Background of Instructor

• BSEE ('85), MS ('92), and PhD ('97) from MIT
• Doctoral work done in the Laboratory for Electromagnetic and Electronic Systems (L.E.E.S.) at MIT in electrodynamic Maglev
• Consultant in analog, magnetics, power electronics, electromagnetics, magnetic braking and failure analysis
• Adjunct Professor at Worcester Polytechnic Institute, Worcester MA teaching graduate courses in analog design, power electronics, electromechanics and power distribution
• Have also taught for University of Wisconsin since 2006
Lens Actuator
• For high speed laser printing
High Power Laser Diode Driver Based on Power Converter Technology

See:
Magnetically-Levitated Flywheel Energy Storage System

- For NASA; $P = 100$W, energy storage = 100 W-hrs
Transrapid Maglev

- Currently in operation from Pudong (Shanghai), connecting airport to subway line
- Operational speed is 431 km/hr
Japanese EDS Maglev

- Slated to run parallel to the high-speed Shinkansen line
- On December 2, 2003, this three-car train set attained a maximum speed of 581 km/h in a manned vehicle run

Reference: http://www.rtri.or.jp
Japanese EDS Guideway

- This view shows levitation, guidance and propulsion coils
MIT Maglev Suspension Magnet

MIT Maglev Test Fixture

MIT EDS Maglev Test Facility

- 2 meter diameter test wheel
- Max. speed 1000 RPM (84 m/s)
- For testing “flux canceling” HTSC Maglev
- Sidewall levitation

References:
Permanent Magnet Brakes

- For roller coasters
- Braking force > 10,000 Newtons per meter of brake

Reference: http://www.magnetarcorp.com
Halbach Permanent Magnet Array

- Special PM arrangement allows strong side (bottom) and weak side (top) fields
- Applicable to magnetic suspensions (Maglev), linear motors, and induction brakes
Halbach Permanent Magnet Array

- 2D FEA modeling
Photovoltaics

Fig. 1. Typical hybrid system layout.

Offline Flyback Power Supply

FIGURE 1:
SIMPLIFIED APPLICATION DIAGRAM

Transcutaneous Energy Transmission

50 KW Inverter Switch
Transformer Failure Analysis
Non-Contact Battery Charger

- Modeled using PSIM
High Voltage RF Supply

[Diagram of a high voltage RF supply system with various components and labels such as 'Setpoint 500V peak per volt', 'Integrator', 'Buck bandwidth', 'Limiter', 'Voltage slew limiter', 'Vo_buck', 'Step-up transformer', 'Vsec', 'Isec', 'Resonant inductor 190e-6', '500:1 load resistor', '1e6 RL', '30p', 'Master clock 1.68e6', 'RMS/DC bandwidth 1 1.68e6 1', 'RMS/DC divide 0.002', and various other electrical symbols and connections.]
60 Hz Transformer Shielding Study

• Modeled using Infolytica 2D and 3D
Course Overview

• Day 1 --- Basics of power, 3 phase power, harmonics, etc.
• Day 2 --- Magnetics, transformers and energy conversion
• Day 3 --- Basic machines

• Numerous examples are done throughout the 3 days, some from Fitzgerald, Kingsley and Umans
### Course Overview --- Day 1

<table>
<thead>
<tr>
<th>Day</th>
<th>Material Covered</th>
<th>Hours</th>
<th>Slides</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Basics</td>
<td>8:00-8:30</td>
<td>1-26</td>
</tr>
<tr>
<td></td>
<td>Attendee sign-in</td>
<td>8:30-8:45</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Introductions, instructor background, and course 3-day overview</td>
<td>8:45-9:15</td>
<td>27-58</td>
</tr>
<tr>
<td></td>
<td>Basic circuit analysis concepts</td>
<td>9:15-10:15</td>
<td>59-96</td>
</tr>
<tr>
<td></td>
<td>Basic single-phase power, complex number review, 3-phase, RMS</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Morning break</td>
<td>10:15-10:30</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Power: real, imaginary and instantaneous; P, Q, S and power factor</td>
<td>10:30-11:00</td>
<td>97-104</td>
</tr>
<tr>
<td></td>
<td>Power cables; AWG and circular mils; cable impedance</td>
<td>11:00-11:45</td>
<td>105-126</td>
</tr>
<tr>
<td></td>
<td>Single line drawings</td>
<td>11:45-12:00</td>
<td>127-130</td>
</tr>
<tr>
<td></td>
<td>Lunch</td>
<td>12:00-1:00</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Power factor and power factor correction issues</td>
<td>1:00-1:30</td>
<td>131-151</td>
</tr>
<tr>
<td></td>
<td>Harmonics, Fourier series; total harmonic distortion (THD)</td>
<td>1:30-2:00</td>
<td>152-174</td>
</tr>
<tr>
<td></td>
<td>Begin 3-phase; 6-pulse rectifiers; 12-pulse rectifiers</td>
<td>2:00-2:45</td>
<td>175-188</td>
</tr>
<tr>
<td></td>
<td>Afternoon break</td>
<td>2:45-3:00</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3-phase circuits and 3-phase power; neutral currents with and without harmonics; nonlinear loads</td>
<td>3:00-3:45</td>
<td>189-201</td>
</tr>
<tr>
<td></td>
<td>Summarize</td>
<td>3:45-4:00</td>
<td>202</td>
</tr>
</tbody>
</table>

