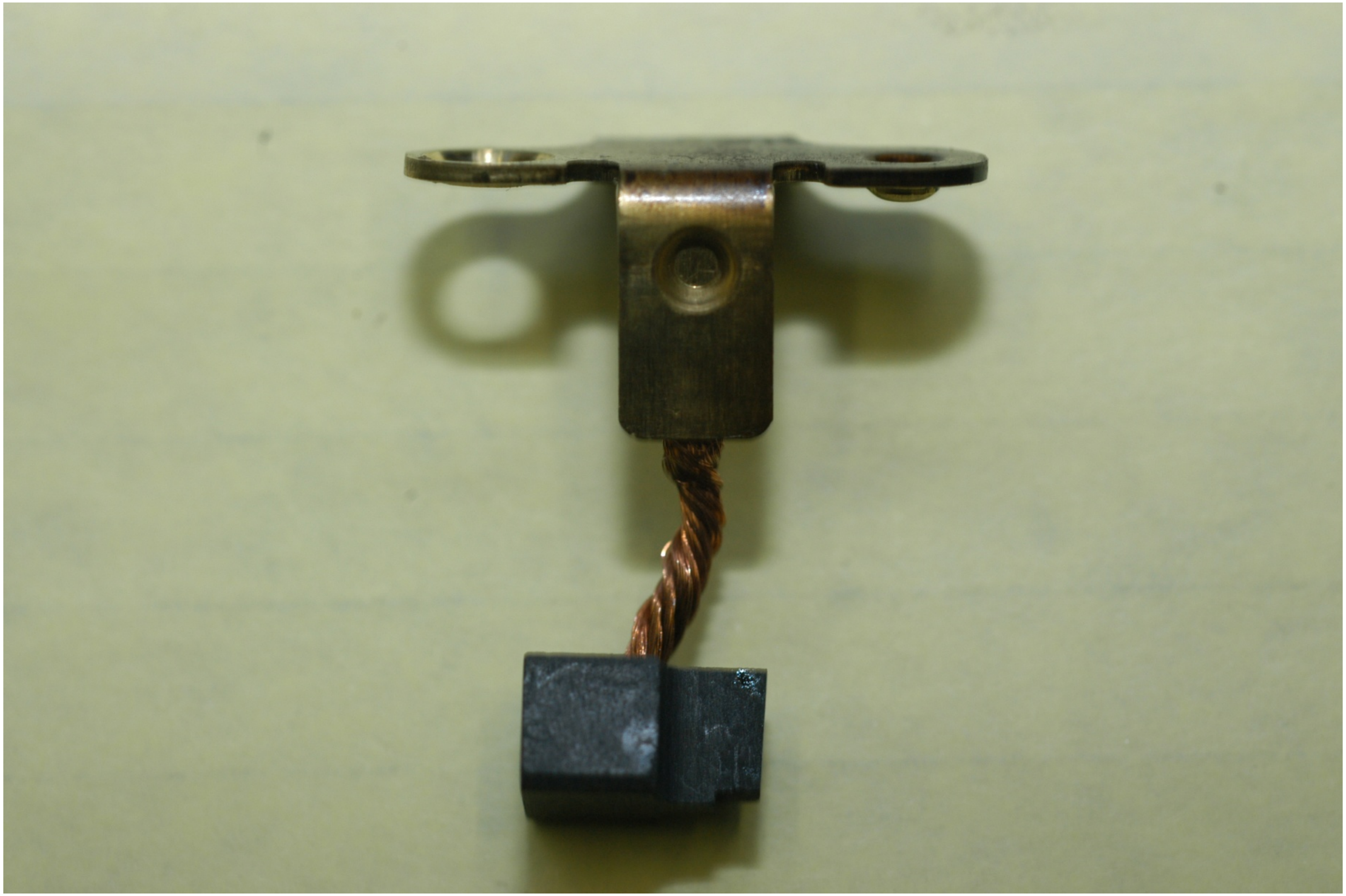
Reference: Staco

Autotransformer Drawing

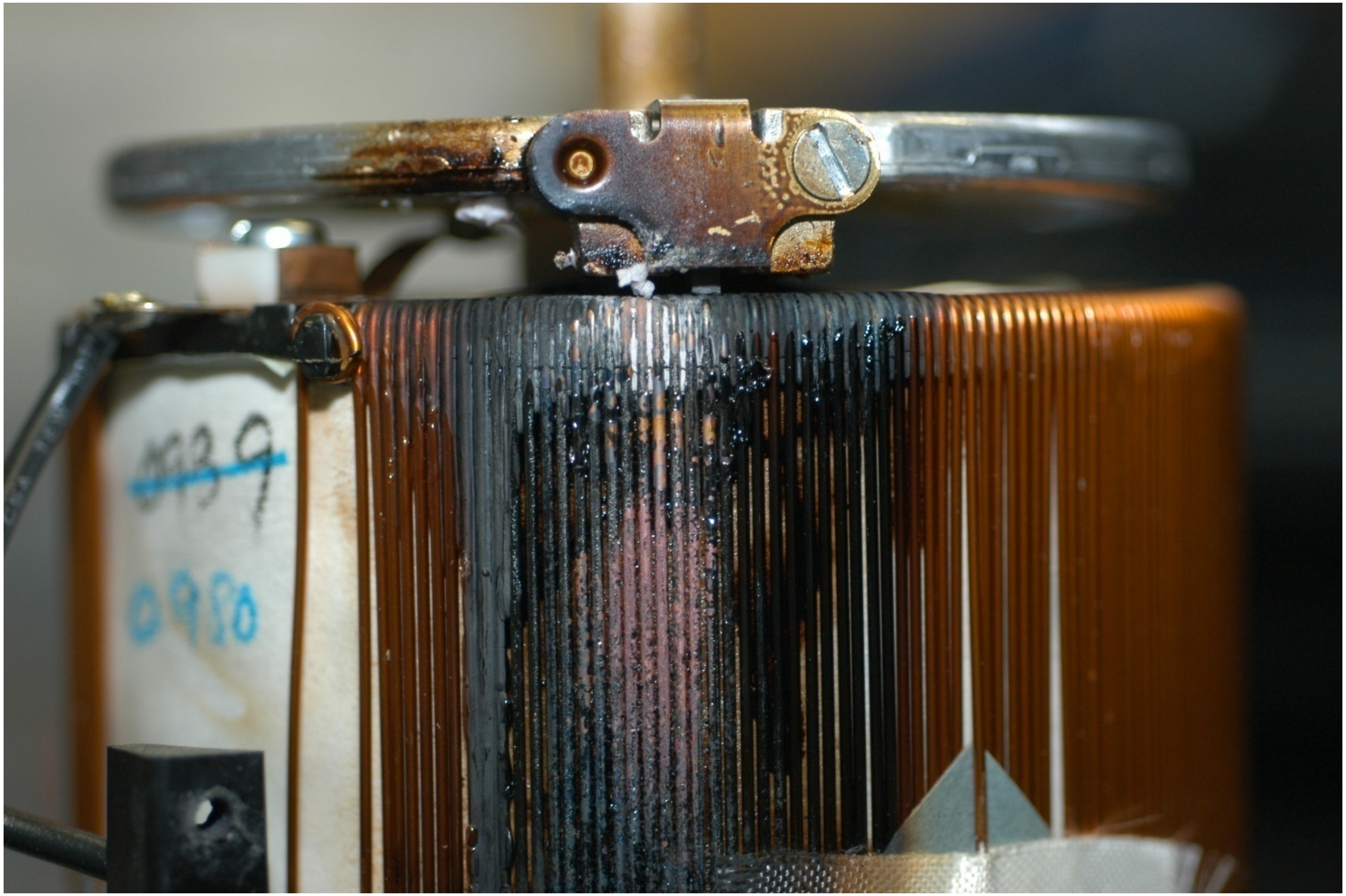


Reference: Staco

Autotransformer Brush



Damaged Autotransformer

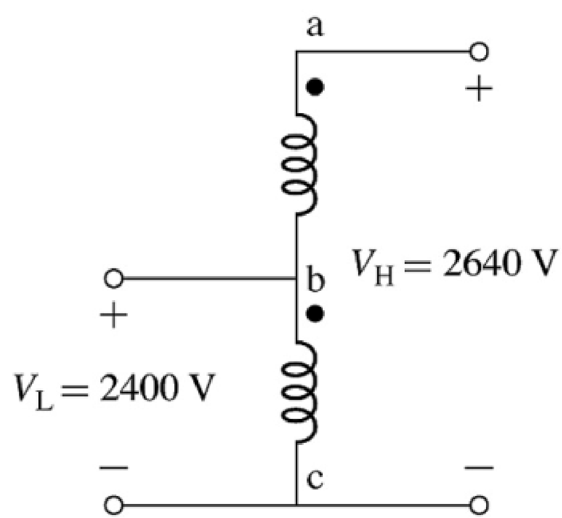


Example: Autotransformer

- *Fitzgerald*, Example 2.7
- 2400:240V 50-kVA transformer is connected as an autotransformer with ab being the 240V winding and bc is the 2400V winding.

(a) Compute the kVA rating

(b) Find currents at rated power



Example: Autotransformer Example --- Solution

For 50 kVA, rating of 240V winding is:

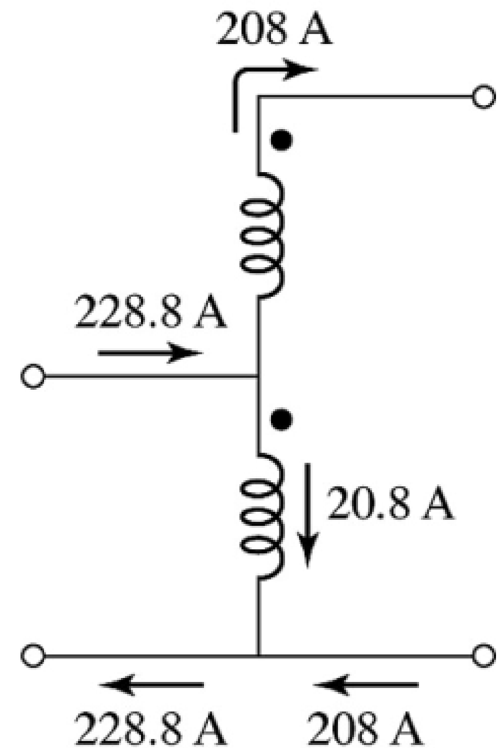
$$50000 / 240 = 208A$$

The autotransformer VA rating is:

$$V_H I_H = (2.64kV)(208A) = 549\ kVA$$

Rated current at low-voltage winding:

$$I_L = I_H \left(\frac{2640}{2400} \right) = 229A$$

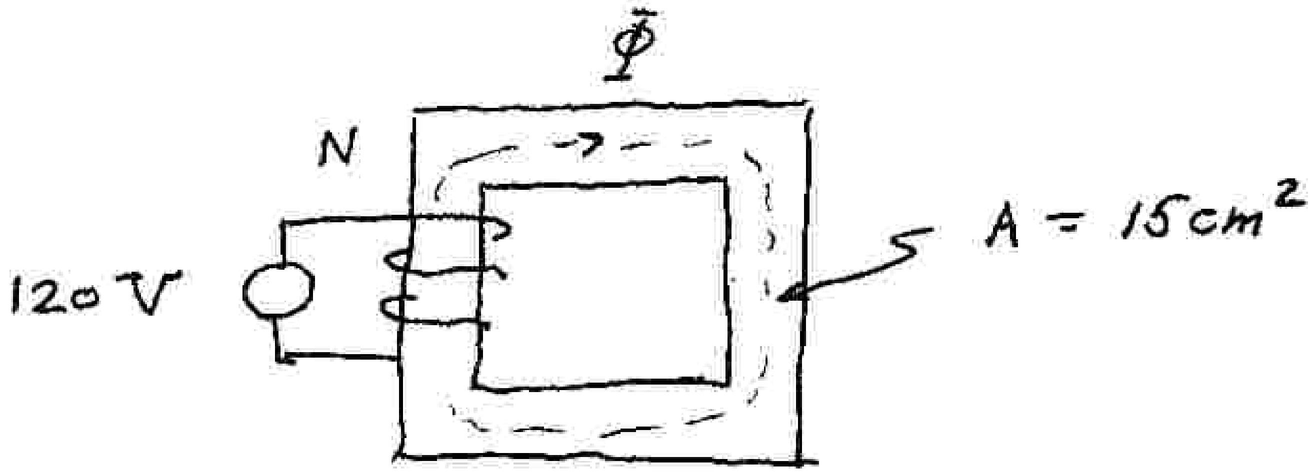


Some Comments on Autotransformers

- Autotransformers differ from isolation transformers in that there is no isolation between primary and secondary
- However, this lack of isolation allows some of the transferred power to be conducted from primary to secondary instead of magnetic induction
- Autotransformers in general require less core material per kVA rating
- Autotransformers used where lack of isolation doesn't pose a safety issue

