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Overview of Magnetics

• Review of Maxwell’s equations
• Ampere’s law, Gauss’ law, Faraday’s law
• Magnetic circuits
• Flux, flux linkage, inductance and energy
Review of Maxwell’s Equations

• First published by James Clerk Maxwell in 1864
• Maxwell’s equations couple electric fields to magnetic fields, and describe:
  – Magnetic fields
  – Electric fields
  – Wave propagation (through the wave equation)
• There are 4 Maxwell’s equations, but in magnetics we generally only need 3:
  – Ampere’s Law
  – Faraday’s Law
  – Gauss’ Magnetic Law
Review of Maxwell’s Equations

• We’ll review Maxwell’s equations in words, followed by a little bit of mathematics and some computer simulations showing the magnetic fields
Ampere’s Law

• Flowing current creates a magnetic field

\[ \oint \vec{H} \cdot d\vec{l} = \int \vec{J} \cdot d\vec{A} + \frac{d}{dt} \int \varepsilon_0 \vec{E} \cdot d\vec{A} \]

• In magnetic systems, generally there is high current and low voltage (and hence low electric field) and we can approximate for low \( d/dt \):

\[ \oint \vec{H} \cdot d\vec{l} \approx \int \vec{J} \cdot d\vec{A} \]

• In words: the magnetic flux density integrated around any closed contour equals the net current flowing through the surface bounded by the contour.

André-Marie Ampère
Finite-Element Analysis (FEA)

- Very useful tool for visualizing and solving shapes and magnitudes of magnetic fields
- FEA is often used to simulate and predict the performance of motors, etc.
- Following we’ll see some 2-dimensional (2D) FEA results to help explain Maxwell’s equations
Field From Current Loop, NI = 500 A-turns

- Coil radius R = 1"; plot from 2D finite-element analysis
Faraday’s Law

- A changing magnetic flux impinging on a conductor creates an electric field and hence a current (eddy current)

\[ \oint_C \vec{E} \cdot d\vec{l} = -\frac{d}{dt} \int_S \vec{B} \cdot d\vec{A} \]

- The electric field integrated around a closed contour equals the net time-varying magnetic flux density flowing through the surface bound by the contour

- In a conductor, this electric field creates a current by:

\[ \vec{J} = \sigma \vec{E} \]

- Induction motors, brakes, etc.
Circular Coil Above Conducting Aluminum Plate

- Flux density plots at DC and 60 Hz
- At 60 Hz, currents induced in plate via magnetic induction create lift force
Demonstration of Faraday’s Law: Electrodynam
Drag (NdFeB Magnet-in-Tube)

• Process:
  – Moving magnet creates changing magnetic field in
copper tube
  – Changing magnetic field creates induced voltage
  – Induced voltage creates current
  – By Lorentz force law, induced current and applied
  magnetic field create drag force
Gauss’ Magnetic Law

• Gauss' magnetic law says that the integral of the magnetic flux density over any closed surface is zero, or:

\[ \oint_S B \cdot dA = 0 \]

• This law implies that magnetic fields are due to electric currents and that magnetic charges (“monopoles”) do not exist.

• Note: similar form to KCL in circuits. (We’ll use this analogy later…)

\[ B_3 A_3 = B_1 A_1 + B_2 A_2 \]
Gauss’ Law --- Continuity of Flux Lines

\[ \phi_1 + \phi_2 + \phi_3 = 0 \]

**Figure 3-13** Continuity of flux.

Lorentz Force Law

• Experimentally derived rule:

\[ F = \int J \times B \, dV \]

• For a wire of length \( l \) carrying current \( I \) perpendicular to a magnetic flux density \( B \), this reduces to:

\[ F = IBl \]
Lorentz Force Law and the Right Hand Rule

\[ F = \int J \times B \, dv \]

Intuitive Thinking about Magnetics

- By Ampere’s Law, the current $J$ and the magnetic field $H$ are generally at right angles to one another.
- By Gauss’ law, magnetic field lines loop around on themselves.
  - No magnetic monopole.
- You can think of high-$\mu$ magnetic materials such as steel as an easy conduit for magnetic flux; i.e. the flux easily flows thru the high-$\mu$ material.
Magnetic Field (H) and Magnetic Flux Density (B)

- **H** is the magnetic field (A/m in SI units) and **B** is the magnetic flux density (Weber/m^2, or Tesla, in SI units)
- **B** and **H** are related by the magnetic permeability μ by \( B = \mu H \)
- Magnetic permeability μ has units of Henry/meter
- You can think of high-μ magnetic materials such as steel as an easy conduit for magnetic flux…. i.e. the flux easily flows thru the high-μ material
- In free space \( \mu_0 = 4\pi \times 10^{-7} \) H/m
- Note that **B** and **H** are vectors; they have both a magnitude and a direction
Right Hand Rule and Direction of Magnetic Field

Reference: http://sol.sci.uop.edu/~jfalward/magneticforcesfields/magneticforcesfields.html
Forces Between Current Loops

Reference: http://sol.sci.uop.edu/~jfalward/magneticforcesfields/magneticforcesfields.html
Inductor Without Airgap

- Magnetic flux is constrained to flow within steel

Constitutive relationships
In free space:
\[ B = \mu_o H \]

Magnetic permeability of free space \( \mu_o = 4\pi \times 10^{-7} \) Henry/meter.
In magnetic material, magnetic permeability is higher than \( \mu_o \):
\[ B = \mu H \]
Example: Flux in Inductor with Airgap

Ampere's law:
\[ \oint H \cdot dl = \int J \cdot dA \Rightarrow H_c l_c + H_g g = NI \]

Let's use constitutive relationships:
\[ \frac{B_c}{\mu_c} l_c + \frac{B_g}{\mu_o} g = NI \]
Example: Flux in Inductor with Airgap

\[ B_c = \frac{\Phi}{A_c} \]

\[ B_g = \frac{\Phi}{A_c} \]

Put this into previous expression:

\[ NI = \Phi \left( \frac{l_c}{\mu A_c} + \frac{g}{\mu_o A_g} \right) \]

Solve for flux:

\[ \Phi = \frac{NI}{\left( \frac{l_c}{\mu A_c} + \frac{g}{\mu_o A_g} \right)} \]

If \( g/\mu_o A_g \gg l_c/\mu A_c \),

\[ \Phi \approx \frac{NI}{g} \frac{A_c}{\mu_o A_g} \]
Magnetic Circuits

• Use Ohm’s law analogy to model magnetic circuits

\[ V \leftrightarrow NI \]
\[ I \leftrightarrow \Phi \]
\[ R \leftrightarrow \mathcal{R} \]

• Use magnetic “reluctance” instead of resistance

\[ R = \frac{l}{\sigma A} \leftrightarrow \mathcal{R} = \frac{l}{\mu A} \]

• This is a very powerful method to get approximate answers in magnetic circuits
Magnetic-Electric Circuit Analogy

- In an electric circuit, voltage $V$ forces current $I$ to flow through resistances $R$
- In a magnetic circuit, MMF $NI$ forces flux $\Phi$ to flow through reluctances $\mathcal{R}$

\[
I = \frac{V}{R_1 + R_2}
\]

\[
\phi = \frac{\mathcal{F}}{(\mathcal{R}_c + \mathcal{R}_g)}
\]
C-Core with Gap --- Using Magnetic Circuits

• Flux in the core is easily found by:

\[ \Phi = \frac{NI}{R_{\text{core}} + R_{\text{gap}}} = \frac{NI}{\frac{l_p}{\mu_c A_c} + \frac{g}{\mu_o A_c}} \]

• Now, note what happens if \( g/\mu_o \gg l_p/\mu_c \): The flux in the core is now approximately independent of the core permeability, as:

\[ \Phi \approx \frac{NI}{R_{\text{gap}}} \approx \frac{NI}{\frac{g}{\mu_o A_c}} \]

• Inductance:

\[ L = \frac{N \Phi}{I} \approx \frac{N^2}{R_{\text{gap}}} \approx \frac{N^2}{\frac{g}{\mu_o A_c}} \]
C-Core with Gap --- FEA
Fringing Fields in Airgap

- If fringing is negligible, $A_c = A_g$
Example: C-Core with Airgap

- **Fitzgerald**, Example 1.1; with $B_c = 1.0 \text{T}$, find reluctances, flux and coil current

\[
\begin{align*}
A_c &= A_g = 9 \text{ cm}^2 \\
g &= 0.05 \text{ cm} \\
l_c &= 30 \text{ cm} \\
N &= 500 \\
\mu_r &= 70,000
\end{align*}
\]

\[
\begin{align*}
\mathcal{R}_c &= \frac{l_c}{A_c \mu_c} = \frac{0.3}{(70,000)(4\pi \times 10^{-7})(9 \times 10^{-4})} = 3.8 \times 10^3 \frac{A - \text{turns}}{Wb} \\
\mathcal{R}_g &= \frac{g}{A_g \mu_0} = \frac{5 \times 10^{-4}}{(4\pi \times 10^{-7})(9 \times 10^{-4})} = 4.4 \times 10^5 \frac{A - \text{turns}}{Wb}
\end{align*}
\]

Note that $\mathcal{R}_g >> \mathcal{R}_c$
Example: C-Core with Airgap

Flux:

\[ \Phi = B_c A_c = (1.0)(9 \times 10^{-4}) = 9 \times 10^{-4} \text{ Weber} \]

Coil current:

\[ NI = \Phi (\mathcal{R}_c + \mathcal{R}_g) \]

\[ \Rightarrow I = \frac{\Phi (\mathcal{R}_c + \mathcal{R}_g)}{N} = \frac{(9 \times 10^4)(4.46 \times 10^5)}{500} = 0.8 \text{ A} \]
Example: C-Core with Airgap --- FEA

- NI = 400 A-turns
Example: C-Core with Airgap --- FEA

- $NI = 400$ A-turns, close up near the core
Example: C-Core with Airgap --- FEA Result

- Flux density in the core is approximately 1 Tesla
Example: C-Core with Airgap --- Gap Detail
Example: Simple Synchronous Machine

- *Fitzgerald*, Example 1.2
- Assuming $\mu \rightarrow \infty$, find airgap flux $\Phi$ and flux density $B_g$
  Assume $I = 10\, \text{A}$, $N = 1000$ turns, $g = 1\, \text{cm}$ and $A_g = 2000\, \text{cm}^2$
Aside: What Does Infinite Core $\mu$ Imply?