**Note:** we’ll start at 7:30 on days #2 and #3

**Supplemental notes:** Inductance calculation methods (if time permits)

**Supplemental notes:** Maglev (if time permits)
# Course Overview --- Day 2

<table>
<thead>
<tr>
<th>Topic</th>
<th>Time</th>
<th>Location</th>
</tr>
</thead>
<tbody>
<tr>
<td>Review of Maxwell’s equations; demo; concepts of magnetic fields; Ampere’s, Faraday’s and Gauss’ magnetic laws; Lorentz force law; magnetic circuits</td>
<td>7:30-9:30</td>
<td>1-54</td>
</tr>
<tr>
<td>Morning break</td>
<td>9:30-9:45</td>
<td></td>
</tr>
<tr>
<td>Soft magnetic materials (steel); hysteresis and core losses.</td>
<td>9:45-10:45</td>
<td>55-76</td>
</tr>
<tr>
<td>Hard magnetic (permanent magnets) and PM circuits</td>
<td>10:45-12:00</td>
<td>77-139</td>
</tr>
<tr>
<td>Lunch</td>
<td>12:00-1:00</td>
<td></td>
</tr>
<tr>
<td>Comments on superconductors; basic transformers: analysis, equivalent circuits</td>
<td>1:00-2:45</td>
<td>140-208</td>
</tr>
<tr>
<td>Afternoon break</td>
<td>2:45-3:00</td>
<td></td>
</tr>
<tr>
<td>Per-unit system</td>
<td>3:00-3:15</td>
<td>209-216</td>
</tr>
<tr>
<td>Begin electromechanical conversion, begin forces and torques</td>
<td>3:15-4:00</td>
<td>217-240</td>
</tr>
<tr>
<td>Summarize</td>
<td>4:00</td>
<td>241</td>
</tr>
</tbody>
</table>
## Course Overview --- Day 3

<table>
<thead>
<tr>
<th>3</th>
<th>Basic machines</th>
</tr>
</thead>
<tbody>
<tr>
<td>Finish forces and torques with demo and 2 examples</td>
<td>7:30-8:30</td>
</tr>
<tr>
<td>Introduction to AC and DC machines</td>
<td>8:30-9:30</td>
</tr>
<tr>
<td>Morning break</td>
<td>9:30-9:45</td>
</tr>
<tr>
<td>MMF of windings; rotating magnetic fields in rotating machinery; generated voltage</td>
<td>9:45-11:00</td>
</tr>
<tr>
<td>Synchronous machines</td>
<td>11:00-12:00</td>
</tr>
<tr>
<td>Lunch</td>
<td>12:00-1:00</td>
</tr>
<tr>
<td>Linear and PM synchronous machines</td>
<td>1:00-1:30</td>
</tr>
<tr>
<td>Induction machines</td>
<td>1:30-2:45</td>
</tr>
<tr>
<td>Afternoon break</td>
<td>2:45-3:00</td>
</tr>
<tr>
<td>Finish induction machines</td>
<td>3:00-3:30</td>
</tr>
<tr>
<td>Induction (eddy current) brakes</td>
<td>3:30-4:00</td>
</tr>
<tr>
<td>Summarize</td>
<td>4:00-4:15</td>
</tr>
</tbody>
</table>
Basic Circuit Analysis Concepts

• Some background in circuits to get us all on the same page
  • First-order and second-order systems
  • Resonant circuits, damping ratio and Q
First-Order Systems

• A system with a single energy-storage element is a “first-order” system
• Step voltage driven RC lowpass filter

\[ v_o(t) = V (1 - e^{-\frac{t}{\tau}}) \]
\[ i_r(t) = \frac{V}{R} e^{-\frac{t}{\tau}} \]
\[ \tau = RC \]
\[ \tau_R = 2.2\tau \]

\[ \omega_h = \frac{1}{\tau} \]
\[ f_h = \frac{\omega_h}{2\pi} \]
\[ \tau_R = \frac{0.35}{f_h} \]
First-Order Systems --- Some Details

- Frequency response:

\[ H(s) = \frac{1}{\tau s + 1} \]