Example: Magnetic Circuit Problem

- *Fitzgerald*, Problem 2.2
- A magnetic circuit with a cross-sectional area of 15 cm^2 is to be operated from a 120V RMS supply. Calculate the number of turns required to achieve a peak magnetic flux density of 1.8 Tesla in the core



Example: Magnetic Circuit Problem --- Solution

Flux is: $\Phi = \Phi_{\max} \sin(\omega t)$

The time rate of change of flux is:

$$\frac{d\Phi}{dt} = \omega\Phi_{\max} \cos(\omega t)$$

The time rate of change of flux linkage is:

$$\frac{d\lambda}{dt} = N\frac{d\Phi}{dt} = N\omega\Phi_{\max} \cos(\omega t) = V$$

Let's relate flux density to flux:

$$\frac{\Phi_{\max}}{A} = B_{\max}$$

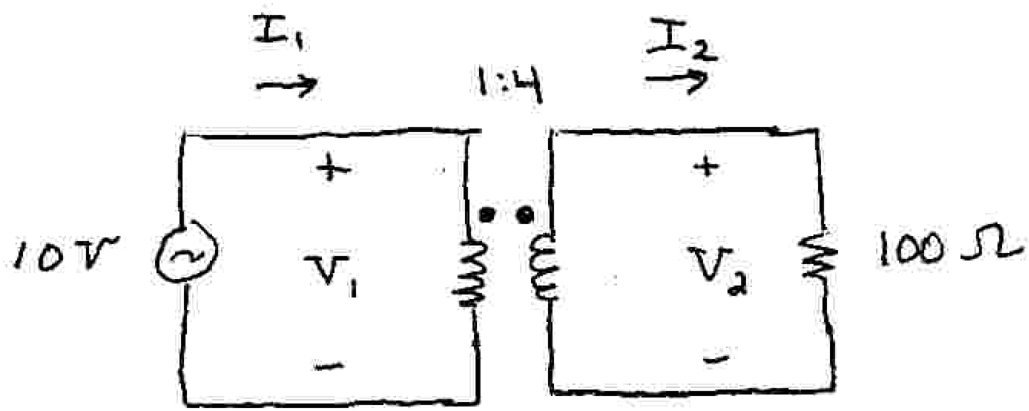
So, we can solve for N

$$N = \frac{\sqrt{2}V}{\omega B_{\max} A} = \frac{(\sqrt{2})(120)}{(2\pi \times 60)(1.8)(1.5\times 10^{-3})} = 166.7$$

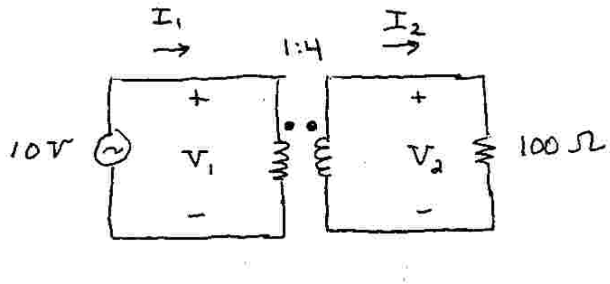
Round up to N = 167

Example: Transformer Problem

- *Fitzgerald*, Problem 2.4
- A 100-Ohm resistor is connected to the secondary of an ideal transformer with a turns ratio of 1:4 (primary to secondary). A 10V RMS, 1-kHz voltage source is connected to the primary. Calculate the primary current and the voltage across the 100-Ohm resistor



Example: Transformer Problem --- Solution



We know that this is a step-up transformer, so V_1 = 10V and V_2 = 40V.

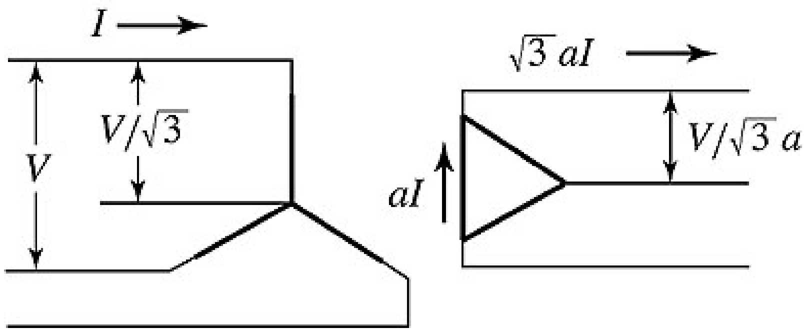
We next find the secondary current I2

$$I_2 = \frac{40V}{100\Omega} = 0.4A$$

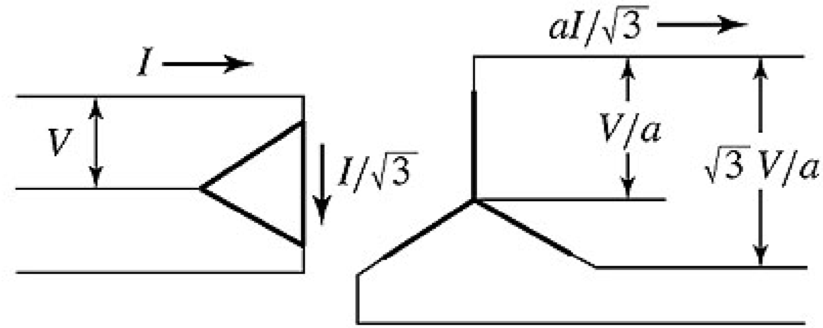
The voltage steps up, so the current steps down; hence

$I_1 = 4I_2 = 1.6A$

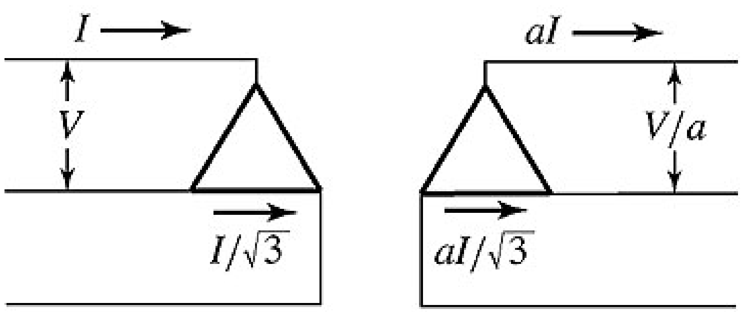
3-Phase Connections of Transformers



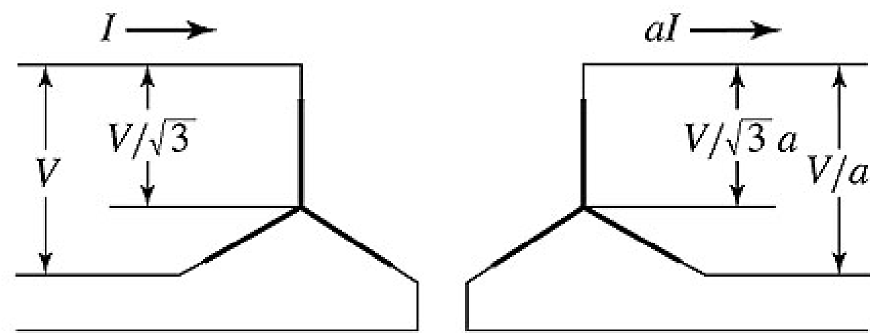
(a) Y-Δ connection



(b) Δ-Y connection



(c) Δ-Δ connection

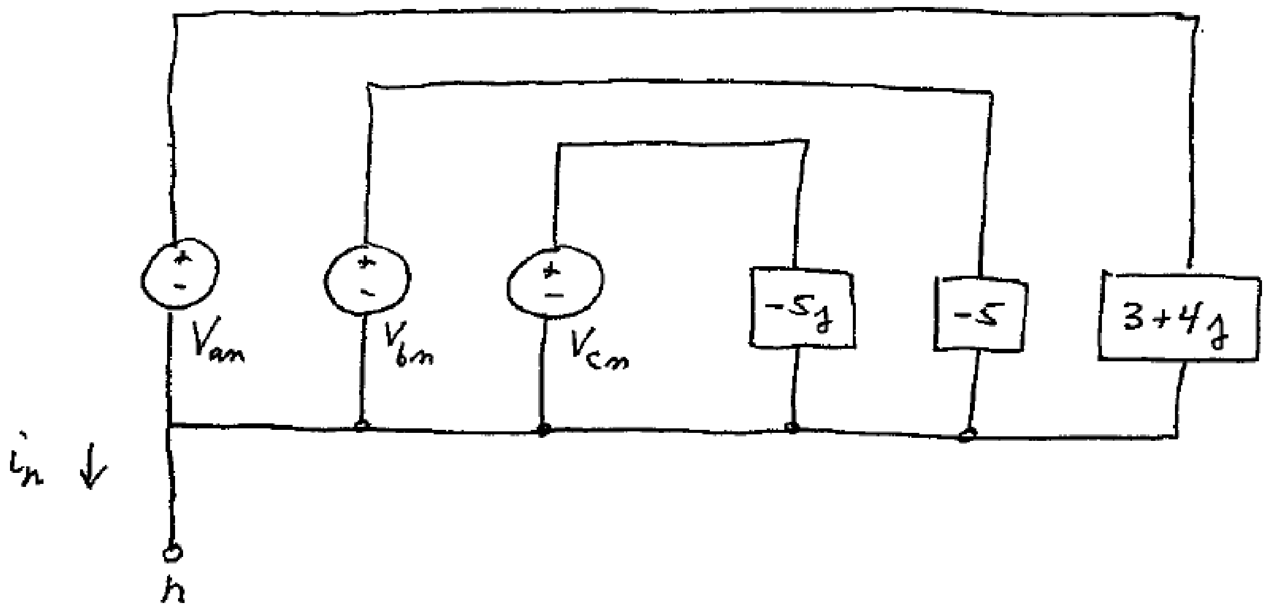


(d) Y-Y connection

Example: Finding 3-Phase Currents

- Problem: A three phase, 208V (line-line) Y connected load has:
  - $Z_{an} = 3 + j4$
  - $Z_{bn} = 5$
  - $Z_{cn} = -5j$
- Find
  - (a) Phase voltages
  - (b) Line and phase currents
  - (c) Neutral currents