In core:
\[ \Phi_c = B_c A_C \]

In airgap:
\[ \Phi_g = B_g A_g \]

In core, $B_c$ is finite; this means that if $\mu \to \infty$, then $H_c \to 0$ for finite $B_c$. Also, infinite $\mu$ implies zero reluctance
Example: Simple Synchronous Machine

- Some initial thoughts, before doing any equations:
  - By symmetry, airgap flux and flux density are the same in both gaps
  - Since permeability is infinite, H inside steel is zero
Example: Simple Synchronous Machine

Reluctances:

\[ R_{g1} = R_{g2} = \frac{g}{\mu_o A_g} = \frac{(0.01)}{(4\pi \times 10^{-7})(2000)(0.01^2)} = 39789 \frac{A-turns}{Wb} \]

Flux:

\[ \Phi = \frac{NI}{R_{g1} + R_{g2}} = \frac{(1000)(10)}{(2)(39789)} = 0.126 \, Wb \]

Magnetic flux density:

\[ B = \frac{\Phi}{A_g} = \frac{0.126 \, Wb}{(2000)(0.01^2)} = 0.63 \, \frac{Wb}{m^2} = 0.63 \, T \]
Flux Linkage, Voltage and Inductance

- By Faraday’s law, changing magnetic flux density creates an electric field (and a voltage)

\[ \oint_C \vec{E} \cdot d\vec{l} = -\frac{d}{dt} \int_S \vec{B} \cdot d\vec{A} \]

- Induced voltage:

\[ v = N \frac{d\Phi}{dt} = \frac{d\lambda}{dt} \]

\[ \lambda = "flux\ linkage" = N\Phi \]

Inductance relates flux linkage to current

\[ L = \frac{\lambda}{I} \]
Finding Inductance Using Magnetic Circuits

- Let’s at first assume infinite core permeability; this means that the reluctance of the core is zero
- Note that inductance always scales as $N^2$ (why is that?)

\[
\Phi \approx \frac{NI}{g \mu_o A_g}
\]

\[
\lambda \approx N\Phi = \frac{\mu_o A_g N^2 I}{g}
\]

\[
L \approx \frac{\lambda}{I} = \frac{\mu_o A_g N^2}{g}
\]
Magnetic Circuit with Two Airgaps

Total flux: \( \Phi = \frac{NI}{R_1 \| R_2} \) (a)

Reluctances: \( R_1 = \frac{g_1}{\mu_o A_1} \) and \( R_2 = \frac{g_2}{\mu_o A_2} \)

Inductance:
\[
L = \frac{\lambda}{I} = \frac{N\Phi}{I} = N^2 \left( \frac{R_1 + R_2}{R_1 \| R_2} \right) = \mu_o N^2 \left( \frac{A_1}{g_1} + \frac{A_2}{g_2} \right)
\]
Magnetic Circuit with Two Airgaps (cont.)

Flux in leg#1: \( \Phi_1 = \frac{NI}{\mathcal{R}_1} = \frac{\mu_o A_1 NI}{g_1} \)

Flux in leg#2: \( \Phi_2 = \frac{NI}{\mathcal{R}_2} = \frac{\mu_o A_2 NI}{g_2} \)

Flux density in leg#1: \( B_1 = \frac{\Phi_1}{A_1} = \frac{\mu_o NI}{g_1} \)

Flux density in leg#2: \( B_2 = \frac{\Phi_2}{A_2} = \frac{\mu_o NI}{g_2} \)
Inductance vs. Relative Permeability

- What happens if we assume **finite** core permeability?
- Reluctance of the core is now finite as well

Reluctances: \( R_c = \frac{l_c}{\mu A_c} \) and \( R_g = \frac{g}{\mu_o A} \)

Let's assume that \( A_c = A_g = A \)

Flux: \( \Phi = \frac{NI}{R_c + R_g} \)

Flux linkage: \( \lambda = N\Phi = \frac{N^2 I}{R_c + R_g} \)

Inductance: \( N = \frac{\lambda}{I} = \frac{N^2}{R_c + R_g} = \frac{N^2}{g + \left(\frac{l_c \mu_o}{\mu}\right)} \)

Inductance is independent of core permeability if:

\( g \gg \frac{l_c \mu_o}{\mu} \)
Inductance vs. Relative Permeability
Example: Effects of Finite Permeability

- *Fitzgerald*, problem 1.5

Cross-sectional area $A_c = 1.8 \times 10^{-3}$ m$^2$
Mean core length $l_c = 0.6$ m
Gap length $g = 2.3 \times 10^{-3}$ m
$N = 83$ turns

1.5 The magnetic circuit of Problem 1.1 has a nonlinear core material whose permeability as a function of $B_m$ is given by

$$
\mu = \mu_0 \left( 1 + \frac{3499}{\sqrt{1 + 0.047(B_m)^{7.8}}} \right)
$$

where $B_m$ is the material flux density.

a. Using MATLAB, plot a dc magnetization curve for this material ($B_m$ vs. $H_m$) over the range $0 \leq B_m \leq 2.2$ T.

b. Find the current required to achieve a flux density of 2.2 T in the core.

c. Again, using MATLAB, plot the coil flux linkages as a function of coil current as the current is varied from 0 to the value found in part (b).
Example: Effects of Finite Permeability

- Relative $\mu$ vs. $B$
Example: Effects of Finite Permeability

- B/H curve
Example: Effects of Finite Permeability

- **Current calculation**

Cross-sectional area \( A_c = 1.8 \times 10^{-3} \, \text{m}^2 \)
Mean core length \( l_c = 0.6 \, \text{m} \)
Gap length \( g = 2.3 \times 10^{-3} \, \text{m} \)
\( N = 83 \) turns

From Ampere’s law:

\[
H_c l_c + H_g g = NI
\]

In core:

\[
H_c = \frac{B_c}{\mu}
\]

In gap (let’s assume \( B_c = B_g = B \)):

\[
H_g = \frac{B}{\mu_o}
\]

Put this back into Ampere’s law:

\[
\frac{Bl_c}{\mu} + \frac{Bg}{\mu_o} = NI
\]

\[
\therefore I = \left( \frac{B}{\mu_o N} \right) \left( \frac{l_c}{\mu_r} + g \right) = 65.8 \, \text{A}
\]
Example: Effects of Finite Permeability

- Coil flux linkage $\lambda$ as a function of coil current
- Note that at low current, $\lambda$-I curve is linear, indicating constant inductance
Example: MATLAB Script

```matlab
% Fitzgerald, problem 1.5
% Constants
mu0=4*pi*1e-7;

% Part a
Bm=0:0.01:2.2;
mu=mu0*(1+3499./(sqrt(1+0.047*Bm.^7.8)));
plot(Bm, mu/mu0, 'k');
xlabel('Bm, Tesla')
ylabel('Relative mu')
grid
title('Relative mu vs. B, problem 1.5')
mu_rel=mu(length(mu))/mu0

figure
Hm=Bm./mu;
plot(Hm, Bm, 'k');
xlabel('H [A/m]')
ylabel('B [T]')
grid
title('Bm vs. Hm, problem 1.5')

% Part b
B=2.2;
N=83;
lc=0.6;
g=2.3e-3;
I=(B/(mu0*N))*(lc/mu_rel + g)

% Part c
Ac=1.8e-3;
figure
I=(Bm/(mu0*N)).*(lc./mu/mu0+g);
flux_linkage=N*Bm*Ac;
plot(I, flux_linkage);
xlabel('I [A]')
ylabel('Flux linkage [Wb]')
title('Coil flux linkage as a function of I, problem 1.5')
grid
```
Inductance and Energy

• Magnetic stored energy (in Joules) is:

\[ W = \frac{1}{2} LI^2 \]

• This is a good thing to remember
Magnetic Circuit with Two Windings

- Note that flux is the sum of flux due to $i_1$ and that due to $i_2$.
Magnetic Circuit with Two Windings