- Phase response:

\[ \angle H(s) = -\tan^{-1}(\omega \tau) \]

- -3 dB bandwidth:

\[ \omega_h = \frac{1}{\tau} \]

\[ f_h = \frac{\omega_h}{2\pi} \]
Step Response 10 - 90% Risetime

- Often-measured figure of merit for systems
- Defined as the time it takes a step response to transition from 10% of final value to 90% of final value
- This plot is for a first-order system with no overshoot or ringing

\[ \tau_R = 2.2 \tau \]
Relationship Between Risetime and Bandwidth

• Exact for a first-order system (with one pole):

\[ \tau_R = \frac{0.35}{f_h} \]

• Approximate for higher-order systems
First-Order Step and Frequency Response

• Single pole at -1 rad/sec.
Second-Order Mechanical System

- 2 energy storage modes in mass and spring

Spring force: \( f_y = -ky \)

Newton’s law for moving mass:
\[
f_y = -ky = M \frac{d^2y}{dt^2}
\]

Differential equation for mass motion:
\[
M \frac{d^2y}{dt^2} + ky = 0
\]

Guess a solution of the form:
\[
y(t) = Y_o \sin(\omega t)
\]
Second-Order Mechanical System

\[ y(t) = Y_0 \sin(\omega t) \]

Put this proposed solution into the differential equation:

\[ M\left(-\omega^2 Y_0 \sin(\omega t)\right) + k\left(Y_0 \sin(\omega t)\right) = 0 \]

This solution works if:

\[ \omega = \sqrt{\frac{k}{M}} \]
Second-Order Mechanical System

- Electromechanical modeling; useful for modeling speakers, motors, acoustics, etc.

Second-Order Electrical System

\[ H(s) = \frac{v_o(s)}{v_i(s)} = \frac{1}{Cs} \cdot \frac{1}{R + Ls + \frac{1}{Cs}} = \frac{1}{LCs^2 + RCs + 1} = \frac{1}{s^2 + \frac{2\zeta\omega_n}{\omega_n}s + \omega_n^2} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \]

Natural frequency \( \omega_n = \frac{1}{\sqrt{LC}} \)

Damping ratio \( \zeta = \frac{\omega_n RC}{2} = \frac{1}{2} \left( \frac{R}{\sqrt{L}} \right) = \frac{1}{2} \frac{R}{Z_o} \)
Second-Order System Frequency Response

\[ H(j\omega) = \frac{1}{\frac{2j\zeta\omega}{\omega_n} + \left(1 - \frac{\omega^2}{\omega_n^2}\right)} \]

\[ |H(j\omega)| = \frac{1}{\sqrt{\left(\frac{2\zeta\omega}{\omega_n}\right)^2 + \left(1 - \frac{\omega^2}{\omega_n^2}\right)^2}} \]

\[ \angle H(j\omega) = -\tan^{-1} \left(\frac{\frac{2\zeta\omega}{\omega_n}}{1 - \frac{\omega^2}{\omega_n^2}}\right) = -\tan^{-1} \left(\frac{2\zeta\omega\omega_n}{\omega_n^2 - \omega^2}\right) \]
Second-Order System Frequency Response

Frequency response for natural frequency = 1 and various damping ratios

![Graph showing frequency response for natural frequency = 1 and various damping ratios.](image)

Electromechanics Basics

1-38
Quality Factor, or “Q”

Quality factor is defined as:

\[
\frac{E_{\text{stored}}}{\omega P_{\text{diss}}}
\]

where \( E_{\text{stored}} \) is the peak stored energy in the system and \( P_{\text{diss}} \) is the average power dissipation.
Q of Series Resonant RLC

\[ Q = \left( \frac{1}{\sqrt{LC}} \right) \left( \frac{1}{2} \frac{LI_{pk}^2}{R} \right) = \frac{\sqrt{L}}{R} = \frac{Z_o}{R} \]

\[ \omega = \frac{1}{\sqrt{LC}} \]

\[ E_{\text{stored}} = \frac{1}{2} LI_{pk}^2 \]

\[ P_{\text{diss}} = \frac{1}{2} I_{pk}^2 R \]
Q of Parallel Resonant RLC

\[
\omega = \frac{1}{\sqrt{LC}} \\
E_{\text{stored}} = \frac{1}{2} CV_{pk}^2 \\
P_{\text{diss}} = \frac{1}{2} \frac{V_{pk}^2}{R} \\
Q = \left( \frac{1}{\sqrt{LC}} \right) \left( \frac{1}{2} \frac{CV_{pk}^2}{1 \frac{V_{pk}^2}{2 \frac{R}{L}}} \right) = \frac{R}{\sqrt{L}} = \frac{R}{Z_o}
\]
Relationship Between Damping Ratio and “Quality Factor” Q