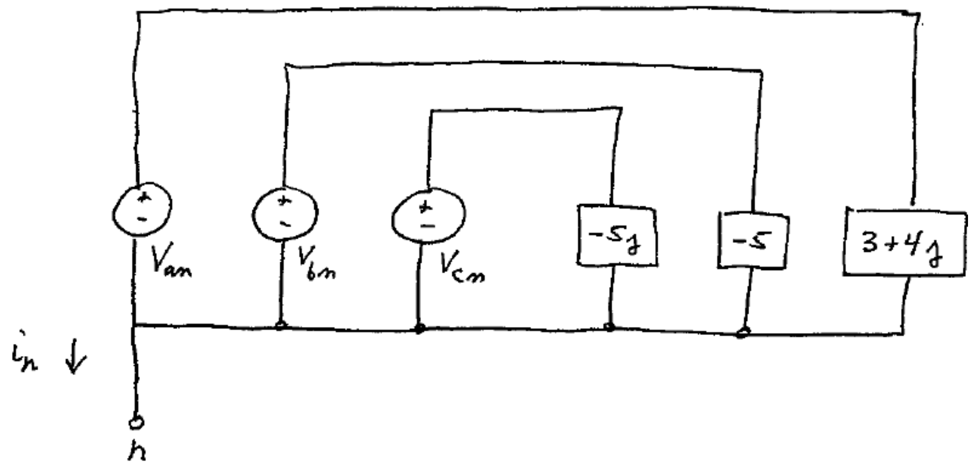
Example: Finding 3-Phase Currents --- Solution



Line-neutral voltages are found by taking the line-line voltage and dividing by $\sqrt{3}$

$$V_{L-N} = \frac{208}{\sqrt{3}} = 120$$

Example: 3-Phase Currents --- Solution



Line (also called phase) currents are found by taking the line-line voltages and dividing by impedance

$$I_a = \frac{120\angle 0^o}{(3+4j)} = 24\angle -53.13^o$$

$$I_b = \frac{120\angle 120^o}{(5)} = 24\angle 120^o$$

$$I_c = \frac{120\angle 240^o}{(-5j)} = 24\angle -30^o$$

Example: 3-Phase Currents --- Solution

- Note that loads are unbalanced so there is a net neutral current

$$I_n = I_a + I_b + I_c = 25.4\angle 155.9^o$$

480V Y System

- Line-line = 480V; line-neutral = 277V

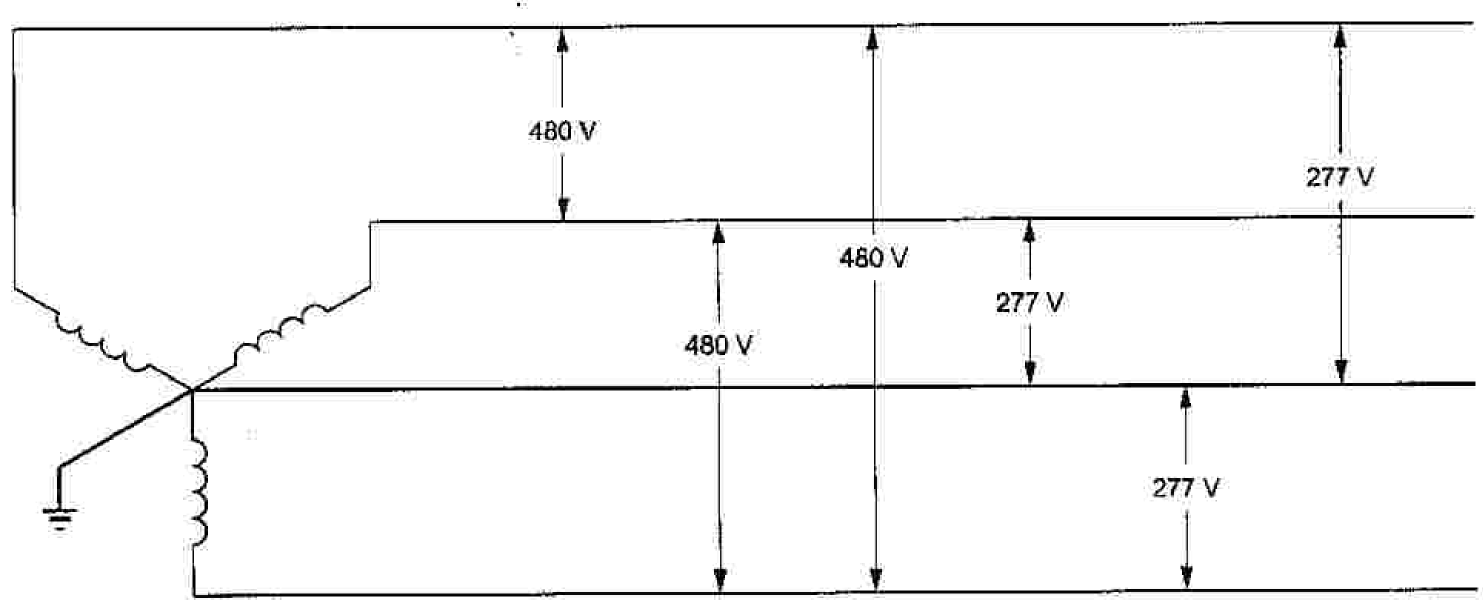


FIGURE 1–1 480Y/277-Volt System

Reference: Ralph Fehr, *Industrial Power Distribution*, Prentice Hall, 2002

Delta System

- Line-line = 480V

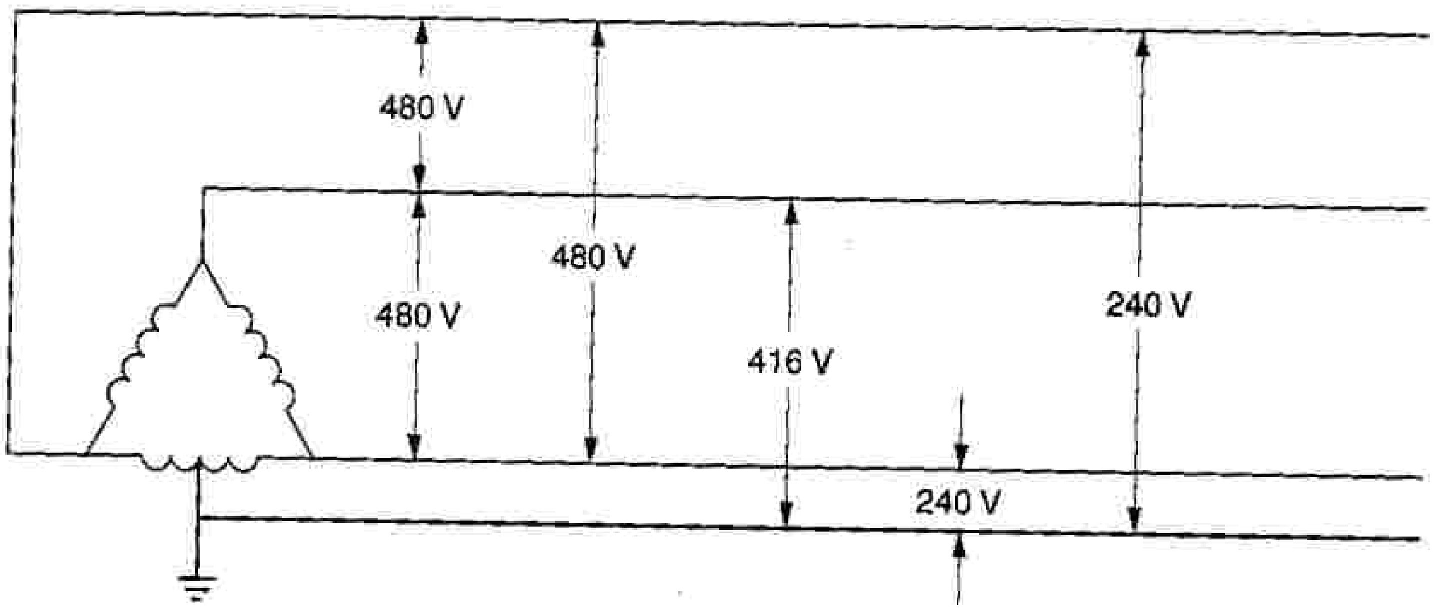
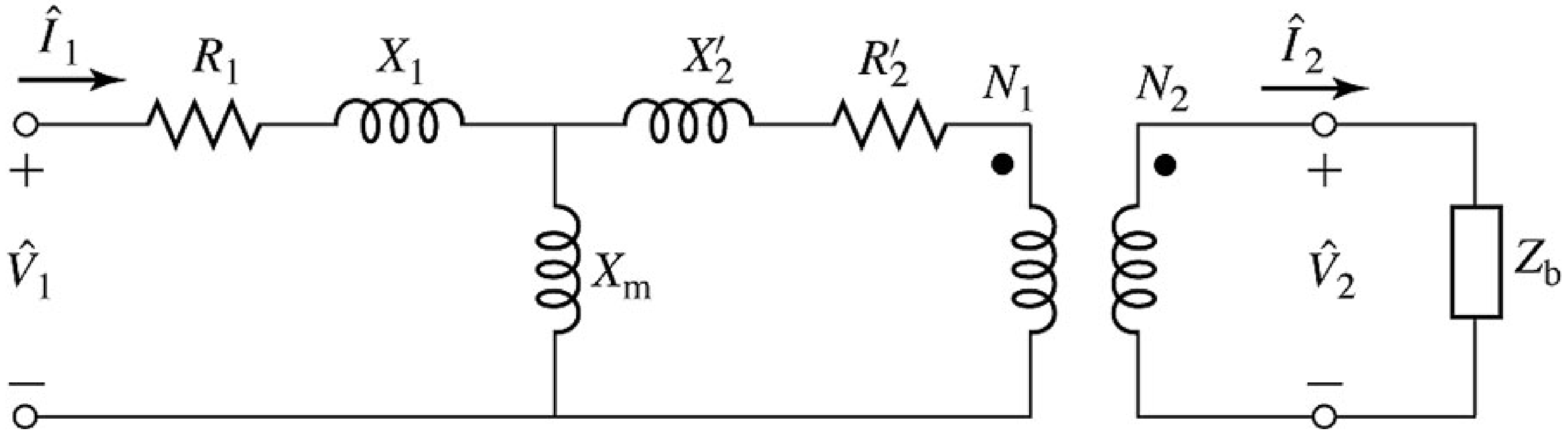


FIGURE 1–2 High-Leg Delta System

Reference: Ralph Fehr, *Industrial Power Distribution*, Prentice Hall, 2002

Instrument Transformer

- Used for protection, relaying, voltage or current monitoring, etc.



Per-Unit System

- Computations in electric machines and transformers are often done using the “per-unit” system
- Actual circuit quantities (Watts, VArs, etc.) are scaled to the per-unit system
- This method allows removal of transformers from diagrams
- To convert to per-unit, 4 base quantities are established
  - Base power VA_{base}
  - Base voltage V_{base}
  - Base current I_{base}
  - Base impedance Z_{base}

Example: Per-Unit System

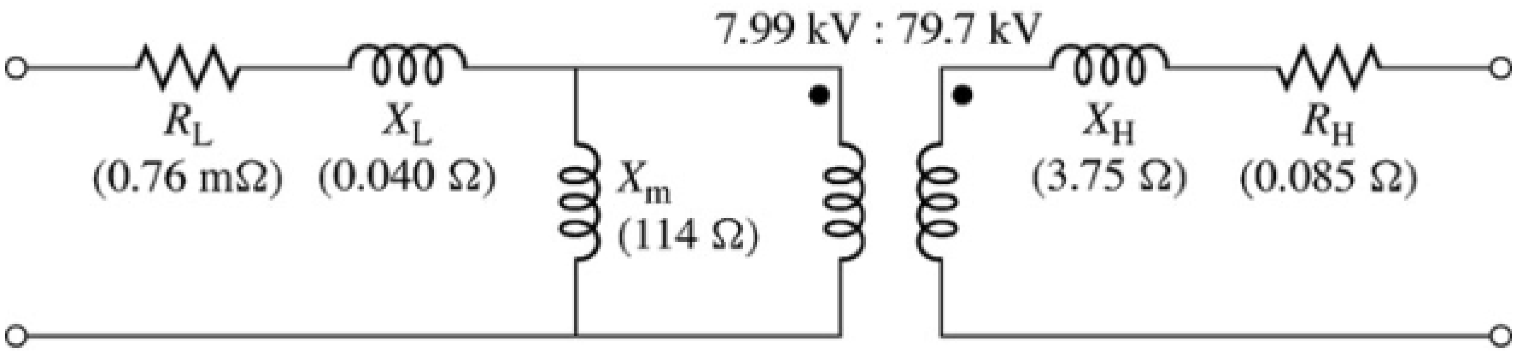
- Example: A system has $Z_{base} = 10\ \Omega$ and $V_{base} = 400V$. Find base VA and I_{base}
- Solution:
  - $I_{base} = V_{base}/Z_{base} = 400/10 = 40A$
  - $(VA)_{base} = V_{base}I_{base} = (400)(40) = 16\ kVA$

Example: Per-Unit System

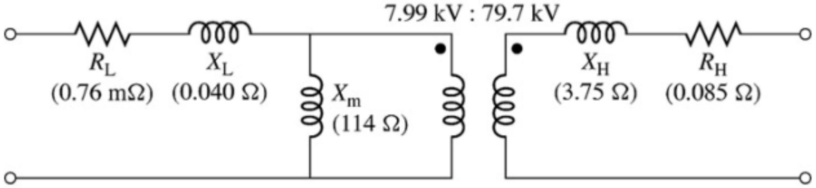
- A 5 kVA, 400/200V transformer has 2 Ω reactance referred to the 200V side. Express the transformer reactance in p.u.
- Solution:
  - $(VA)_{base} = 5000$
  - $V_{base} = 200$
  - $I_{base} = (VA)_{base}/V_{base} = 5000/200 = 25A$
  - $Z_{base} = V_{base}/I_{base} = 200/25 = 8\ \Omega$
  - $Z = 2.0/8.0 = 0.25$ p.u.