- Note that by the right-hand rule the flux due to $i_1$ and $i_2$ are additive given the current directions shown.
Magnetic Circuit with Two Windings

- Note that by the right-hand rule the flux due to $i_1$ and $i_2$ are additive given the current directions shown

Flux: $\Phi = (N_1 i_1 + N_2 i_2) \left( \frac{\mu_o A_c}{g} \right)$

Flux linkage for coil #1:

$\lambda_1 = N_1 \Phi = N_1^2 i_1 \left( \frac{\mu_o A_c}{g} \right) + N_1 N_2 i_2 \left( \frac{\mu_o A_c}{g} \right)$

We can write this as:

$\lambda_1 = L_{11} i_1 + L_{12} i_2$

$L_{11}$ is “self inductance” of coil #1
$L_{12}$ is “mutual inductance” between coils #1 and #2
Magnetic Circuit with Two Windings

Flux linkage for coil #2:
\[
\lambda_2 = N_2 \Phi = N_1 N_2 i_1 \left( \frac{\mu_o A_c}{g} \right) + N_2^2 i_2 \left( \frac{\mu_o A_c}{g} \right)
\]

Or rewriting:
\[
\lambda_2 = L_{21} i_1 + L_{22} i_2
\]

Or, in matrix form:
\[
\begin{bmatrix}
\lambda_1 \\
\lambda_2
\end{bmatrix} =
\begin{bmatrix}
L_{11} & L_{12} \\
L_{21} & L_{22}
\end{bmatrix}
\begin{bmatrix}
i_1 \\
i_2
\end{bmatrix}
\]
“Soft” Magnetic Materials

- Materials with a small B/H curve, such as steels, etc.
- Much of the previous analysis assumed that steel had infinite permeability ($\mu \to \infty$) or that permeability was constant and large.
- However, soft magnetic materials exhibit both saturation and losses
B-H Curve and Saturation

- Definition of magnetic permeability: slope of B-H curve

\[ \mu = \frac{\Delta B}{\Delta H} = \frac{B}{H} \]

**Figure 3-12** Relation between B- and H-fields.

BH Curve for M-5 Steel

- Note horizontal scale is logarithmic
Hysteresis Loop

- Real-world magnetic materials have a “hysteresis loop”
- Hysteresis loss is proportional to shaded area
B-H Loop for M-5 Grain-Oriented Steel

- Only the top half of the loops shown for steel 0.012” thick
BH Curves for Various Soft Magnetic Materials

FIGURE 1.27 The B-H curves of various soft magnetic materials.

Relationship Between Voltage, Flux and Current
Exciting RMS VA per kg at 60 Hz
Hysteresis Loop Size Increases with Frequency

- Hysteresis loss increases as frequency increases

Total Core Loss

- Total core loss is the sum of:
  - Hysteresis loss
  - Eddy current losses
- Eddy current losses are due to induced currents (via Faraday’s law)
- Eddy current losses are minimized by laminating magnetic cores
Laminated Core

- Cores made from conductive magnetic materials must be made of many thin laminations. Lamination thickness < skin depth.
Eddy Current Loss in Lamination
Total Core Loss

- M-5 steel at 60 Hz

![Graph showing the relationship between $B_{\text{max}}$, Wb/m$^2$, and $P_c$, W/kg]
Total Core Loss vs. Frequency and $B_{max}$

- Core loss depends on peak flux density and excitation frequency
- This is the curve for a high frequency core material

Process to Find Core Loss

- Find maximum B
- From this B and switching frequency, find core loss per kg
- Total loss is power density $\times$ mass of core
Example: C-Core with Airgap --- Current

- *Fitzgerald*, Example 1.7; find the current necessary to produce $B_c = 1 \text{T}$

\[ A_c = A_g = 9 \text{ cm}^2 \]
\[ g = 0.05 \text{ cm} \]
\[ l_c = 30 \text{ cm} \]
\[ N = 500 \]
\[ \mu_c = 70,000 \]
**Example: C-Core with Airgap --- Solution**

The value of \( H_c \) needed for \( B_c = 1 \) Tesla is read from the chart:

\[
H_c = 11 \text{ A-turns/meter}
\]

The MMF drop in the core is:

\[
H_c l_c = (11)(0.3) = 33 \text{ A-turns}
\]

The MMF drop in the airgap is:

\[
H_g g = \frac{B_g g}{\mu_o} = \frac{(1.0)(5 \times 10^{-4})}{4\pi \times 10^{-7}} = 396 \text{ A-turns}
\]

The winding current is:

\[
I = \frac{\sum\text{MMF}}{N} = \frac{33 + 396}{500} = 0.8A
\]
Example: Inductor

- *Fitzgerald*, Example 1.8

Material: M-5 steel

f = 60 Hz

N = 200

\( B_c = 1.5 \sin \omega t \) Tesla

Steel is 94% of cross section

Density of steel = 7.65 g/cm³

Find:

(a) Applied voltage

(b) Peak current

(c) RMS current

(d) Core loss
Example: Excitation Voltage

From Faraday’s law:

\[ e = \frac{d\lambda}{dt} = N \frac{d\Phi}{dt} = NA_c \frac{dB_c}{dt} \]

\[ A_c = 2in \times 2in \times 0.94 = 3.76in^2 = 2.4 \times 10^{-3} \, m^2 \]

\[ \frac{dB_c}{dt} = (1.5\omega)\cos(\omega t) = 565\cos(\omega t) \]

\[ e = (200)(2.4 \times 10^{-3})(565\cos(\omega t)) = 274\cos(\omega t) \]
Example: Peak Current in Winding

From Figure 1.10, $B = 1.5T$ requires $H = 36$ A-turns/m

From Ampere’s law, $Hl_c = NI$

$l_c = 2 \times (8” + 6”) = 28” = 0.71m$

$I = \frac{Hl_c}{N} = \frac{(36)(0.71)}{200} = 0.13A$
Example: RMS Winding Current

From Figure 1.12, at $B_{\text{max}} = 1.5\, \text{T}$, $P_a = 1.5\, \text{VA/kg}$

Core volume:

$$V_c = (4in^2)(0.94)(28in) = 105.5\, \text{in}^3 = 1.7 \times 10^{-3}\, \text{m}^3$$

Core mass:

$$M = V_c \rho_c = (1.7 \times 10^{-3}\, \text{m}^3)(7650\, \frac{\text{kg}}{\text{m}^3}) = 13.2\, \text{kg}$$

Core Volt-

Amperes: $P_c = 1.5\, \frac{\text{VA}}{\text{kg}} \times 13.2\, \text{kg} = 19.8\, \text{VA}$

Current: $I_{\text{RMS}} = \frac{VA}{E_{\text{RMS}}} = \frac{19.7}{\left(\frac{274}{\sqrt{2}}\right)} = 0.10\, \text{A}$
Example: Core Loss

From Figure 1.14, core loss density = 1.5 W/kg at \( B_{\text{max}} = 1.5 \) Tesla. Total core loss is:

\[
P_c = M \times 1.5 \frac{W}{kg} = (13.2)(1.5) = 20W
\]
Permanent Magnets

- “Soft” magnetic materials such as magnetic steel can behave as very weak permanent magnets
- Permanent magnets, or “hard” magnetic materials, have a high coercive force $H_c$ and can produce significant flux in an airgap; they also have a “wide” hysteresis loop

Brief History of Permanent Magnets

• c. 1000 BC: Chinese compasses using lodestone
  – Later used to cross the Gobi desert

Brief History of Permanent Magnets

Brief History of Permanent Magnets (cont.)

• c. 200 BC: Lodestone (magnetite) known to the Greeks
  – Touching iron needles to magnetite magnetized them
• 1200 AD: French troubadour de Provins describes use of a primitive compass to magnetize needles
• 1600: William Gilbert publishes first journal article on permanent magnets
• 1819: Oersted reports that an electric current moves compass needle

References:
Brief History of Permanent Magnets (cont.)

- c. 1825: Sturgeon invents the electromagnet, resulting in a way to artificially magnetize materials
- 7-ounce magnet was able to lift 9 pounds

References:
2. Britannica Online
   Electromechanics
   Magnetics and Energy Conversion
Brief History of Permanent Magnets (cont.)

• c. 1830: Joseph Henry (U.S.) constructs electromagnets

Photo reference: Smithsonian Institute archives
Brief History of Permanent Magnets (cont.)

- 1917: Cobalt magnet steels developed by Honda and Takagi in Japan
- 1940: Alnico --- first “modern” material still in common use
  – Good for high temperatures
- 1960: SmCo (samarium cobalt) rare earth magnets
  – Good thermal stability
- 1983: GE and Sumitomo develop neodymium iron boron (NdFeB) rare earth magnet
  – Highest energy product, but limited temperature range

References:
Magnetizing Permanent Magnets

- Material is placed inside magnetizing fixture
- Magnetizing coil is energized with a current producing sufficient field to magnetize the PM material

Pictorial View of Magnetization Process


Figure 3.6  Pictorial explanation of magnetization curve in a ferromagnetic bar.
Permanent Magnets

- External effects of permanent magnets can be modeled as surface current

Figure 1.4. Uniformly magnetized magnet (a), which may be modeled by a current density over its boundary (b).