- A second order system can also be characterized by its “Quality Factor” or Q.

\[
|H(s)|_{\omega=\omega_n} = \frac{1}{2\zeta} = Q
\]

- Use Q in transfer function of series resonant circuit:

\[
H(s) = \frac{1}{\frac{s^2}{\omega_n^2} + \frac{2\zeta s}{\omega_n} + 1} = \frac{1}{\frac{s^2}{\omega_n^2} + \frac{s}{\omega_n Q} + 1}
\]
Second-Order System Step Response

- Shown for varying values of damping ratio.
Undamped Resonant Circuit

We find the resonant frequency by guessing that the voltage \( v(t) \) is sinusoidal with \( v(t) = V_o \sin \omega t \). Putting this into the equation for capacitor voltage results in:

\[
-\omega^2 \sin(\omega t) = -\frac{1}{LC} \sin(\omega t)
\]

This means that the resonant frequency is the standard (as expected) resonance:

\[
\omega_r^2 = \frac{1}{LC}
\]
# Energy Methods

<table>
<thead>
<tr>
<th>Storage Mode</th>
<th>Relationship</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capacitor/electric field storage</td>
<td>$E_{elec} = \frac{1}{2} CV^2$</td>
<td></td>
</tr>
<tr>
<td>Inductor/magnetic field storage</td>
<td>$E_{mag} = \frac{1}{2} LI^2 = \int \frac{B^2}{2\mu_0}dV$</td>
<td></td>
</tr>
<tr>
<td>Kinetic energy</td>
<td>$E_k = \frac{1}{2} Mv^2$</td>
<td></td>
</tr>
<tr>
<td>Rotary energy</td>
<td>$E_r = \frac{1}{2} I\omega^2$</td>
<td>$I \equiv$ mass moment of inertia (kg·m²)</td>
</tr>
<tr>
<td>Spring</td>
<td>$E_{spring} = \frac{1}{2} kx^2$</td>
<td>$k \equiv$ spring constant (N/m)</td>
</tr>
<tr>
<td>Potential energy</td>
<td>$\Delta E_p = Mg\Delta h$</td>
<td>$\Delta h \equiv$ height change</td>
</tr>
<tr>
<td>Thermal energy</td>
<td>$\Delta E_T = C_{TH}\Delta T$</td>
<td>$C_{TH} \equiv$ thermal capacitance (J/K)</td>
</tr>
</tbody>
</table>
By using energy methods we can find the ratio of maximum capacitor voltage to maximum inductor current. Assuming that the capacitor is initially charged to $V_o$ volts, and remembering that capacitor stored energy $E_c = \frac{1}{2}CV^2$ and inductor stored energy is $E_L = \frac{1}{2}LI^2$, we can write the following:

$$\frac{1}{2}CV_o^2 = \frac{1}{2}LI_o^2$$
Energy Methods

What is the inductor current? We can solve for the ratio of $V_o/I_o$ resulting in:

$$\frac{V_o}{I_o} = \sqrt{\frac{L}{C}} \equiv Z_o$$

The term “$Z_o$” is defined as the characteristic impedance of a resonant circuit. E.g. with $C = 1$ microFarad and $L = 1$ microHenry, the resonant frequency is $10^6$ radians/second (or 166.7 kHz) and that the characteristic impedance is 1 Ohm.
Simulation

Initial conditions at $t = 0$:
capacitor voltage = 1;
inductor current = 0.
Typical Resonant Circuit

- Model of a MOSFET gate drive circuit

\[ H(s) = \frac{v_o(s)}{v_i(s)} = \frac{1}{Cs} \frac{1}{R + Ls + \frac{1}{Cs}} = \frac{1}{LCs^2 + RCS + 1} = \frac{1}{s^2 + \frac{2\zeta s}{\omega_n} + 1} \]

\[ \omega_n = \frac{1}{\sqrt{LC}} \]

\[ \zeta = \frac{\omega_n RC}{2} = \frac{1}{2} \frac{R}{\sqrt{L}} = \frac{1}{2} \frac{R}{Z_o} \]

\[ \omega_n \sqrt{LC} \]
Resonant Circuit --- Underdamped

- With “small” resistor, circuit is underdamped

\[ \omega_n = \frac{1}{\sqrt{LC}} = 200 \text{ Mrad / sec} \]

\[ f_n = 31.8 \text{ MHz} \]

\[ Z_o = \sqrt{\frac{L}{C}} = 5\Omega \]

\[ \zeta = \frac{\omega_n RC}{2} = \frac{1}{2} \frac{R}{\sqrt{\frac{L}{C}}} = \frac{1}{2} \frac{R}{Z_o} = 0.001 \]
Resonant Circuit --- Underdamped Results