Example: Per-Unit Applied to Transformer

- *Fitzgerald*, Example 2.12
- Convert this circuit showing a 100 MVA transformer to the per-unit system



Example: Low-Voltage Side



$$(VA)_{BASE} = 100\ MVA$$

$$V_{BASE} = 7.99\ kV$$

$$R_{BASE} = X_{BASE} = Z_{BASE} = \frac{V_{BASE}^2}{(VA)_{BASE}}$$

$$= \left(\frac{(7.99 \times 10^3)^2}{100 \times 10^{-6}} \right) = 0.638\ \Omega$$

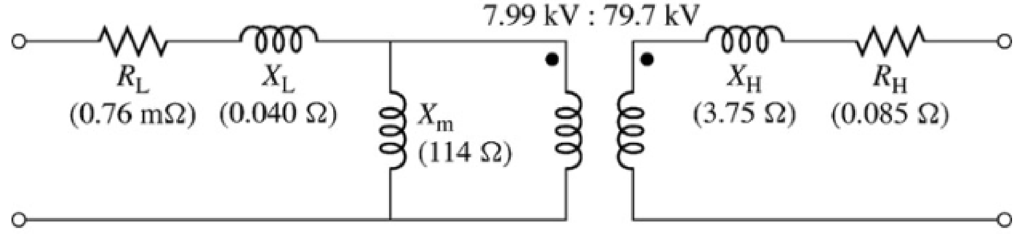
In per-unit system:

$$R_L = \frac{0.76 \times 10^{-3}}{0.638} = 0.0012\ p.u.$$

$$X_L = \frac{0.04}{0.638} = 0.063\ p.u.$$

$$X_m = \frac{114}{0.638} = 178.7\ p.u.$$

Example: High-Voltage Side



$$(VA)_{BASE} = 100\ MVA$$

$$V_{BASE} = 79.7\ kV$$

$$R_{BASE} = X_{BASE} = Z_{BASE} = \frac{V_{BASE}^2}{(VA)_{BASE}}$$

$$= \left(\frac{(79.7 \times 10^3)^2}{100 \times 10^{-6}} \right) = 63.5\ \Omega$$

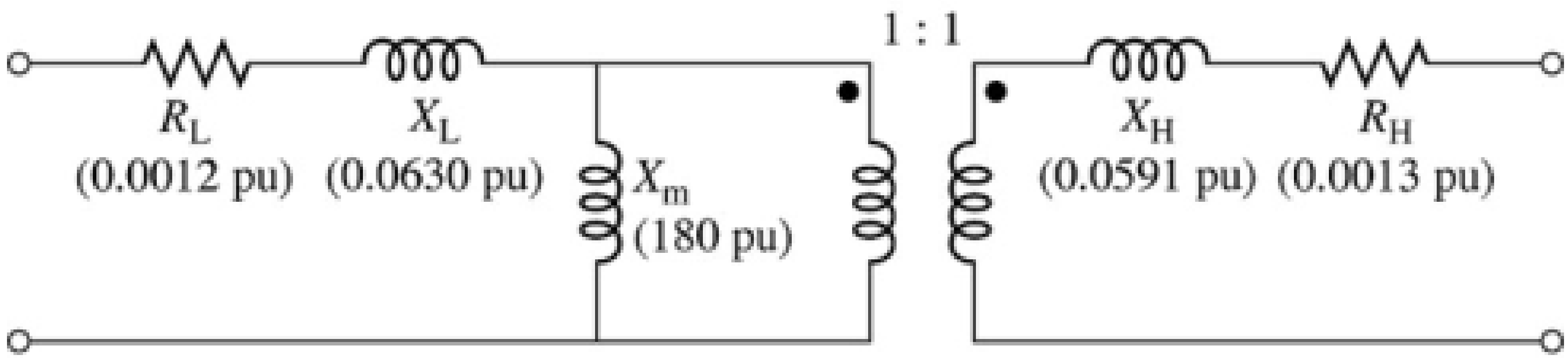
In per-unit system:

$$R_H = \frac{0.085}{63.5} = 0.00133\ p.u.$$

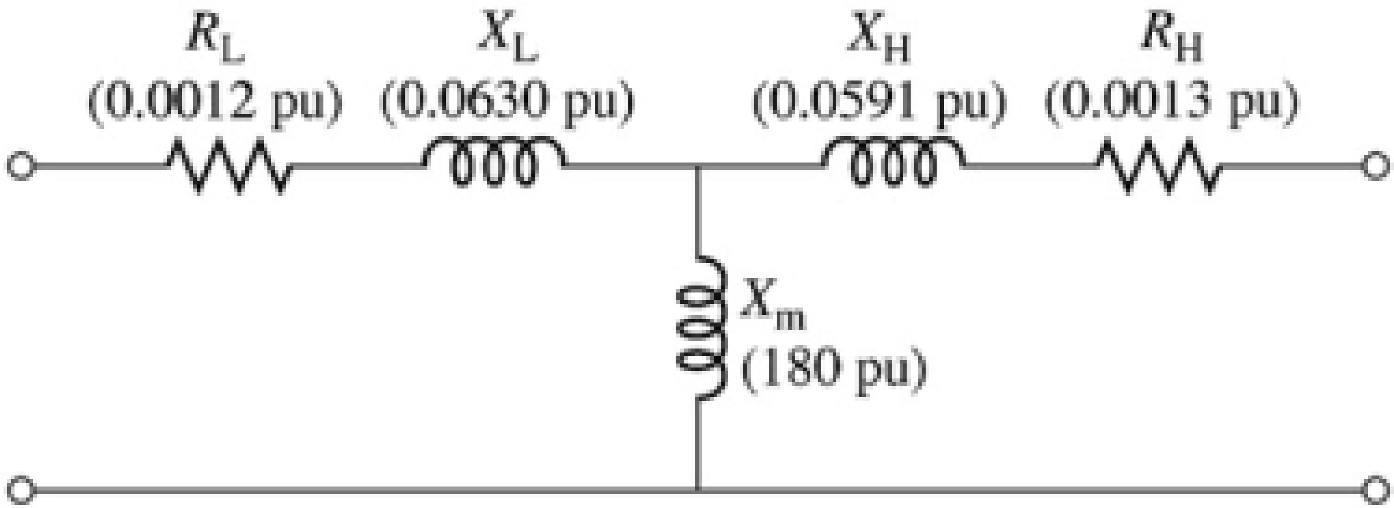
$$X_H = \frac{3.75}{63.5} = 0.059\ p.u..$$

Example: Model

- Turns ratio is 1:1, so we can remove transformer



Example: Get Rid of 1:1 Transformer

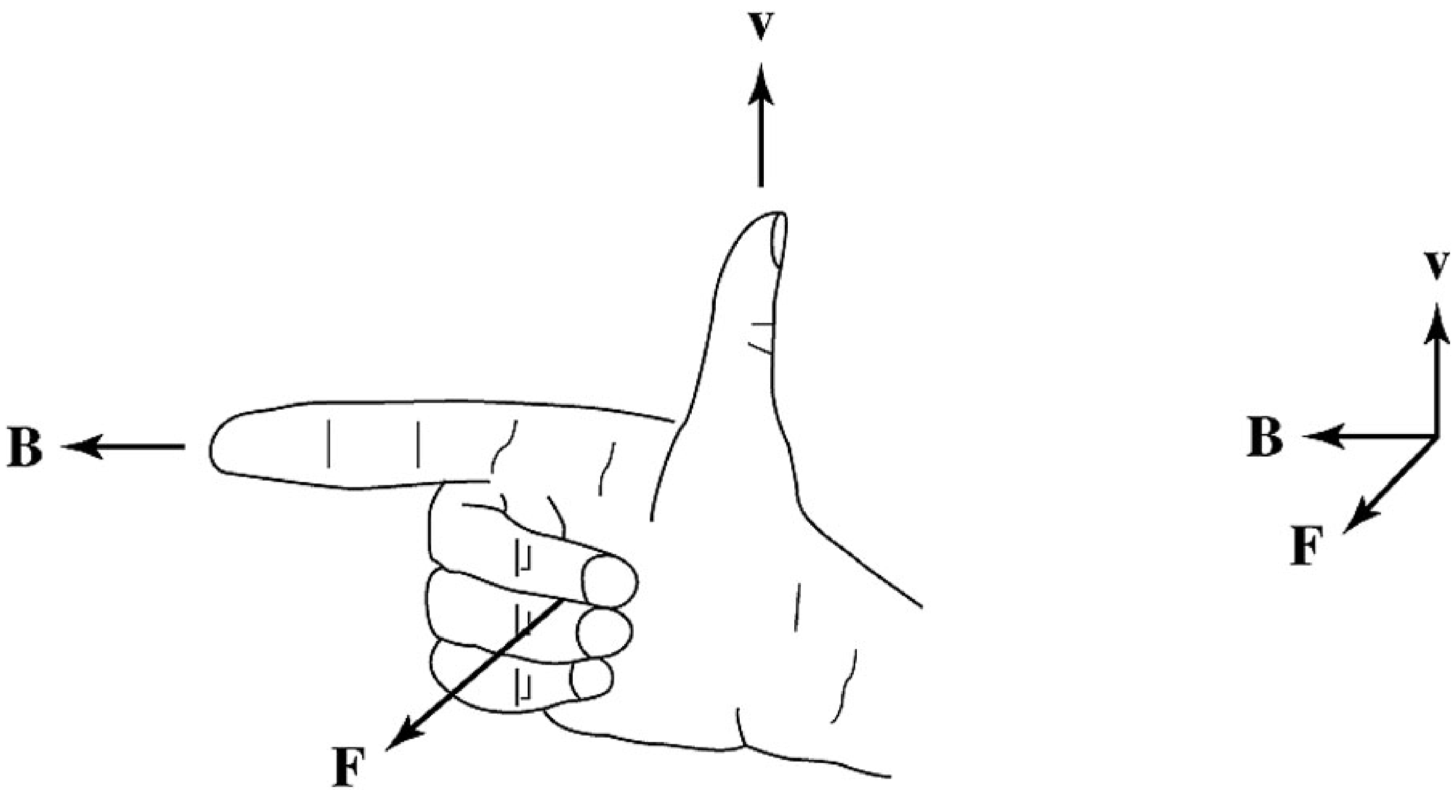


Energy Conversion --- Overview

- Forces and torques
- Energy balance
- Determining magnetic forces and torques from energy
- Multiply-excited systems
- Forces and torques in systems with permanent magnets