Permanent Magnets

- After magnetization, $\mathbf{M}$ has values of either $+M_{sat}$ or $-M_{sat}$

Figure 1.10. Intrinsic magnetization characteristic for an elemental volume of a magnet.

Permanent Magnets

- By a constitutive relationship, $B = \mu_0(H+M)$
- Since $M$ has values of either $+M_{sat}$ or $-M_{sat}$, it follows that the slope of the BH curve for the permanent magnet is $\mu_0$

Demagnetization Curves of Ceramic 8

• Typical sintered ceramic magnet

Demagnetization Curves for NdFeB

- Strong neodymium-iron-boron

Permanent Magnets vs. Steel

- Note that PM has much higher coercive force

**Permanent magnet: Alnico 5**

**M-5 steel**
Lines of Force

- Iron filings follow magnetic field lines
Cast Alnico

Example: PM in Magnetic Circuits

- *Fitzgerald*, Example 1.9

\[
g = 0.2 \text{ cm} \\
l_m = 1.0 \text{ cm} \\
A_m = A_g = 4 \text{ cm}^2
\]

Find flux in airgap $B_g$ for magnetic materials

(a) Alnico 5

(b) M-5 steel
Example: Solution with Alnico PM

NI = 0, so by Ampere’s law:

\[ H_m l_m + H_g g = 0 \]

Solve for \( H_g \):

\[ H_g = -H_m \left( \frac{l_m}{g} \right) \]

Continuity of flux (Gauss’ law):

\[ A_g B_g = A_m l_m \Rightarrow B_g = B_m \left( \frac{A_m}{A_g} \right) \]

Next, solve for \( B_m \) as a function of \( H_m \):

\[ B_m = B_g \left( \frac{A_g}{A_m} \right) = \mu_0 H_g \left( \frac{A_g}{A_m} \right) \]

\[ \Rightarrow B_m = \mu_0 \left( -H_m \frac{l_m}{g} \right) \left( \frac{A_g}{A_m} \right) = -6.28 \times 10^{-6} H_m \]

Plot this load line on Alnico BH curve
Example: Solution with Alnico

- Result: $B_g = 0.3$ Tesla

Example: Load Line Solution with M-5 Steel

- Use same load line; $B_g = 0.38$ Gauss (much lower than with Alnico)
- Note: Earth’s magnetic field $\sim 0.5$ Gauss
Some Common Permanent Magnet Materials

- Other tradeoffs not shown here include: mechanical strength, temperature effects, etc.
Typical NdFeB B-H Curve

- Neodymium-iron-boron (NdFeB) is the highest strength permanent magnet material in common use today.
- Good material for applications with temperature less than approximately 80 - 150°C.
- Cost per pound has reduced greatly in the past few years.
- B/H curve below for “grade 35” or 35 MGOe material.

![B-H Curve Diagram]

- \( B_m \), Tesla
- \( B_r \)
- \( (BH)_{max} \)
- \( H_c \), kA/m
- \( H_m \), kA/m
NdFeB B-H Curves for Different Grades

Maximum Energy Product

- BH has units of Joules per unit volume

![Diagram](a)
Maximum Energy Product

Why is maximum energy product important?

(1) \[ B_g = B_m \left( \frac{A_m}{A_g} \right) \]

(2) \[ \frac{H_m l_m}{H_g g} = -1 \]

Let’s find \( B_g^2 \)

\[ B_g^2 = \left( B_m \frac{A_m}{A_g} \right) \times \left( \mu_0 H_g \right) \]

\[ = - \left( B_m \frac{A_m}{A_g} \right) \left( \mu_0 \frac{H_m l_m}{g} \right) \]

\[ = \mu_0 \left( \frac{Vol_{mag}}{Vol_{gap}} \right) \left( - B_m H_m \right) \]

Solve for magnet volume \( Vol_{mag} \)

\[ Vol_{mag} = B_g^2 \left( \frac{Vol_{gap}}{-B_m H_m} \right) \]

To use minimum volume of magnet for a given \( B_g \), operate magnet at \((BH)_{max}\) point
Progress in PM Specs

- One figure of merit is $(BH)_{\text{max}}$ product

Progress in PM Specs


Electromechanics Magnetics and Energy Conversion 2-104
Applications for Permanent Magnets

- Disk drives
- Speakers
- Motors
  - Rotary motors (Toyota Prius)
  - Linear motors (Maglev, people moving)
- Refrigerator magnets
- Proximity sensors and switches
- Compasses
- Magnetic bearings and magnetic suspensions (Maglev)
- Water filtration
- Plasma fusion research, NMR
- Eddy current brakes (ECBs)
- Etc.

References:
Example: Maximum Energy Product

- *Fitzgerald*, Example 1.10: Find magnet dimensions for desired airgap flux density $B_g = 0.8$ Tesla

At maximum (BH), $B_m=1.0$ T and $H_m = -40$ kA/m

$$A_m = A_g \left( \frac{B_g}{B_m} \right) = (2\text{cm}^2) \left( \frac{0.8}{1.0} \right) = 1.6\text{cm}^2$$

$$l_m = -g \left( \frac{H_g}{H_m} \right) = -g \left( \frac{B_g}{\mu_0 H_m} \right) = (-0.2\text{cm}) \left( \frac{0.8}{(4\pi \times 10^{-7})(-40000)} \right) = 3.18\text{cm}$$

So, magnet is 3.18 cm long and 1.6 cm$^2$ in area
Open-Circuited Permanent Magnet

Open Circuited Permanent Magnet --- FEA
Short-Circuited Permanent Magnet

- Find $B$ inside core, ignoring any leakage and assuming infinite permeability in core
Short Circuited Permanent Magnet

- For infinite permeability, load line is vertical
- Intersection of load lines occurs at $B \approx B_r$

Short Circuited Permanent Magnet --- FEA
Circuit Modeling of Permanent Magnets

Circuit Modeling of Permanent Magnets

Let's use this in a magnetic circuit with airgap:

\[
\Phi
\]

\[
l_m
\]

\[
\text{AMPERE'S LAW}
\]

\[
4 \quad H_m l_m + H_g g = 0
\]
Circuit Modeling of Permanent Magnets

\[ \begin{align*}
\text{put } (3) \text{ into (4)} & \\
\left( \frac{B_m}{\mu_m} - H_c \right) l_m + H_g g &= 0 \\
\Rightarrow & \\
5. & \quad H_g g + \frac{B_m}{\mu_m} l_m = H_c l_m \\
\text{or} & \\
\frac{B_g g}{\mu_0} + \frac{B_m l_m}{\mu_0} &= H_c l_m \\
\text{Total flux: } & \\
\Phi &= B_g A_g + B_m A_m \\
\text{so, (5) reduces to: } & \\
\Phi \left( R_g + R_m \right) &= H_c l_m
\end{align*} \]
Circuit Modeling of Permanent Magnets

WE CAN MODEL THIS AS
FOLLOWS:

PM MODEL:
Example: Circuit Modeling of Permanent Magnets

- **Example:**
  - Grade 37 NdFeB
  - \( H_c = 950,000 \text{ A/m} \)
  - \( \mu_m = 1.048 \mu_0 \)
  - \( l_m = 2 \text{ cm} \)
  - \( g = 1 \text{ cm} \)
  - \( A_m = A_g = 25 \text{ cm}^2 \)

Find \( B_g \)
Example: Circuit Modeling of Permanent Magnets

This example is particularly simple because $A_m = A_g$

Solution:

$$H_c l_m = (980,000)(0.02) = 19,000$$

$$R_m = \frac{l_m}{M_A l_m} = \frac{0.02}{(1.048)(\pi)(10^{-7})(25)(10^{-4})}$$

$$= 6.07 \times 10^6$$

$$R_g = \frac{q}{M_o A_g} = \frac{0.01}{(\pi)(10^{-7})(25)(10^{-4})}$$

$$= 3.2 \times 10^6$$

$$\Phi = \frac{H_c l_m}{R_m + R_g} = 2.04 \times 10^{-3} \text{ Wb}$$

$$B_g = \frac{\Phi}{A_g} = \frac{2.04 \times 10^{-3}}{25 \times 10^{-4}}$$

$$\therefore B_g = 0.082 \text{ Wb/m}^2$$

$$0.82 \text{ Tesla}$$
Example: Circuit Modeling of Permanent Magnets---FEA
Example: Magnetic Circuit With Steel

- Estimate airgap field $B_g$ assuming grade 37 NdFeB
- Airgap $g$, magnet thickness $t_m$

Assume no saturation in steel
Example: Magnetic Circuit With Steel

- This analysis also ignores leakage

\[ 2H_m \tau_m + H_g g = 0 \quad \text{(Ampere's Law)} \]

\[ H_g = \frac{\mu g}{\mu_0} = \frac{B_m}{\mu_0} \quad \text{(constitutive relation)} \]

\[ 2H_m \tau_m + \frac{B_m g}{\mu_0} = 0 \]

\[ \frac{B_m g}{\mu_0} = -2H_m \tau_m \]

\[ \frac{B_m}{H_m} = -\frac{2\tau_m \mu_0}{g} \]

Assume no saturation in steel
Example: Magnetic Circuit With Steel --- Operating Point vs. Magnet Thickness $t_m$

\[ t_m = \frac{q}{2} \rightarrow B_g \sim 0.65 \text{T} \]
\[ t_m = q \rightarrow B_g \sim 0.85 \text{T} \]
\[ t_m = 2q \rightarrow B_g \sim 1.05 \text{T} \]
Example: FEA with Magnet Thickness $t_m = g/2$
Example: FEA with Magnet Thickness $t_m = g$
Example: FEA with Magnet Thickness $t_m = 2g$
Example: Comparison of Different Magnet Thicknesses

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<td>$tm = 2g$</td>
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PM and a Winding

- Many motors have permanent magnets, steel and windings

Analysis of PM in closed core, with excitation:

\[ H_m l_m = NI \]

\[ \therefore H_m = \frac{NI}{l_m} \]
PM and a Winding --- Load Line

- Note that demagnetization can occur if current is sufficiently high.