• This circuit is very underdamped, so we expect step response to oscillate at around 31.8 MHz
• Expect peaky frequency response with peak near 31.8 MHz

\[ \omega_n = \frac{1}{\sqrt{LC}} = 200 \text{ Mrad / sec} \]

\[ f_n = 31.8 \text{ MHz} \]

\[ Z_o = \sqrt{\frac{L}{C}} = 5\Omega \]

\[ \zeta = \frac{\omega_n RC}{2} = \frac{1}{2} \frac{R}{\sqrt{\frac{L}{C}}} = \frac{1}{2} \frac{R}{Z_o} = 0.001 \]
Underdamped Resonant Circuit, Step Response

- PSPICE result: rings at around 31.8 MHz
Underdamped Resonant Circuit, Frequency Response

- Frequency response peaks at 31.8 MHz
Resonant Circuit --- Critical Damping

• Now, let’s employ “critical damping” by increasing value of resistor to $R = 2Z_o = 10$ Ohms
• This is also a typical MOSFET gate drive damping resistor value
Critical Damping, Step Response

- Note that response is still relatively fast (< 100 ns response time) but with no overshoot
- “Critical damping” results in fastest step response without overshoot
Critical Damping, Frequency Response

- No overshoot in the transient response corresponds to no peaking in the frequency response
Impedance of a Series-Resonant Circuit

- The impedance is capacitive below the resonant frequency
- Impedance is inductive above resonance

Impedance of a Parallel-Resonant Circuit

- The impedance is inductive below the resonant frequency

Figure 9-9  Frequency characteristics of a parallel-resonant circuit.

Power, Complex Numbers and RMS

- Basic single-phase circuits
- AC voltage, current and power
- Complex numbers
- 3-phase power
- Root-mean square (RMS)
Sinewaves

- A pure sinewave can be expressed as $v(t) = V_{pk}\sin(\omega t)$
- $V_{pk} =$ peak voltage
- $\omega =$ radian frequency
- In Hz, $\omega = 2\pi f$ where $f$ is in Hz
- $V_{RMS} = V_{pk}/\sqrt{2} = 120V$ for sinewave with peaks at $\pm 170V$
- More on RMS later
Sinewave Voltage Source with Resistive Load

- $v(t)$ and $i(t)$ are in phase and have the same shape; i.e. no harmonics in current
  
  - Time representation
  
  - Phasor representation
  
  - In this case, $V$ and $I$ have the same phase
Sinewave with Inductive Load

- For an inductor, remember that $v = L \frac{di}{dt}$
- So, $i(t)$ lags $v(t)$ by $90^\circ$ in an inductor

![Diagram of a sinewave with an inductive load](image)
Sinewave with L/R Load

- Phase shift (also called angle) between $v$ and $i$ is somewhere between $0^\circ$ and $-90^\circ$

$$
\angle = -\tan^{-1}\left(\frac{\omega L}{R}\right)
$$
Sinewave with Capacitive Load

- Remember that $i = C\frac{dv}{dt}$ for a capacitor
- Current leads voltage by $+90^\circ$

- Phasor representation
Phasor Representation of L and C

In inductor, current lags voltage by 90 degrees

In capacitor, voltage lags current by 90 degrees


Electromechanics Basics 1-65
Response of $L$ and $C$ to pulses


**Figure 3-7** Inductor and capacitor response.
Review of Complex Numbers

- In “rectangular” form, a complex number is written in terms of real and imaginary components
- \( A = \text{Re}(A) + j \times \text{Im}(A) \)

- Angle

\[
\theta = \tan^{-1}\left(\frac{\text{Im}(A)}{\text{Re}(A)}\right)
\]

- Magnitude of \( A \)

\[
|A| = \sqrt{(\text{Re}(A))^2 + (\text{Im}(A))^2}
\]
Find Polar Form

• Assume that current $I = -3.5 + j(4.2)$

$$|I| = \sqrt{(-3.5)^2 + (4.2)^2} = 5.5 \text{A}$$

$$\theta = 180^\circ - \gamma$$

$$\gamma = \tan^{-1}\left(\frac{4.2}{3.5}\right) = 50.2^\circ$$

$$\theta = 180^\circ - 50.2^\circ = 129.8^\circ$$

$\therefore I = 5.5 \text{A} \angle 129.8^\circ$
Converting from Polar to Rectangular Form

\[ \text{Re}\{A\} = |A| \cos(\theta) \]
\[ \text{Im}\{A\} = |A| \sin(\theta) \]
National Electrical Code (NEC)

• Put out by National Fire Protection Agency (NFPA) as NFPA 70
• Initially developed in 1897, and is updated every 3 years
• Sets out requirements for wire sizing (phase conductors, neutrals, grounds) and other rules for installation of power in residential single phase and industrial 3 phase systems