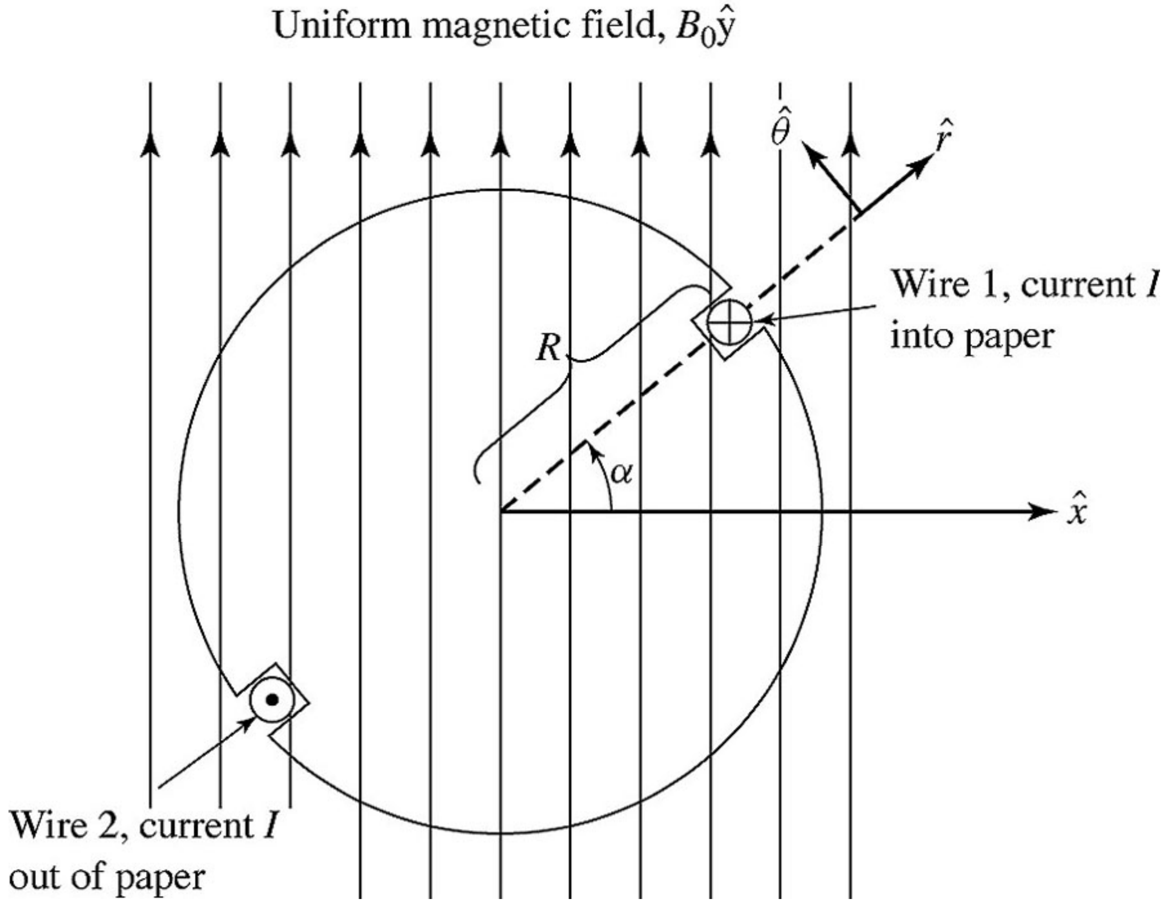
Right-Hand Rule

• For determining the direction magnetic-field component of the Lorentz force F=*q*(v x B) = JxB.

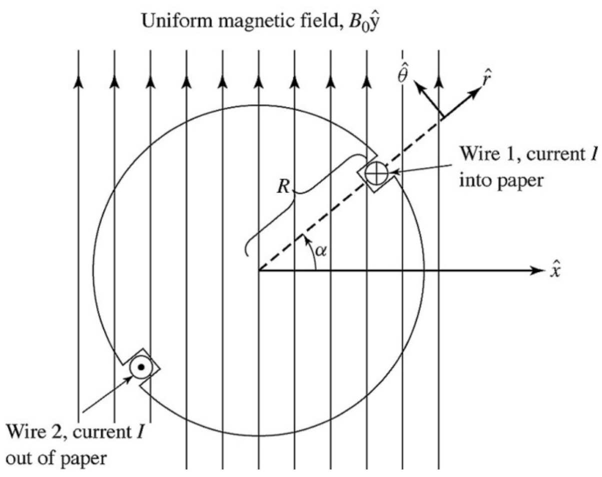


Example: Single-Coil Rotor

- *Fitzgerald*, Example 3.1
- Find θ-directed torque as a function of α



Example: Single-Coil Rotor



For wire #1:

$$F_{1\theta} = -IlB_o \sin\alpha$$

For wire #2:

$$F_{2\theta} = -IlB_o \sin\alpha$$

Total torque (T = force × distance):

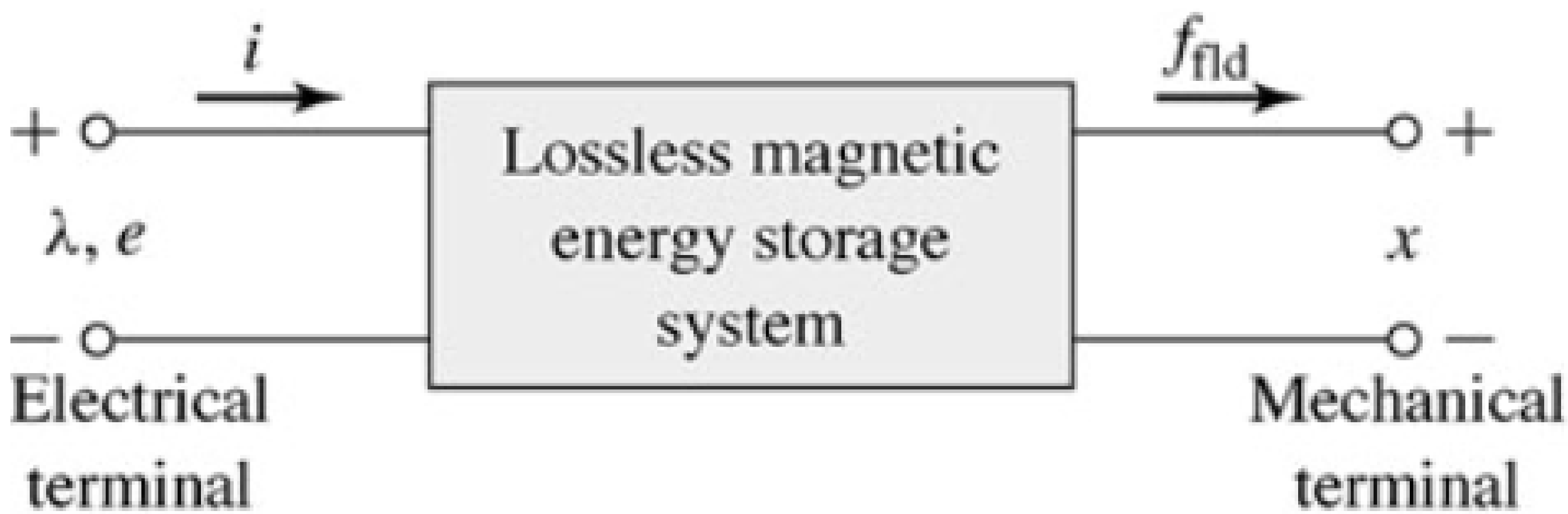
$$T_\theta = -2IlB_o \sin\alpha R$$

What happens if B points left-right instead
of up-down?

$$T_\theta = -2IlB_o \cos\alpha R$$

Electromechanical Energy Conversion Device

- This box can be used to model motors, actuators, lift magnets, etc.
- Note 2 electrical terminals (voltage and current) and 2 mechanical terminals (force f_{fld} and position x)
- The lossless magnetic energy storage system converts electrical energy to mechanical energy



Interaction Between Electrical and Mechanical Terminals

W_{FLD} = stored magnetic energy

In words, the rate of change of magnetic energy equals the power in minus the mechanical work out

$$\frac{dW_{FLD}}{dt} = ei - f_{fld}\frac{dx}{dt}$$

By Faraday's law, e = dλ/dt, so let's rework:

$$\frac{dW_{FLD}}{dt} = i\frac{d\lambda}{dt} - f_{fld}\frac{dx}{dt}$$

Multiply through everywhere by dt:

$$dW_{FLD} = id\lambda - f_{fld}dx$$

Interaction Between Electrical and Mechanical Terminals

In a lossless system, we can rewrite the energy balance:

$$dW_{ELEC} = dW_{MECH} + dW_{FLD}$$

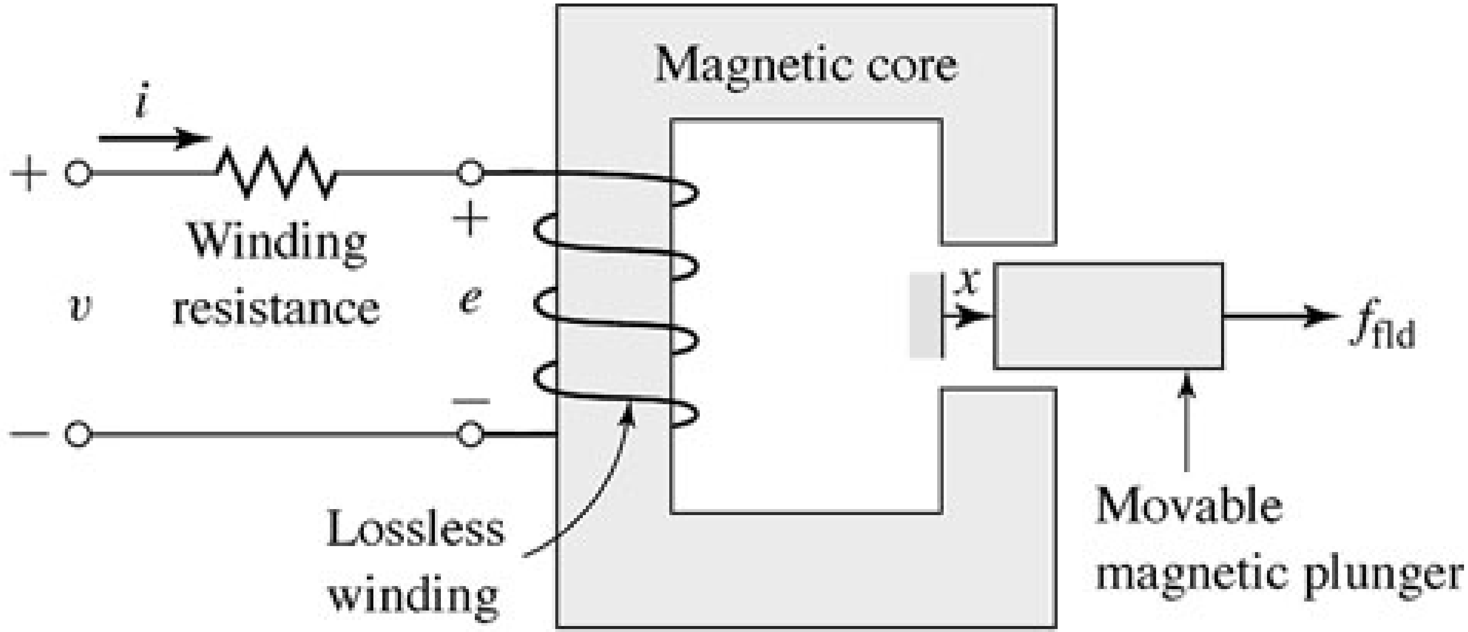
Differential energy in: $dW_{ELEC} = i d\lambda$