![Diagram of PM and a Winding --- Load Line](image-url)
Another Example --- Excitation and Airgap

(1) Ampere’s law:
\[ H_m l_m + H_g g = NI \]

(2) Constitutive law: \( B_g = \mu_o H_g \)

(3) Gauss’ law: \( B_m A_m = B_g A_g \)

Solve for \( B_m - H_m \) load line:

\[
B_m = -\mu_o \left( \frac{A_m l_m}{A_m g} \right) \left( H_m - \frac{NI}{l_m} \right)
\]
Another Example --- Excitation and Airgap --- Load Line
Interesting Calculation Tool

Reference: www.magnetsales.com
### NdFeB Rounds

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Reference: www.dextermag.com

Electromechanics

Magnetics and Energy Conversion
What Can You Buy?

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Reference: www.dextermag.com
Electromechanics Magnetics and Energy Conversion
## What Can You Buy?

Reference: www.magnetsales.com

### Rectangular Magnets - Finished Sizes sorted in ascending order of Length, Width, and then Thickness.

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<td>0.500</td>
<td>Die Pressed, Machined, Nickel Plated</td>
</tr>
<tr>
<td>35NE805820</td>
<td>35</td>
<td>1.500</td>
<td>0.305</td>
<td>0.305</td>
<td>Die Pressed, Machined</td>
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<tr>
<td>35NE881107</td>
<td>35</td>
<td>2.000</td>
<td>0.170</td>
<td>0.170</td>
<td>Die Pressed, Machined</td>
</tr>
<tr>
<td>35NE8812832</td>
<td>35</td>
<td>2.000</td>
<td>2.000</td>
<td>0.500</td>
<td>Die Pressed, Machined</td>
</tr>
<tr>
<td>35NE8812832</td>
<td>35</td>
<td>2.000</td>
<td>2.000</td>
<td>0.500</td>
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<tr>
<td>35NE8812864</td>
<td>35</td>
<td>2.000</td>
<td>2.000</td>
<td>1.000</td>
<td>Die Pressed, Machined</td>
</tr>
</tbody>
</table>

The T* dimension represents the orientation direction.

Tolerances on *machined* blocks are the greater of ±1.5% of the dimension or ±0.015" on cross sectional dimensions, and ±0.005" on the orientation direction.
Magnetization Patterns

Different Magnetizing Patterns Give Different Results

Except for the regular flexible and Ceramic 1 materials, all magnet materials are “pre-oriented” and can only be magnetized in a particular direction.

Standard (or “conventional”) magnetization is straight through the orientation direction, and produces one North pole and one South pole. The Rare Earth magnets are extremely difficult to magnetize in non-standard ways. However, the Flexible and Ceramic types can be magnetized in many non-standard ways to give special results.

Reference: www.magnetsales.com
Comparison

Reference: www.magnetsales.com
Comparison of Maximum Operating Temperatures

Reference:  http://www.electronenergy.com/media/Magnetics%202005.pdf
Comparison of Maximum Operating Temperatures

Reference: http://www.electronenergy.com/media/Magnetics%202005.pdf
# Magnet Comparisons

<table>
<thead>
<tr>
<th>Material</th>
<th>Approx. Maximum Operating Temperatures</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>°C</td>
</tr>
<tr>
<td>NdFeB</td>
<td>140</td>
</tr>
<tr>
<td>SmCo</td>
<td>300</td>
</tr>
<tr>
<td>Ferrite</td>
<td>300</td>
</tr>
<tr>
<td>Alnico</td>
<td>540</td>
</tr>
<tr>
<td>Flexible</td>
<td>100</td>
</tr>
</tbody>
</table>

Reference: [www.magnetsales.com](http://www.magnetsales.com)

<table>
<thead>
<tr>
<th>Material</th>
<th>BHmax</th>
<th>Relative Cost ($ / pound)</th>
<th>Relative Cost ($ / BHmax)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flexible</td>
<td>1</td>
<td>$1.00</td>
<td>$0.60</td>
</tr>
<tr>
<td>Ceramic</td>
<td>3</td>
<td>$2.00</td>
<td>$0.50</td>
</tr>
<tr>
<td>Alnico</td>
<td>5</td>
<td>$20.00</td>
<td>$4.30</td>
</tr>
<tr>
<td>SmCo</td>
<td>20</td>
<td>$100.00</td>
<td>$6.00</td>
</tr>
<tr>
<td>NdFeB</td>
<td>40</td>
<td>$50.00</td>
<td>$1.40</td>
</tr>
</tbody>
</table>
PM Online Resources

- [http://members.aol.com/marctt/](http://members.aol.com/marctt/)
- [www.dextermag.com](http://www.dextermag.com)
- [www.magnetsales.com](http://www.magnetsales.com)
- [http://www.grouparnold.com/](http://www.grouparnold.com/)
- Magnetic Materials Producers Association (MMPA standard)
Some Very Brief Comments on Superconductors

• Superconductors have zero resistance if the temperature is low enough, the field acting on the superconductor is low enough, and the current through the superconductor is low enough
• Superconductors are classified as “low temperature” (NbTi, NbSn) or “high temperature” (YBCO, BSCCO)
• Low-Tc superconductors are usually chilled with liquid helium (4.2K)
• High-Tc superconductors are usually used in the 20K-77K range

Some Data on Low-Tc Material

• Shown for niobium titanium
• This type of superconductor is used in the Japanese MLX 500 km/hr Maglev
Some Data on High-Tc Material

• Some superconductors are anisotropic; i.e. superconducting tapes

Quotes

*It is well to observe the force and virtue and consequence of discoveries, and these are to be seen nowhere more conspicuously than in printing, gunpowder, and the magnet.*
--- Sir Francis Bacon

*The mystery of magnetism, explain that to me! No greater mystery, except love and hate.*
--- John Wolfgang von Goethe
Transformers --- Overview

• Selected history
• Types of transformers
• Voltages and currents
• Equivalent circuits
• Voltage and current transformers
• Per-unit system
Selected History

• 1831 --- Transformer action demonstrated by Michael Faraday
• 1880s: modern transformer invented

Early Transformer (Stanley, c. 1880)
Faraday’s Law

• A changing magnetic flux impinging on a conductor creates an electric field and hence a current (eddy current)

\[ \oint_C E \cdot dl = -\frac{d}{dt} \int_S B \cdot dA \]

• The electric field integrated around a closed contour equals the net time-varying magnetic flux density flowing through the surface bound by the contour

• In a conductor, this electric field creates a current by:

\[ J = \sigma E \]

• Induction motors, brakes, etc.
Magnetic Circuit with Two Windings

- Note that flux is the sum of flux due to $i_1$ and that due to $i_2$
Types of Transformers

• There are many different types and power ratings of transformers: single phase and multi-phase, signal transformers, current transformers, etc.
Instrument Transformer

- Instrument transformers (voltage and current) provide line current and line voltage information to protective relays and control systems
- Current transformer shown below

200A Current Transformer (CT)

Applications
- Sensing Overload Current
- Ground fault detection
- Metering
- Analog to Digital Circuits

Electrical Specifications @ 20°C ambient

<table>
<thead>
<tr>
<th>Electrical Specifications</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Primary Current</td>
<td>200A nom., 500A max.</td>
</tr>
<tr>
<td>Turns Ratio</td>
<td>1000:1 nominal</td>
</tr>
<tr>
<td>Load (per Amp Ratio) at 200A for 100 ohm</td>
<td>0.100 V/A</td>
</tr>
<tr>
<td>Volt per Amp Ratio at 20A for 100 ohm load</td>
<td>0.0991 V/A</td>
</tr>
<tr>
<td>DC Resistance at 20°C</td>
<td>11 ohms</td>
</tr>
<tr>
<td>Dielectric Withstanding Voltage (Hi-pot)</td>
<td>4KVrms</td>
</tr>
</tbody>
</table>

Mechanical Specifications

<table>
<thead>
<tr>
<th>Case</th>
<th>Polycarbonate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Encapsulant</td>
<td>Epoxy</td>
</tr>
<tr>
<td>Flammability</td>
<td>Conforms to UL94-VO</td>
</tr>
<tr>
<td>Terminals</td>
<td>Pins Ø 1.0mm</td>
</tr>
<tr>
<td>Marking</td>
<td>TALEMA Date Code (W/Y)</td>
</tr>
<tr>
<td></td>
<td>AC1200, Dot at start pin</td>
</tr>
<tr>
<td>Approximate Weight</td>
<td>150 grams</td>
</tr>
<tr>
<td>Tolerance</td>
<td>±0.2mm</td>
</tr>
</tbody>
</table>

Voltage Instrument Transformer

Voltage Transformers

Applications
- For single phase voltage measurement in AC power system

Features
- Resin cast moulded plastic cases
- Integral fuses available on some models
- Designed to meet IEEE C57.13
- 50Hz design available
- UL Recognized and CSA Approved

Audio Transformer

P.C. BOARD MOUNT - EPOXY POTTED
BROADCAST QUALITY
AUDIO TRANSFORMERS

- Rugged black epoxy potted case with 9 pin connections (.025" square by 0.5" long).
- Frequency response @ 0 dbm +/- 1 db max. (+/- 0.5 db is typical) of 30 Hz. to 30 Khz., except 560Q which is 30 Hz. to 15 Khz.