National Electrical Code® Softbound 2005 Edition (NFPA 70)
120V, 2-Wire System

- Older system, largely (hopefully) replaced in residences
- Neutral = “grounded conductor”
- Voltage between hot and neutral is 120 VAC (or 120 VRMS)
- Voltage stepped down at pole-mount or pad-mount transformer; enters house service entrance at 120V between HOT and NEUTRAL
120/240, 3-Wire System

- Most common residential service; 3 wires are derived by center-tapping the distribution transformer secondary.
- Voltage between one phase and neutral is 120V.
- Voltage between phases is 240V: used for hot water heaters, dryers, air conditioners, etc.
- Neutral is grounded at the service entrance.
Typical Single-Phase Residential Service

• Typical 7.2 kV distribution voltage
• Output voltage is 120V with 2 single phases
• You’ll run low-power loads (i.e. lights, electronics) from a single phase
• High power loads (i.e. clothes dryer, hot water heater) run phase-to-phase
Single-Phase Service --- Simulation Results

- Transformer step-down
200A Breaker Panel

- Main breaker at top (200A)
- Multiple branch circuits below: black = hot wire, white = neutral
Single-Phase 60 Hz Power

Three-Phase Power

- Three-phase is used in higher power applications
- You can carry more power with less copper using 3 phase
- Large generators and motors are more efficient and smaller using 3-phase compared to single-phase
- 3-phase voltages are created by 3-phase generators
Three-Phase Generators

- More on 3-phase motor/generators later
Three-Phase, 480V 60 Hz Power

- Line-line voltage is 480V RMS
- Line-neutral voltage is 277V (or 480/sqrt(3)), which has peaks at ± 392V

208Y/120, 3-Phase, 4 Wire

- Can drive small 3 phase loads or single phase loads
- Voltage phase-phase = 208V (i.e. from A to B)
- Voltage phase-neutral = \( \frac{208}{\sqrt{3}} \) = 120V
480/277, 3-Phase, 4 Wire

- Typical industrial voltage for larger 3-phase loads
- Large 3-phase loads can be connected to the three phases
- Smaller 277V single-phase loads can be connected between phase and ground
- Phase-phase = 480V; phase-neutral = 277V
480V 4-Wire System

- Line-line = 480V; line-neutral = 277V

Some Comments on Delivery Voltage

• A typical factory utilization voltage is 480Y/277V, meaning that a 4-wire Y connected service is provided with 480V line-line and a line-neutral voltage of 277V
• Service for industrial facilities can be supplied by a utility at distribution voltage (2.4kV to 34.5 kV) or at subtransmission or transmission voltages (46 kV to 230 kV)
• 1888: Polyphase AC system invented by Tesla
• First three-phase system was installed between Lauffen and Frankfurt am Main in Germany in 1891
Root Mean Square (RMS)

• Used for description of periodic, often multi-harmonic, waveforms

• Calculation of RMS is done by taking the square root of the average over a cycle (mean) of the square of a waveform

\[ I_{RMS} = \sqrt{\frac{1}{T} \int_{0}^{T} i^2(t)dt} \]

• RMS current of any waveshape will dissipate the same amount of heat in a resistor as a DC current of the same value
  – DC waveform: \( V_{rms} = V_{DC} \)
  – Symmetrical square wave: \( I_{RMS} = I_{pk} \)
  – Pure sine wave: \( I_{RMS} = 0.707 I_{pk} \)

• Example: 120 VRMS line voltage has peaks of ±169.7 V
Intuitive Description of RMS

- The RMS value of a sinusoidal or other periodic waveform dissipates the same amount of power in a resistive load as does a battery of the same RMS value.
- So, 120V RMS into a resistive load dissipates as much power in the load as does a 120V battery.
RMS Value of Various Waveforms

• Following are waveforms typically found in power electronics, power systems, and motors, and their corresponding RMS values
DC Voltage

- Battery

\[ i(t) \]

\[ I \rightarrow t \]

\[ \text{RMS} = I \]
Pure Sinewave

- AC line

\[ \text{RMS} = \frac{I_{pk}}{\sqrt{2}} \]
Square Wave

• This type of waveform can be put out by a square wave converter or full-bridge converter

\[ i(t) \]

\[ I_{pk} \]

\[ t \]

\[ \text{RMS} = I_{pk} \]
DC with Ripple

• Buck converter inductor current (DC value + ripple)

\[ \text{RMS} = I_0 \sqrt{1 + \left(\frac{1}{3}\right)\left(\frac{\Delta I_{pp}}{2I_0}\right)^2} \]
Triangular Ripple, No DC Value