Differential work out: $dW_{MECH} = f_{fld} dx$

Change in magnetic energy: dW_{FLD}

Force-Producing Device

- This solenoid is an example of a force-producing device



(b)

Energy

Thinking about energy, we start out with:

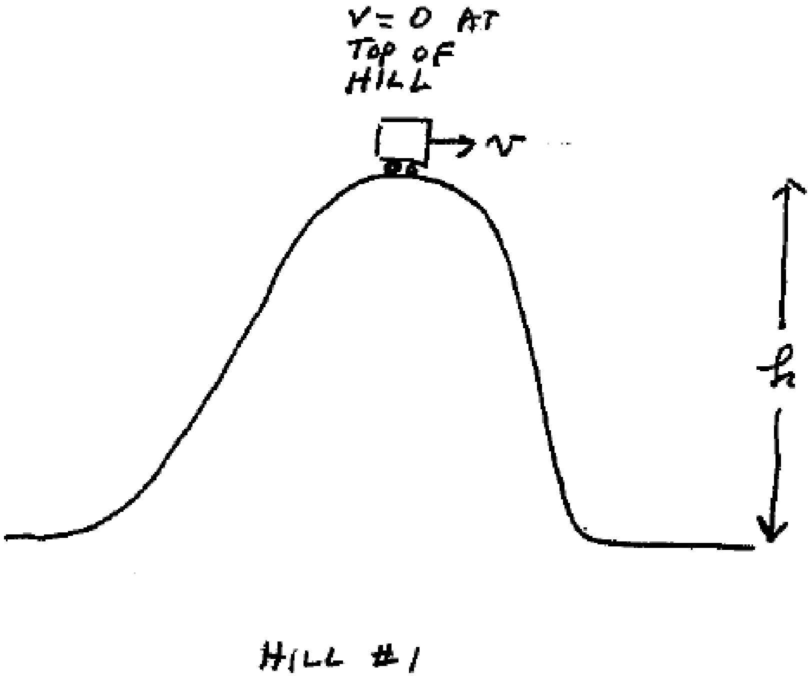
$$dW_{FLD} = id\lambda - f_{fld}dx$$

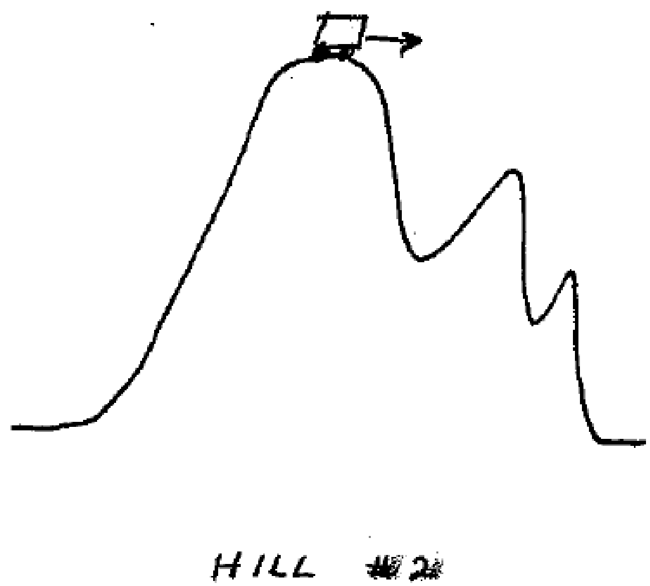
If magnetic energy storage is lossless, this is a conservative system and W_{fld} is determined by state variables λ and x

In a conservative system, the path you take to do this integration doesn't matter

Another Conservative System

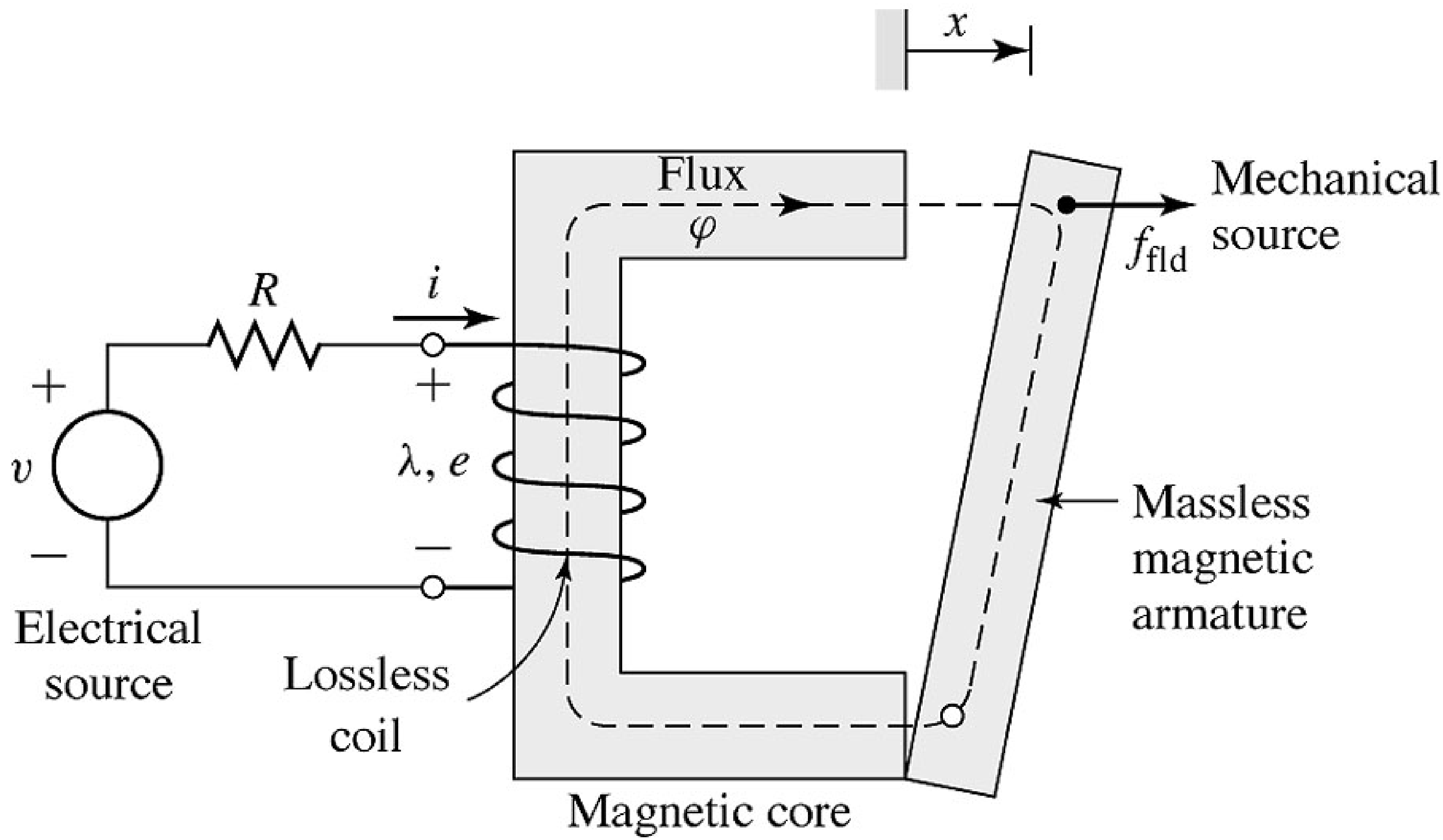
- Roller coaster, ignoring friction, the path doesn't matter. Speed of both coasters is the same at the bottom of the hill



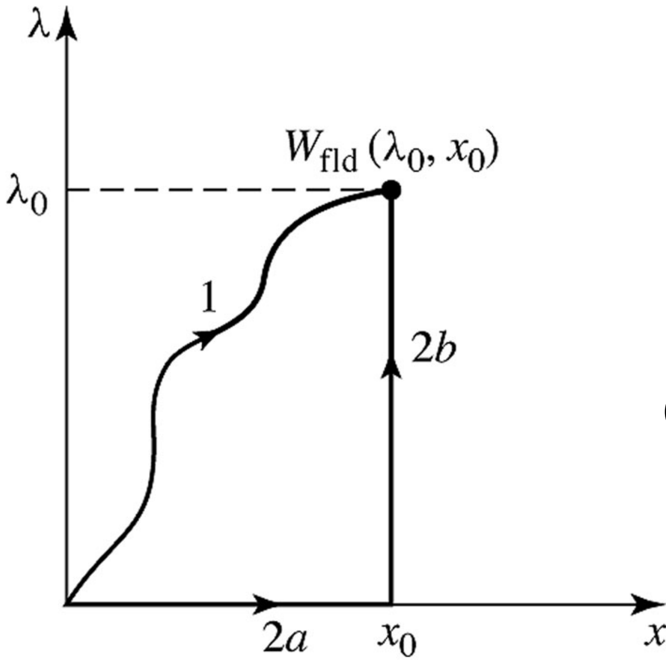


Magnetic Relay

- This illustrates lossless magnetic structure with external losses due to resistance



Integration Path for Finding Magnetic Stored Energy

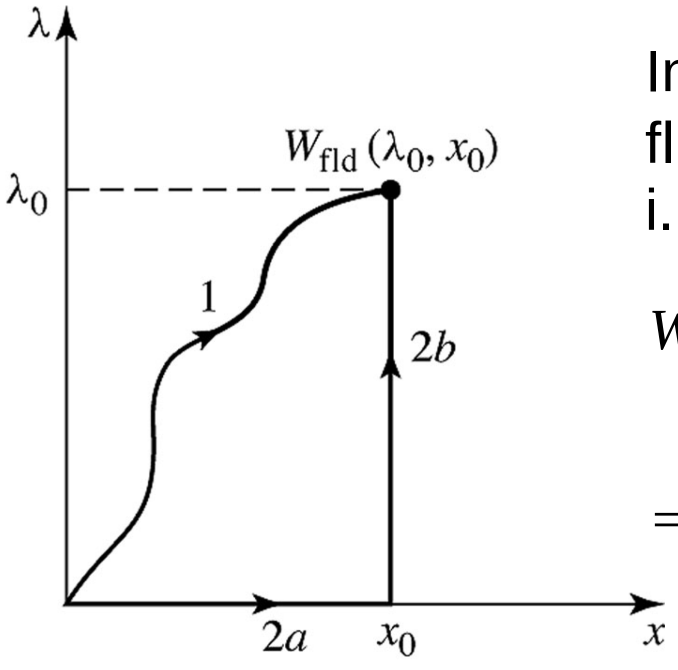


$$W_{FLD}(\lambda_o, x_o) = \int_{path2a} dW_{FLD} + \int_{path2b} dW_{FLD}$$

On path 2a, $d\lambda = 0$ and $f_{fld} = 0$ since zero λ means zero magnetic force, therefore:

$$W_{FLD}(\lambda_o, x_o) = \int_0^{\lambda} i(\lambda, x_o)\, d\lambda$$

Special Case --- Linear System



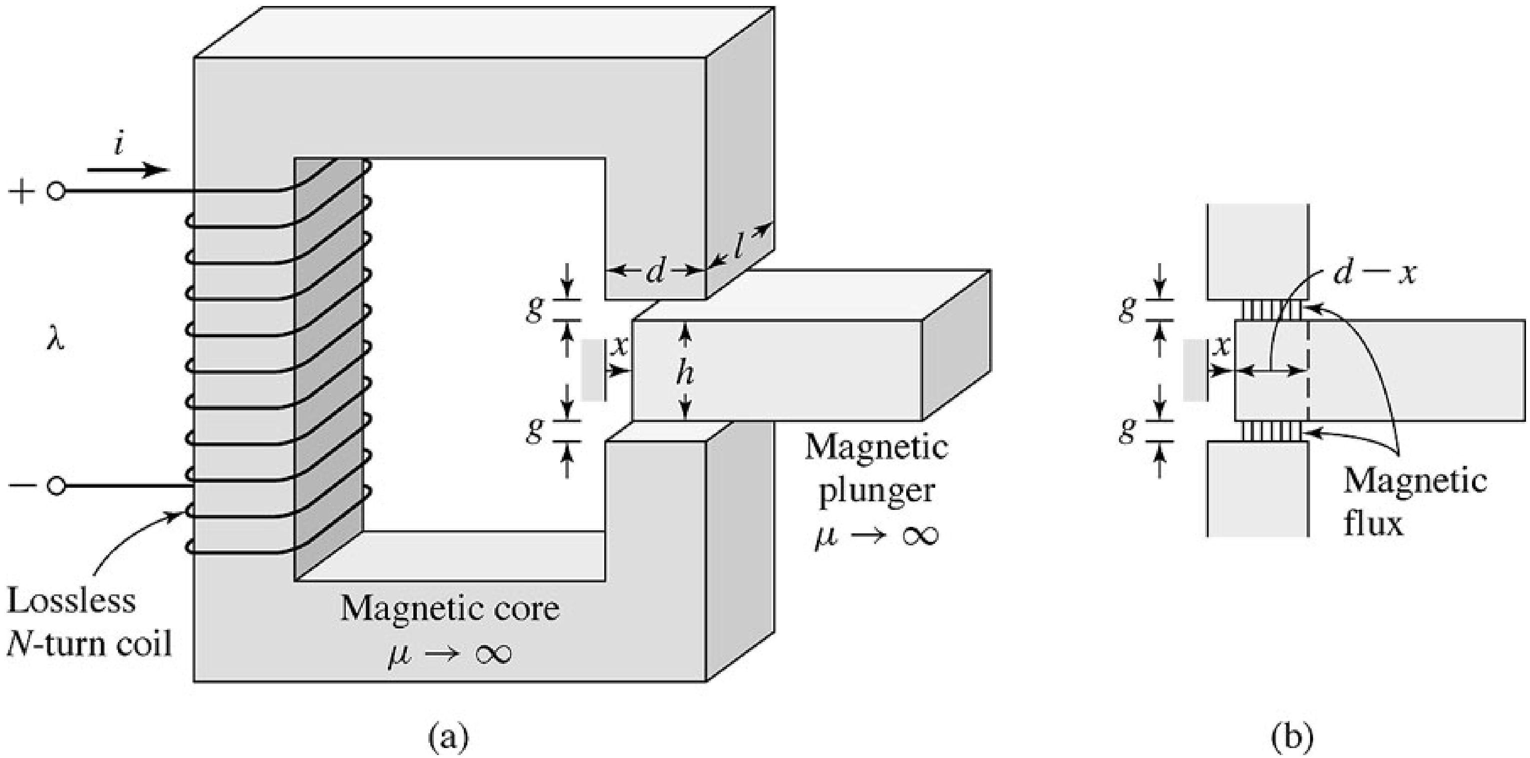
In the special case of a linear system, the flux linkage is proportional to current, or $\lambda \propto$ i.

$$W_{FLD}(\lambda, x) = \int_0^{\lambda} i(\lambda', x)\, d\lambda'$$

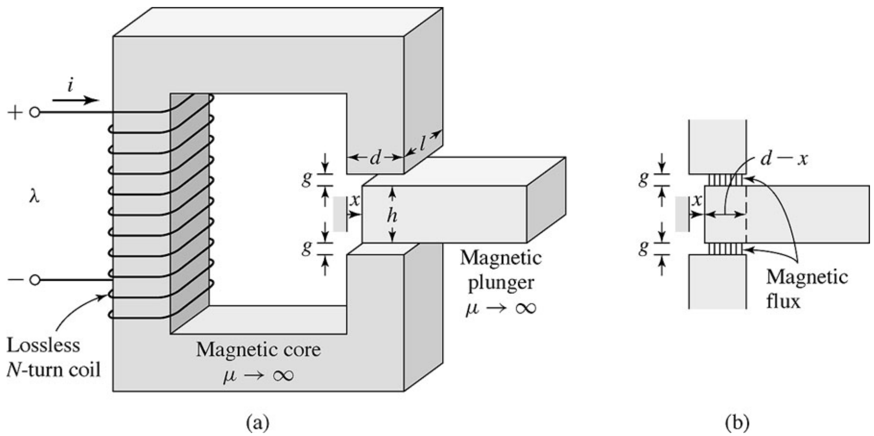
$$= \int_0^{\lambda} \frac{\lambda'}{L(x)}\, d\lambda' = \frac{1}{2}\frac{\lambda^2}{L(x)}$$

Example: Relay with Movable Plunger

- *Fitzgerald*, Example 3.2
- Find magnetic stored energy W_{FLD} as a function of x with I = 10A



Example: Relay with Movable Plunger



$$W_{FLD} = \frac{1}{2}\frac{\lambda^2}{L(x)}$$ with I constant.

We know that $\lambda = L(x)I$, so

$$W_{FLD} = \frac{1}{2}\frac{L^2(x)I^2}{L(x)} = \frac{1}{2}L(x)I^2$$

I'll give you that $L(x) = \frac{\mu_o N^2}{2g} ld(1 - \frac{x}{d})$,

so magnetic stored energy is:

$$W_{FLD} = \frac{\mu_o N^2}{4g} ld(1 - \frac{x}{d})I^2$$

Determining Magnetic Force from Stored Energy

- Next, if we go to all this trouble to find stored energy, let's figure out how to find forces from the energy (very important!)

Remember $dW_{FLD} = id\lambda - f_{fld}dx$

Next, remember the "total differential" from calculus:

$$dF(x_1, x_2) = \left.\frac{\partial F}{\partial x_1}\right|_{x_2 = const.} dx_1 + \left.\frac{\partial F}{\partial x_2}\right|_{x_1 = const.} dx_2$$

Let's rewrite the stored energy expression:

$$dW_{FLD} = \left.\frac{\partial W_{FLD}}{d\lambda}\right|_{x} d\lambda + \left.\frac{\partial W_{FLD}}{dx}\right|_{\lambda}$$

From this, we see that

$i = \left.\frac{\partial W_{FLD}}{d\lambda}\right|_{x}$ and $f_{fld} = -\left.\frac{\partial W_{FLD}}{\partial x}\right|_{\lambda}$

Determining Magnetic Force from Energy

For linear systems with $\lambda = L(x)I$

Energy $W_{FLD} = \dfrac{1}{2}\dfrac{\lambda^2}{L(x)}$

Force:

$$f_{fld} = -\frac{\partial}{\partial x}\left(\frac{1}{2}\frac{\lambda^2}{L(x)}\right)\bigg|_{\lambda=\text{constant}}$$

$$= \frac{\lambda^2}{2L^2(x)}\frac{dL(x)}{dx}$$

With $\lambda = L(x)I$

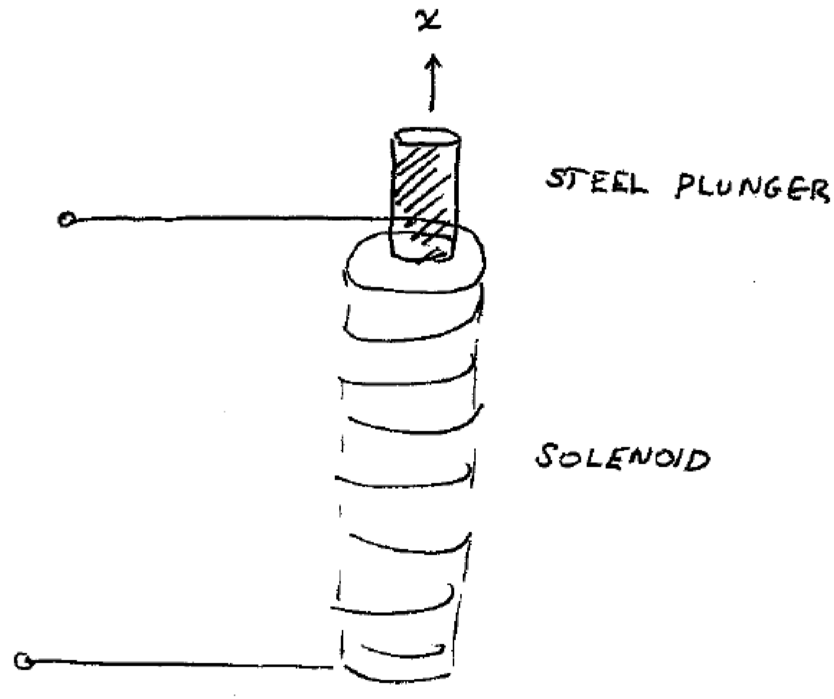
$$f_{fld} = \frac{I^2}{2}\frac{dL(x)}{dx}$$

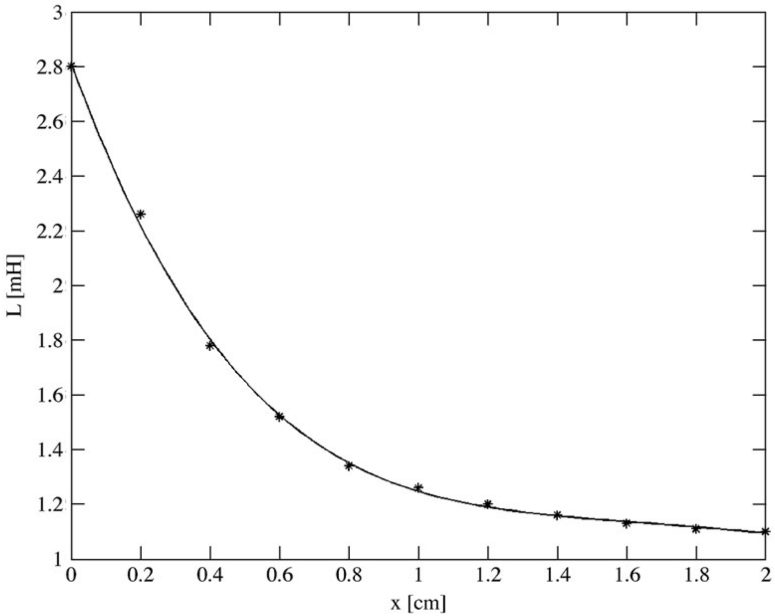
Determining Magnetic Force from Energy

- The bottom line: <u>if</u> your system is linear, and <u>if</u> you can calculate inductance as a function of position, then finding the force is pretty easy

Example: Curve Fit for Inductance of Solenoid with Plunger

- *Fitzgerald*, Example 3.3
- Assume that the following inductance vs. plunger position was measured. We then run the solenoid with 0.75A current

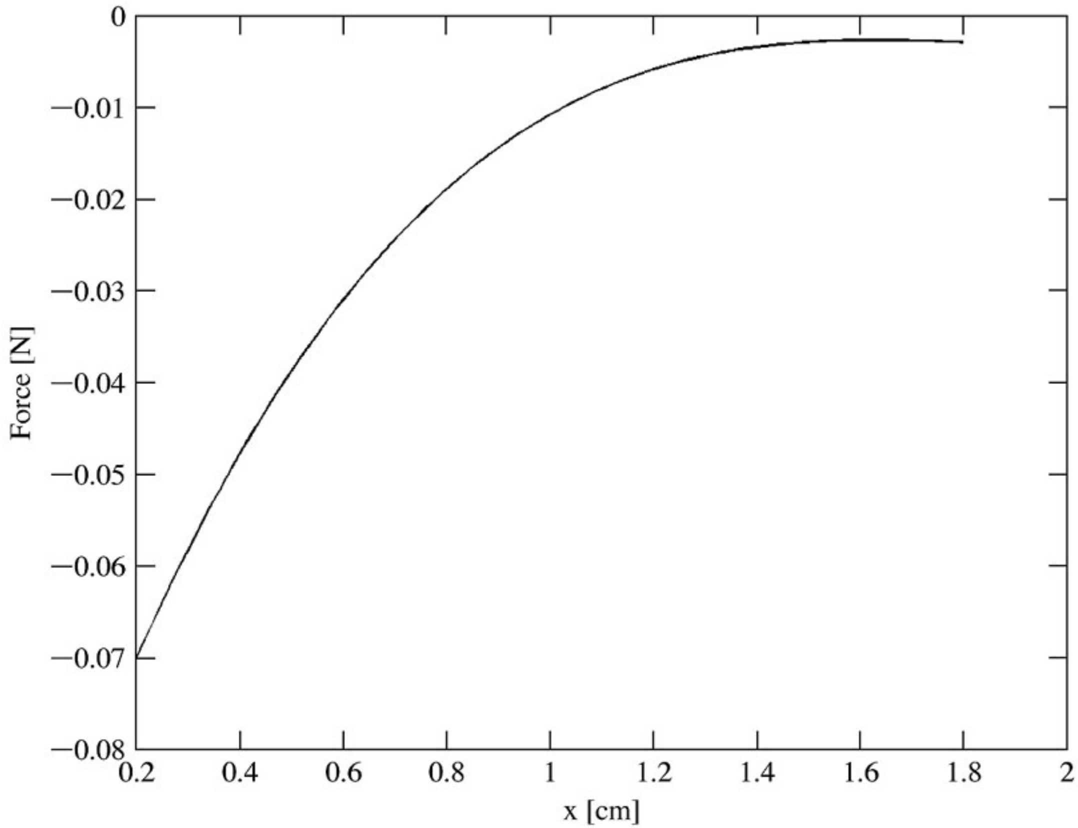




(a)