Pulse Transformer

• Used for triggering SCRs, etc. where isolation is needed

Power Distribution Transformer

- Provides voltage for the customer
- Typical voltages are 2.3-34.5kV primary, and 480Y/277V or 208Y/120V 3-phase or 240/120V single phase
- Pole-top transformers typically 15-100 kVA

Figure 30-6  Dimensioned diagram of (a) a double-E core (b) bobbin, and (c) assembled core with winding.
Offline Flyback Power Supply

FIGURE 1:
SIMPLIFIED APPLICATION DIAGRAM

Transcutaneous Energy Transmission

Fig. 1. Transcutaneous energy transmission system for an implantable artificial heart.

Fig. 2. Transcutaneous transformer.


Electromechanics, Magnetics and Energy Conversion
Power Transformer

Superconducting Transformer

Fig. 13. Schematic of the major system components of a 5/10 MVA HTS transformer.

Fig. 15. Completed 5/10 MVA WES/SP/ORNL HTS transformer under test at the Waukesha Electric test facility.

Transformer Under No-Load Condition

By Faraday’s law:

\[
e_1 = \frac{d\lambda_1}{dt} = N \frac{d\Phi}{dt} = \omega N \Phi_{\text{max}} \cos(\omega t)
\]

RMS value of \(e_1\):

\[
E_{1,\text{rms}} = \frac{2\pi f N \Phi_{\text{max}}}{\sqrt{2}} = \sqrt{2}\pi f N \Phi_{\text{max}}
\]

If resistive drop in winding is negligible:

\[
\Phi_{\text{max}} = \frac{E_{1,\text{rms}}}{\sqrt{2}\pi f N}
\]
No-Load Phasor Diagram

- Winding current has harmonics, and fundamental is generally out of phase with respect to flux.
- In-phase component is from core losses.

\[
P_c = E_1 I_\phi \cos \theta_c
\]

- Magnetizing current is 90 degrees out of phase.
Example: Transformer Calculations

- *Fitzgerald*, Example 2.1. In Example 1.8, the core loss and VA at $B_{\text{max}} = 1.5\text{T}$ and 60 Hz were found to be: $P_c = 16\text{W}$ and $VI = 20\text{ VA}$ with induced voltage 194V. Find power factor, core loss current $I_c$ and magnetizing current $I_m$. 

![Diagram of a transformer with labels: $i$, $e$, $N = 200$ turns, $2\text{ in}$, $8\text{ in}$, $2\text{ in}$, $10\text{ in}$, and $\theta_c$.](image1.png)

**Figure 1.15** Laminated steel core with winding for Example 1.8.
Example: Transformer Calculations --- Solution

Power factor:

\[ PF = \cos(\theta_c) = \frac{16}{20} = 0.8 \]

Exciting current

\[ I_\phi = \frac{VA}{V} = \frac{20}{194} = 0.1A \]

Core loss component

\[ I_c = \frac{16}{194} = 0.082A \]

Magnetizing component

\[ I_m = I_\phi |\sin \theta_c| = 0.060 \text{ } A \]
Ideal Transformer with Load

Ideal transformer:
\[ v_1 = N_1 \frac{d\Phi}{dt} \]
\[ v_2 = N_2 \frac{d\Phi}{dt} \]
\[ \frac{v_2}{v_1} = \frac{N_2}{N_1} \]

By Ampere’s law:
\[ N_1 i_1 - N_2 i_2 = 0 \Rightarrow \frac{i_1}{i_2} = \frac{N_2}{N_1} \]

(Also, by power balance,
\[ v_1 i_1 = v_2 i_2 \])
Impedance Transformation

\[
\hat{V}_1 = \frac{N_1}{N_2} \hat{V}_2
\]

\[
\hat{I}_1 = \frac{N_2}{N_1} \hat{I}_2 = \frac{N_2}{N_1} \frac{\hat{V}_2}{Z_2} = \left( \frac{N_2}{N_1} \right) \left( \frac{N_2}{N_1} \right) \hat{V}_1 \left( \frac{1}{Z_2} \right)
\]

Therefore, impedance at input terminals is:

\[
\frac{\hat{V}_1}{\hat{I}_1} = Z_2 \left( \frac{N_1}{N_2} \right)^2
\]
Equivalent Circuits

• These 3 circuits have the same impedance as seen from the a-b terminals
Example: Use of Equivalent Circuits

- *Fitzgerald*, Example 2.2
- (a) Draw equivalent circuit with series impedance referred to primary
- (b) For a primary voltage of 120VAC and a short at the output, find the primary current and the short circuit current at the output
Example: Impedance Transformation

\[
R'_2 + jX'_2 = \left( \frac{N_1}{N_2} \right)^2 (R_2 + jX_2)
\]

\[
= 25 + j100
\]
Example: Input Current with Output Short Circuit

\[ \hat{I}_1 = \frac{\hat{V}_1}{R'_2 + jX'_2} = \frac{120}{25 + j100} \left( \frac{25 - j100}{25 - j100} \right) \]

\[ \hat{I}_1 = 0.28 - j(1.13) \]

\[ I_{1,rms} = \sqrt{0.28^2 + 1.13^2} = 1.16 \, A \]

\[ \hat{I}_2 = 5\hat{I}_1 = 1.4 - j(5.65) \]

\[ I_{2,rms} = \sqrt{1.4^2 + 5.65^2} = 5.8 \, A \]
Non-Ideal Effects in Transformers --- Magnetizing Inductance

- A real-world transformer doesn’t pass DC
- From either set of terminals, the impedance looks like an inductor if the other set of terminals is open-circuited
- Can model this as an ideal transformer with a magnetizing inductance added.
- The magnetizing current $i_m$ produces the mutual flux which couples to the secondary
Mutual and Leakage Flux

- Not all of the flux created by winding #1 links with winding #2.
Mutual and Leakage Flux

Resultant mutual flux, $\varphi$

Primary leakage flux

Secondary leakage flux
Non-Ideal Effects in Transformers --- Leakage

- Not all of the flux created by winding #1 links with winding #2.
- Therefore, real-world voltage transformation is not exactly equal to the turns ratio, due to the voltage drops on $L_{k1}$ and $L_{k2}$.
Transformer Equivalent Circuits

(a) 

(b) 

(c) 

(d)
Example: Use of Equivalent Circuits

• Fitzgerald, Example 2.3: A 50-kVA 2400:240V 60 Hz distribution transformer has a leakage impedance of 0.72+j0.92Ω in the high voltage winding and 0.0070 + j0.0090 Ω in the low-voltage winding. The impedance $Z_\phi$ of the shunt branch (equal to $R_c + jX_m$ in parallel) is 6.32 + j43.7 Ω when viewed from the low voltage side.

Draw the equivalent circuits referred to the high voltage side and the low voltage sides.