- Capacitor ripple current in some converters (no DC value)

\[
RMS = \frac{\Delta i_{pp}}{2\sqrt{3}}
\]
Pulsating Waveform

- Buck converter input switch current (assuming small ripple)

\[ i(t) \]

\[ I_{pk} \]

\[ DT \]

\[ T \]

\[ \text{RMS} = I_{pk} \sqrt{D} \]

\[ 0 < D < 1 \]
Pulsating with Ripple

- i.e. buck converter switch current
- We can use this result to get RMS value of buck diode current

\[
\text{RMS} = I \sqrt{D} \sqrt{1 + \left(\frac{1}{3}\right)\left(\frac{\Delta i_{pp}}{2I}\right)^2}
\]
Triangular

\[ \text{RMS} = I_{pk} \sqrt{\frac{D}{3}} \]
Piecewise Calculation

- For $I_1$, $I_2$, etc. at different frequencies

\[
F or \ I = I_1 + I_2 + I_3 \ldots
\]

then

\[
I_{RMS} = \sqrt{I_{1,RMS}^2 + I_{2,RMS}^2 + \cdots}
\]
Piecewise Calculation --- Example

• What is RMS value of DC + ripple (shown before)?

\[
RMS = \sqrt{I_o^2 + \left(\frac{\Delta i_{pp}}{2\sqrt{3}}\right)^2} = I_o \sqrt{1 + \left(\frac{1}{3}\right)\left(\frac{\Delta i_{pp}}{2I_o}\right)^2}
\]
Apparent, Real and Reactive Power

• “Power” has many shapes and forms
  – Real power (P, in Watts). This is the power that does work
  – Apparent power (S, in VA). This is the total “apparent” power seen by the source. It’s the product of \( V_{\text{RMS}} \) and \( I_{\text{RMS}} \)
  – Reactive power (Q, expressed in VAr)
    • Reactive power does not do real work

  – Instantaneous power \( p(t) = v(t)i(t) \)

  – Average power \( \langle p(t) \rangle = \frac{1}{T} \int_{0}^{T} v(t)i(t)dt \)
Vector Relationship Between P, Q and S

- P = real power (Watts)
- Q = reactive power (VAr)
- S = apparent power (VA)
- PF = power factor (unitless)

\[ S = \sqrt{P^2 + Q^2} \]

\[ PF = \frac{P}{S} = \cos(\theta) \]
Apparent Power from Voltage and Current

- We’ll use the “Poynting vector” \( S = V \times I^* \)
- \( V = \) complex phase voltage, \( I^* = \) complex conjugate of phase current
- If \( A = \alpha + j\beta \), then \( A^* = \alpha - j\beta \)
- \( A \times A^* = \alpha^2 + \beta^2 \)
Apparent Power from Voltage and Current

• Result:

\[ P = |V||I|\cos(\theta) \]
\[ Q = |V||I|\sin(\theta) \]
Example: Finding P, Q and S

Let’s find P, Q, S and Z given V and I

- Find apparent power $S$ using Poynting vector

$$S = V \times I^* = (120 \angle 30^\circ)(7 \angle -10^\circ) = 840 \angle 20^\circ$$

$$S = 789 + j(287) \text{ VA}$$
Example: Finding P, Q and S

- Find real power $P$
  \[ P = VI \cos(\theta) = (120)(7) \cos(20^\circ) = 798 \text{ W} \]

- Find imaginary power $Q$
  \[ Q = VI \sin(\theta) = (120)(7) \sin(20^\circ) = 287 \text{ VAR} \]

- Find load impedance $Z$
  \[ Z = \frac{V}{I} = \frac{120 \angle 30^\circ}{7 \angle 10^\circ} = 17.14 \Omega \angle 20^\circ \]
Finding Load Current in Single Phase Systems

- We must know the current magnitude to properly size conductors. The NEC (National Electrical Code) gives guidance on sizing of conductors once we know the RMS conductor current.
- Current magnitude results in $I^2R$ loss and temperature rise in conductors.

\[ VI = VA = \frac{P}{PF} \]

\[ I = \frac{P}{V \times PF} \]
Examples: Finding Load Current

• P=10 kVA, PF=1, 120V single phase

\[ I = \frac{10000}{120} = 83.3 \text{A} \]

• P=50 kVA, PF=0.9 lagging, 240V single phase

\[ I = \frac{50000}{(240)(0.9)} = 231.5 \text{A} \]
Sizing of Power Cables

- Large power cables are available in copper and aluminum.
- Wire is sized by AWG (American Wire Gage) or thousands of circular mils (kcmils).
- The smallest wire used in power distribution is #14 AWG, typically a solid conductor with an outside diameter of 0.0641”, or 64.1 mils.
- The largest AWG is #4/0, with a diameter of approximately 0.522” for a 7-strand conductor.
AWG