Example: Force as a Function of Position

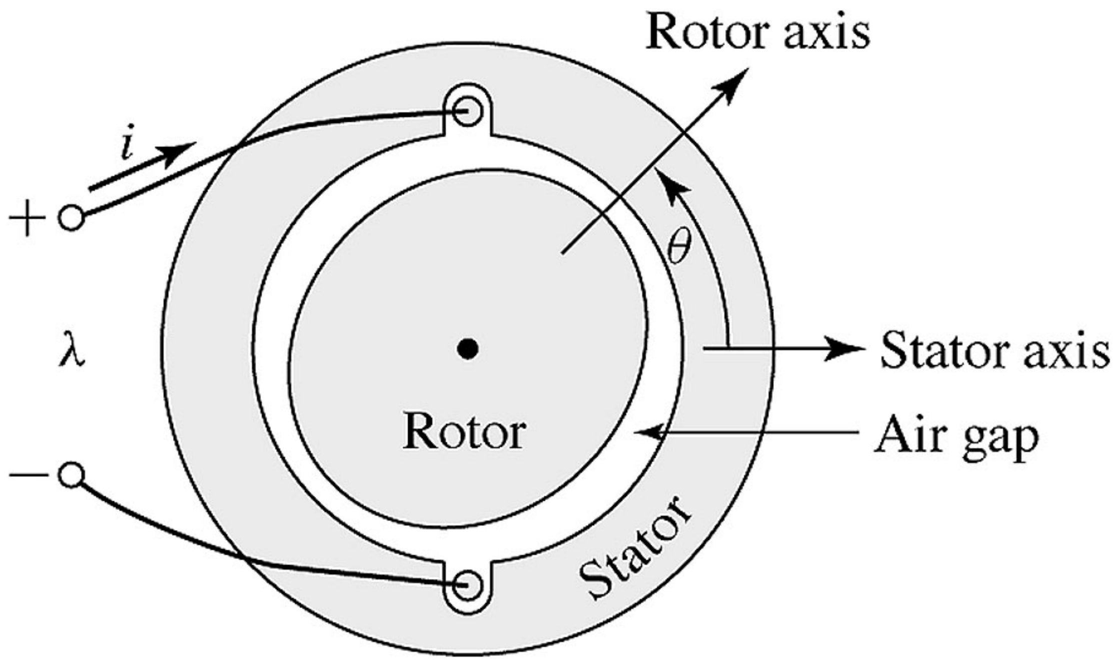
• We can fit a polynomial to the inductance, and use energy methods to find the plunger force as a function of position



(b)

Torque in Magnetic Circuit

- Can solve this as before, by analogy

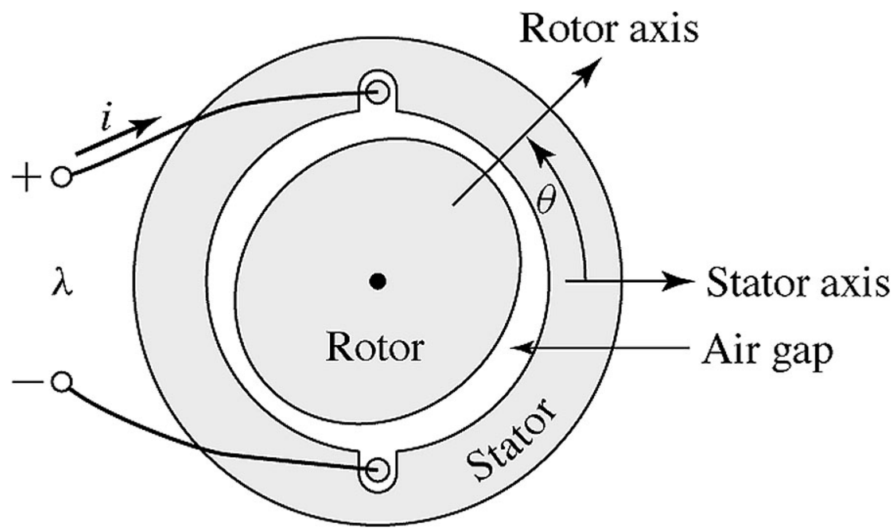


By analogy:

$$T_{fld} = \frac{I^2}{2} \frac{dL(\theta)}{d\theta}$$

Example: Finding Torque

- *Fitzgerald*, Example 3.4



Assume $L(\theta) = L_o + L_2\cos(2\theta)$ with $L_o = 10.6$ mH and $L_2 = 2.7$ mH. Find the torque with I = 2A.

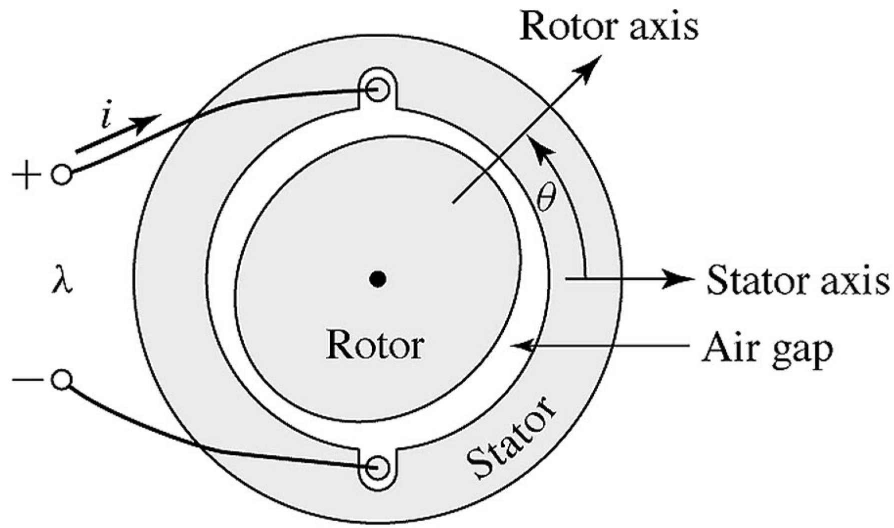
Solution:

$$T(\theta) = \frac{I^2}{2}\frac{dL(\theta)}{d\theta} = +\frac{I^2}{2}\left(-2L_2\sin(2\theta)\right)$$

$$= -1.08\times 10^{-2}\sin(2\theta)\ N-m$$

Example: Finding Torque

- *Fitzgerald*, Practice Problem 3.4

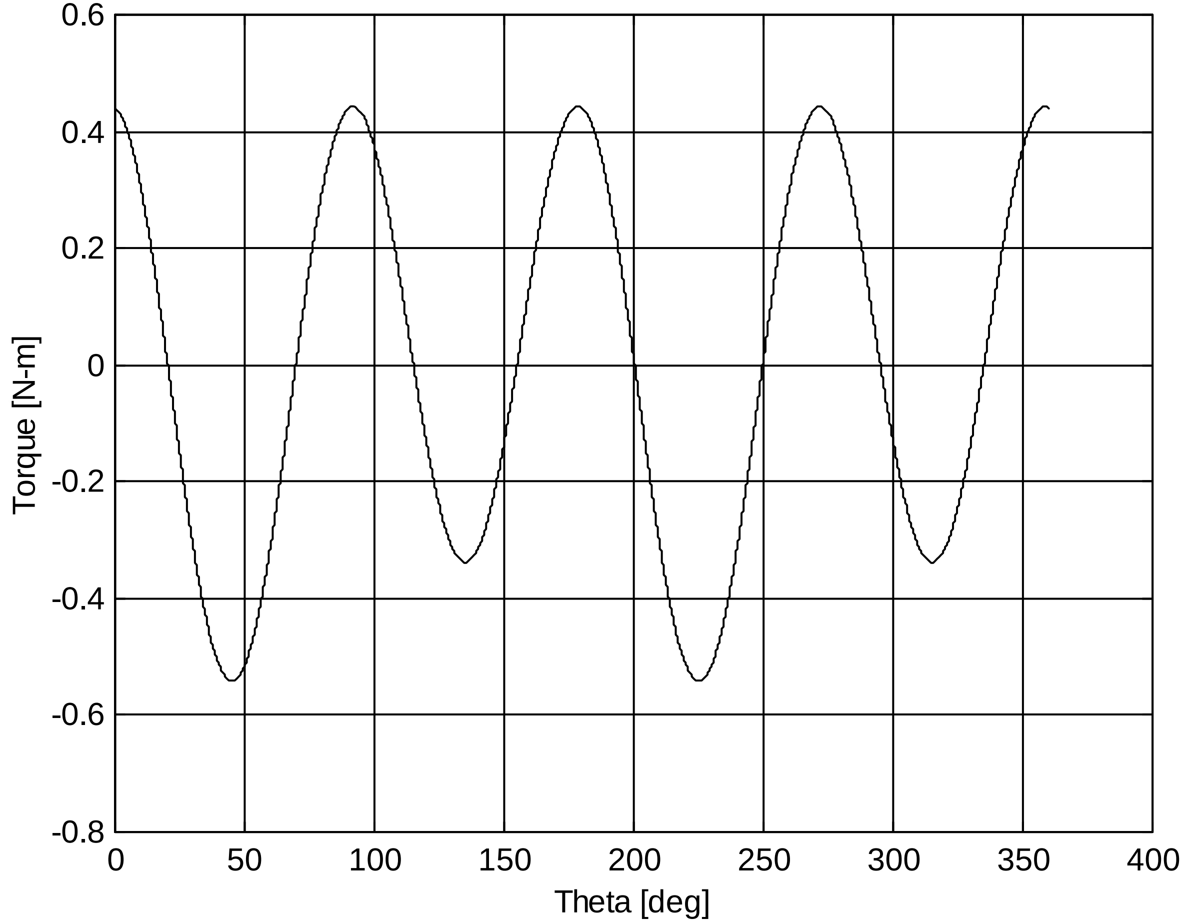


Assume $L(\theta) = L_o + L_2\cos(2\theta) + L_4\sin(4\theta)$ with $L_o = 25.4$ mH, $L_2 = 8.3$ mH and $L_4 = 1.8$ mH. Find the torque with I = 3.5A.

Solution:

$$T(\theta) = -0.1017\sin(2\theta) + 0.044\cos(4\theta)\ N-m$$

Another Example --- Torque vs. Rotor Angle



Today's Summary

- Today we've covered:
  - Maxwell's equations: Ampere's, Faraday's and Gauss' laws
  - Soft magnetic materials (steels, etc.)
  - Hard magnetic materials (permanent magnets)
  - Basic transformers
  - The per-unit system
  - We started electromechanical conversion basics