SOLUTION:  
Note that N1:N2 is 1:10, so impedances step up and down by 100
Example: Solution

--- Leakage impedance of 0.72 + j0.92 Ω in the high voltage winding and 0.0070 + j0.0090 Ω in the low-voltage winding.
--- The impedance $Z_\phi$ of the shunt branch (equal to $R_c + jX_m$ in parallel) is 6.32 + j43.7 Ω when viewed from the low voltage side.
Approximate Transformer Equivalent Circuits

- “Cantilever circuits”
- Ignoring voltage drop in primary or secondary leakage impedances
Approximate Transformer Equivalent Circuits

- Circuit if we ignore the magnetizing inductance and core resistance

- Circuit if we further ignore the winding resistance
Example: Use of Cantilever Circuit

- *Fitzgerald*, Example 2.4

Using T-model:

\[
\hat{V}_2 = (2400) \left( \frac{Z_\varphi}{Z_{l1} + Z_\varphi} \right) \left( \frac{1}{10} \right) = 239.9 + j0.0315
\]

Using cantilever model:

\[
\hat{V}_2 = (2400) \left( \frac{1}{10} \right) = 240
\]
Example: Find $V_2$

- *Fitzgerald*, Example 2.5
Example: Find $V_2$

From node equations:

$$\hat{V}_s = \hat{V}_2 + \hat{I}_L R + jX\hat{I}_L$$
Example: Find $V_2$

- We need length of vector $Oa$ (which is $V_2$)
- We know length of vector $Oc$ (which is 2400V)

\[
ab = IR \cos \theta + IX \sin \theta = (20.8)(1.72)(0.8) + (20.8)(3.42)(0.6) = 71.4
\]

\[
bc = IX \cos \theta - IR \sin \theta = 35.5
\]

Solve for $V_2$:

\[
(V_2 + ab)^2 + (bc)^2 = V_s^2 \quad \Rightarrow \quad V_2 = 233V
\]
Transformer Testing to Determine Parameters

• By doing various tests on a transformer, we can determine the equivalent circuit parameters
• Testing includes open-circuit and short-circuit testing
Short-Circuit Test

\[ \hat{I}_{sc} \]

\[ \hat{V}_{sc} \]

\[ R_1 \]
\[ X_{l1} \]
\[ X_{l2} \]
\[ R_2 \]

\[ R_c \]
\[ X_m \]

\[ \hat{I}_{sc} = \frac{R_{eq}}{R_1 + R_2} \]
\[ X_{eq} = \frac{X_{eq}}{X_{l1} + X_{l2}} \]

\[ \hat{V}_{sc} \]

(b)
Short-Circuit Testing

Normally:
\( L_m \gg L_{k1}, L_{k2} \) and \( R_c \gg R_{w1}, R_{w2} \)

Using the simplified circuit, we can approximate:

\[
\left| Z_{eq} \right| \approx \frac{V_{TEST}}{I_{TEST}}
\]

\[
R_{SC} = R_{w1} + R_{w2}' = \frac{P_{SC}}{I_{TEST}^2}
\]

\[
X_{SC} \approx \sqrt{\left| Z_{eq} \right|^2 - R_{SC}^2}
\]
Open-Circuit Testing

(a)

(b)
Open-Circuit Testing

• There is no secondary current

\[ R_c \approx \frac{V_{TEST}^2}{P_{OC}} \]

\[ |X_m| \approx \frac{V_{TEST}}{I_{TEST}} \]
Autotransformer

Conventional transformer

Autotransformer redrawn

Connection as autotransformer
Staco Autotransformer

Reference: Staco

Electromechanics  Magnetics and Energy Conversion  2-190
Autotransformer Brush
Damaged Autotransformer
Example: Autotransformer

- *Fitzgerald*, Example 2.7
- 2400:240V 50-kVA transformer is connected as an autotransformer with ab being the 240V winding and bc is the 2400V winding.

(a) Compute the kVA rating
(b) Find currents at rated power
Example: Autotransformer Example --- Solution

For 50 kVA, rating of 240V winding is:
\[
\frac{50000}{240} = 208 \, A
\]

The autotransformer VA rating is:
\[
V_H I_H = (2.64 \, kV)(208 \, A) = 549 \, kVA
\]

Rated current at low-voltage winding:
\[
I_L = I_H \left(\frac{2640}{2400}\right) = 229 \, A
\]
Some Comments on Autotransformers

- Autotransformers differ from isolation transformers in that there is no isolation between primary and secondary.
- However, this lack of isolation allows some of the transferred power to be conducted from primary to secondary instead of magnetic induction.
- Autotransformers in general require less core material per kVA rating.
- Autotransformers used where lack of isolation doesn’t pose a safety issue.
Example: Magnetic Circuit Problem

• *Fitzgerald*, Problem 2.2
• A magnetic circuit with a cross-sectional area of 15 cm$^2$ is to be operated from a 120V RMS supply. Calculate the number of turns required to achieve a peak magnetic flux density of 1.8 Tesla in the core
Example: Magnetic Circuit Problem

--- Solution

Flux is: \( \Phi = \Phi_{\text{max}} \sin(\omega t) \)

The time rate of change of flux is:
\[
\frac{d\Phi}{dt} = \omega \Phi_{\text{max}} \cos(\omega t)
\]

The time rate of change of flux linkage is:
\[
\frac{d\lambda}{dt} = N \frac{d\Phi}{dt} = N \omega \Phi_{\text{max}} \cos(\omega t) = V
\]

Let's relate flux density to flux:
\[
\frac{\Phi_{\text{max}}}{A} = B_{\text{max}}
\]

So, we can solve for \( N \)

\[
N = \frac{\sqrt{2}V}{\omega B_{\text{max}} A} = \frac{(\sqrt{2})(120)}{(2\pi \times 60)(1.8)(1.5 \times 10^{-3})} = 166.7
\]

Round up to \( N = 167 \)
Example: Transformer Problem

- *Fitzgerald*, Problem 2.4
- A 100-Ohm resistor is connected to the secondary of an ideal transformer with a turns ratio of 1:4 (primary to secondary). A 10V RMS, 1-kHz voltage source is connected to the primary. Calculate the primary current and the voltage across the 100-Ohm resistor
Example: Transformer Problem --- Solution

We know that this is a step-up transformer, so $V_1 = 10V$ and $V_2 = 40V$.

We next find the secondary current $I_2$

$$I_2 = \frac{40V}{100\Omega} = 0.4A$$

The voltage steps up, so the current steps down; hence

$$I_1 = 4I_2 = 1.6A$$
3-Phase Connections of Transformers

(a) Y-Δ connection

(b) Δ-Y connection

(c) Δ-Δ connection

(d) Y-Y connection
Example: Finding 3-Phase Currents

- Problem: A three phase, 208V (line-line) Y connected load has:
  - \( Z_{an} = 3 + j4 \)
  - \( Z_{bn} = 5 \)
  - \( Z_{cn} = -5j \)
- Find
  - (a) Phase voltages
  - (b) Line and phase currents
  - (c) Neutral currents
Example: Finding 3-Phase Currents --- Solution

Line-neutral voltages are found by taking the line-line voltage and dividing by $\sqrt{3}$

$$V_{LN} = \frac{208}{\sqrt{3}} = 120$$
Example: 3-Phase Currents --- Solution

Line (also called phase) currents are found by taking the line-line voltages and dividing by impedance

\[ I_a = \frac{120 \angle 0^\circ}{3 + 4j} = 24 \angle -53.13^\circ \]

\[ I_b = \frac{120 \angle 120^\circ}{5} = 24 \angle 120^\circ \]

\[ I_c = \frac{120 \angle 240^\circ}{-5j} = 24 \angle -30^\circ \]
Example: 3-Phase Currents --- Solution

- Note that loads are unbalanced so there is a net neutral current

\[ I_n = I_a + I_b + I_c = 25.4 \angle 155.9^\circ \]
480V Y System

- Line-line = 480V; line-neutral = 277V

Delta System

- Line-line = 480V

Instrument Transformer

- Used for protection, relaying, voltage or current monitoring, etc.
Per-Unit System

- Computations in electric machines and transformers are often done using the “per-unit” system.
- Actual circuit quantities (Watts, VArS, etc.) are scaled to the per-unit system.
- This method allows removal of transformers from diagrams.
- To convert to per-unit, 4 base quantities are established:
  - Base power $VA_{\text{base}}$
  - Base voltage $V_{\text{base}}$
  - Base current $I_{\text{base}}$
  - Base impedance $Z_{\text{base}}$
Example: Per-Unit System

Example: A system has $Z_{\text{base}} = 10 \ \Omega$ and $V_{\text{base}} = 400V$. Find base VA and $I_{\text{base}}$

Solution:
- $I_{\text{base}} = \frac{V_{\text{base}}}{Z_{\text{base}}} = \frac{400}{10} = 40A$
- $(VA)_{\text{base}} = V_{\text{base}}I_{\text{base}} = (400)(40) = 16 \text{ kVA}$
Example: Per-Unit System

• A 5 kVA, 400/200V transformer has 2 Ω reactance referred to the 200V side. Express the transformer reactance in p.u.

• Solution:
  – (VA)\text{base} = 5000
  – V\text{base} = 200
  – I\text{base} = (VA)\text{base}/V\text{base} = 5000/200 = 25A
  – Z\text{base} =V\text{base}/I\text{base} = 200/25 = 8 Ω
  – Z = 2.0/8.0 = 0.25 p.u.
Example: Per-Unit Applied to Transformer

- *Fitzgerald*, Example 2.12
- Convert this circuit showing a 100 MVA transformer to the per-unit system
Example: Low-Voltage Side

\[ (VA)_{BASE} = 100 \text{ MVA} \]

\[ V_{BASE} = 7.99 \text{ kV} \]

\[ R_{BASE} = X_{BASE} = Z_{BASE} = \frac{V_{BASE}^2}{(VA)_{BASE}} \]

\[ = \left( \frac{(7.99 \times 10^3)^2}{100 \times 10^{-6}} \right) = 0.638 \Omega \]

In per-unit system:

\[ R_L = \frac{0.76 \times 10^{-3}}{0.638} = 0.0012 \text{ p.u.} \]

\[ X_L = \frac{0.04}{0.638} = 0.063 \text{ p.u.} \]

\[ X_m = \frac{114}{0.638} = 178.7 \text{ p.u.} \]
Example: High-Voltage Side

\[
(VA)_{BASE} = 100 \text{ MVA}
\]

\[
V_{BASE} = 79.7 \text{ kV}
\]

\[
R_{BASE} = X_{BASE} = Z_{BASE} = \frac{V_{BASE}^2}{(VA)_{BASE}}
\]

\[
= \left( \frac{(79.7 \times 10^3)^2}{100 \times 10^{-6}} \right) = 63.5 \text{ } \Omega
\]

In per-unit system:

\[
R_H = \frac{0.085}{63.5} = 0.00133 \text{ } \text{p.u.}
\]

\[
X_H = \frac{3.75}{63.5} = 0.059 \text{ } \text{p.u.}
\]
Example: Model

- Turns ratio is 1:1, so we can remove transformer
Example: Get Rid of 1:1 Transformer

\[ R_L \quad X_L \quad X_H \quad R_H \]

(0.0012 pu) (0.0630 pu) (0.0591 pu) (0.0013 pu)

\[ X_m \quad (180 \text{ pu}) \]
Energy Conversion --- Overview

• Forces and torques
• Energy balance
• Determining magnetic forces and torques from energy
• Multiply-excited systems
• Forces and torques in systems with permanent magnets
Right-Hand Rule

- For determining the direction magnetic-field component of the Lorentz force $F = q(v \times B) = J \times B$. 
Example: Single-Coil Rotor

- *Fitzgerald*, Example 3.1
- Find $\theta$-directed torque as a function of $\alpha$
Example: Single-Coil Rotor

For wire #1:
\[ F_{1\theta} = -IlB_0 \sin \alpha \]

For wire #2:
\[ F_{2\theta} = -IlB_0 \sin \alpha \]

Total torque (\( T = \text{force} \times \text{distance} \)):
\[ T_\theta = -2IlB_0 \sin \alpha R \]

What happens if \( B \) points left-right instead of up-down?
\[ T_\theta = -2IlB_0 \cos \alpha R \]
Electromechanical Energy Conversion Device

• This box can be used to model motors, actuators, lift magnets, etc.
• Note 2 electrical terminals (voltage and current) and 2 mechanical terminals (force \( f_{\text{fld}} \) and position \( x \))
• The lossless magnetic energy storage system converts electrical energy to mechanical energy
Interaction Between Electrical and Mechanical Terminals

\( W_{FLD} \) = stored magnetic energy

In words, the rate of change of magnetic energy equals the power in minus the mechanical work out

\[
\frac{dW_{FLD}}{dt} = ei - f_{fld} \frac{dx}{dt}
\]

By Faraday’s law, \( e = \frac{d\lambda}{dt} \), so let’s rework:

\[
\frac{dW_{FLD}}{dt} = i \frac{d\lambda}{dt} - f_{fld} \frac{dx}{dt}
\]

Multiply through everywhere by \( dt \):

\[
dW_{FLD} = id\lambda - f_{fld} dx
\]
Interaction Between Electrical and Mechanical Terminals

In a lossless system, we can rewrite the energy balance:

\[ dW_{ELEC} = dW_{MECH} + dW_{FLD} \]

Differential energy in: \( dW_{ELEC} = id\lambda \)
Differential work out: \( dW_{MECH} = f_{fld} dx \)
Change in magnetic energy: \( dW_{FLD} \)
Force-Producing Device

• This solenoid is an example of a force-producing device
Energy

Thinking about energy, we start out with:

\[ dW_{FLD} = i d\lambda - f_{fld} dx \]

If magnetic energy storage is lossless, this is a **conservative** system and \( W_{fld} \) is determined by **state variables** \( \lambda \) and \( x \)

In a conservative system, the path you take to do this integration doesn’t matter
Another Conservative System

- Roller coaster, ignoring friction, the path doesn’t matter. Speed of both coasters is the same at the bottom of the hill
Magnetic Relay
• This illustrates lossless magnetic structure with external losses due to resistance
Integration Path for Finding Magnetic Stored Energy

\[ W_{FLD}(\lambda_o, x_o) = \int_{path\,2a} dW_{FLD} + \int_{path\,2b} dW_{FLD} \]

On path 2a, \( d\lambda = 0 \) and \( f_{fld} = 0 \) since zero \( \lambda \) means zero magnetic force, therefore:

\[ W_{FLD}(\lambda_o, x_o) = \int_0^\lambda i(\lambda, x_o) \, d\lambda \]
Special Case --- Linear System

In the special case of a linear system, the flux linkage is proportional to current, or \( \lambda \propto i \).

\[
W_{FLD}(\lambda, x) = \int_0^\lambda i(\lambda', x)d\lambda'
\]

\[
= \int_0^\lambda \lambda' L(x) d\lambda' = \frac{1}{2} \frac{\lambda^2}{L(x)}
\]
Example: Relay with Movable Plunger

- *Fitzgerald*, Example 3.2
- Find magnetic stored energy $W_{\text{FLD}}$ as a function of $x$ with $I = 10\text{A}$
Example: Relay with Movable Plunger

We know that $\lambda = L(x)I$, so

$$W_{FLD} = \frac{1}{2} \frac{\lambda^2}{L(x)}$$

I’ll give you that $L(x) = \frac{\mu_o N^2}{2g} ld(1 - \frac{x}{d})$, so magnetic stored energy is:

$$W_{FLD} = \frac{\mu_o N^2}{4g} ld(1 - \frac{x}{d})I^2$$
Determining Magnetic Force from Stored Energy

• Next, if we go to all this trouble to find stored energy, let’s figure out how to find forces from the energy (very important!)

Remember \( dW_{FLD} = i d\lambda - f_{fld} dx \)

Next, remember the “total differential” from calculus:

\[
dF(x_1, x_2) = \left. \frac{\partial F}{\partial x_1} \right|_{x_2=\text{const.}} dx_1 + \left. \frac{\partial F}{\partial x_2} \right|_{x_1=\text{const.}} dx_2
\]

Let’s rewrite the stored energy expression:

\[
dW_{FLD} = \left. \frac{\partial W_{FLD}}{\partial \lambda} \right|_x d\lambda + \left. \frac{\partial W_{FLD}}{\partial x} \right|_\lambda dx
\]

From this, we see that

\[
i = \left. \frac{\partial W_{FLD}}{\partial \lambda} \right|_x \quad \text{and} \quad f_{fld} = -\left. \frac{\partial W_{FLD}}{\partial x} \right|_\lambda
\]
Determining Magnetic Force from Energy

For linear systems with $\lambda = L(x)I$

Energy $W_{FLD} = \frac{1}{2} \frac{\lambda^2}{L(x)}$

Force:

$$f_{fld} = -\frac{\partial}{\partial x} \left( \frac{1}{2} \frac{\lambda^2}{L(x)} \right) \bigg|_{\lambda=\text{cons} \tan t}$$

$$= \frac{\lambda^2}{2L^2(x)} \frac{dL(x)}{dx}$$

With $\lambda = L(x)I$

$$f_{fld} = \frac{I^2}{2} \frac{dL(x)}{dx}$$
Determining Magnetic Force from Energy

• The bottom line: if your system is linear, and if you can calculate inductance as a function of position, then finding the force is pretty easy
Example: Curve Fit for Inductance of Solenoid with Plunger

- *Fitzgerald*, Example 3.3
- Assume that the following inductance vs. plunger position was measured. We then run the solenoid with 0.75A current.
Example: Force as a Function of Position

- We can fit a polynomial to the inductance, and use energy methods to find the plunger force as a function of position.
Torque in Magnetic Circuit

- Can solve this as before, by analogy

By analogy:

$$T_{fld} = \frac{I^2}{2} \frac{dL(\theta)}{d\theta}$$
Example: Finding Torque

• *Fitzgerald*, Example 3.4

Assume \( L(\theta) = L_0 + L_2 \cos(2\theta) \) with \( L_0 = 10.6 \text{ mH} \) and \( L_2 = 2.7 \text{ mH} \). Find the torque with \( I = 2A \).

Solution:

\[
T(\theta) = \frac{I^2}{2} \frac{dL(\theta)}{d\theta} = \frac{I^2}{2} (-2L_2 \sin(2\theta))
\]

\[
= -1.08 \times 10^{-2} \sin(2\theta) \text{ Nm}
\]
Example: Finding Torque

• *Fitzgerald*, Practice Problem 3.4

Assume $L(\theta) = L_0 + L_2 \cos(2\theta) + L_4 \sin(4\theta)$ with $L_0 = 25.4 \text{ mH}$, $L_2 = 8.3 \text{ mH}$ and $L_4 = 1.8 \text{ mH}$. Find the torque with $I = 3.5A$.

Solution:

$T(\theta) = -0.1017 \sin(2\theta) + 0.044 \cos(4\theta) \text{ N} \cdot \text{m}$
Another Example --- Torque vs. Rotor Angle
Today’s Summary

• Today we’ve covered:
  • Maxwell’s equations: Ampere’s, Faraday’s and Gauss’ laws
  • Soft magnetic materials (steels, etc.)
  • Hard magnetic materials (permanent magnets)
  • Basic transformers
  • The per-unit system
  • We started electromechanical conversion basics