- Wire size is denoted by AWG (American Wire Gage)
- Wire diameter varies by a factor of 2 every 6 AWG
- #36 AWG is defined to be 0.005” (5 mil) diameter
- “Circular mils” is the diameter in mils, squared. (1 mil = 0.001”)

To find wire diameter $d$ based on AWG number:

In inches:  
$$d = (0.005^{\text{"}}) \times \frac{92}{39}$$

In millimeters:  
$$d = (25.4) \times (0.005^{\text{"}}) \times \frac{92}{39}$$
Circular Mils

The area of larger conductors is expressed in circular mils. To find the area in cmils, square the diameter in mils by itself. For example, a #10 solid conductor with a diameter of 162 mils has an area of $(162 \text{ mils})^2 = 26244 \text{ cmils}$, or $26.2 \text{ kcmil}$
DC Resistance of Power Cables

• Resistance is a function of length, cross-sectional area, and electrical conductivity

\[ R = \frac{l}{\sigma A} \]

• For copper, \( \sigma \approx 5.8 \times 10^7 \text{ Ohm}^{-1}\text{m}^{-1} \) at room temperature. For aluminum, \( \sigma \approx 3.5 \times 10^7 \text{ Ohm}^{-1}\text{m}^{-1} \) at room temperature
Copper Temperature Coefficient of Resistivity

- Copper resistivity goes up as temperature goes up.
- So, the total resistance of a piece of wire goes up as it heats up.
- Temperature coefficient of resistivity is about $+0.004/\degree C$, or about 0.4% per degree C.
- Resistance goes up as temperature goes up.

$$R_{T_2} = R_{25}[1 + \alpha(T_2 - 298)]$$

- $R_{25}$ = resistance at 25C (298K), $\alpha$ = temperature coefficient
  $\approx 0.00385$ for copper and $\approx 0.00395$ for aluminum; rule of thumb for copper is $+0.4\%$ per degree C.
Issues With Aluminum Wire

- Aluminum has higher resistivity than copper (about 70% higher), with resistivity \( \rho = 2.9 \times 10^{-8} \ \Omega\cdot\text{m} \)
- A 15-amp branch circuit made with #14 copper would require #12 aluminum, by the NEC
- Similar TC to copper
- Aluminum is much lighter than copper (about 33% of the weight per unit volume)
- Often used in power transmission (good ratio of resistivity to weight)
- Corrosion when joining aluminum to copper lugs; differing thermal expansion
Power Cables and Cable Impedance

• Power cables have a finite resistance (due to the resistivity of the conductor) and a finite inductance (due to the geometry of the cable)
• Cable impedance results in voltage drops, power loss, and heating in the cable
AC Cable Resistance

- DC resistance of a piece of wire:
  \[ R = \frac{l}{\sigma A} \]
  - \( A \) = cross sectional area of wire, \( l \) = length, and \( \sigma \) = electrical conductivity
- However, AC resistance is higher than DC resistance, due to skin effect and proximity effect
- Wire resistance can be found in a “wire chart” for an isolated wire (i.e. in air, no metallic conduit, no other current-carrying wires nearby)
## Copper Wire Data

**Table 20.1 Copper Wire Data**

<table>
<thead>
<tr>
<th>AWG SIZE</th>
<th>DIAMETER (MM)</th>
<th>Ω/KM (75°C)</th>
<th>KG/KM</th>
<th>TURNS/CMM²</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>8.25</td>
<td>0.392</td>
<td>475</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>7.35</td>
<td>0.494</td>
<td>377</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>6.54</td>
<td>0.624</td>
<td>299</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>5.83</td>
<td>0.786</td>
<td>237</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>5.19</td>
<td>0.991</td>
<td>188</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>4.62</td>
<td>1.25</td>
<td>149</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>4.12</td>
<td>1.58</td>
<td>118</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>3.67</td>
<td>1.99</td>
<td>93.8</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>3.26</td>
<td>2.51</td>
<td>74.4</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>2.91</td>
<td>3.16</td>
<td>59.0</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>2.59</td>
<td>3.99</td>
<td>46.8</td>
<td>14</td>
</tr>
<tr>
<td>11</td>
<td>2.31</td>
<td>5.03</td>
<td>37.1</td>
<td>17</td>
</tr>
<tr>
<td>12</td>
<td>2.05</td>
<td>6.34</td>
<td>29.4</td>
<td>22</td>
</tr>
<tr>
<td>13</td>
<td>1.83</td>
<td>7.99</td>
<td>23.3</td>
<td>27</td>
</tr>
<tr>
<td>14</td>
<td>1.63</td>
<td>10.1</td>
<td>18.5</td>
<td>34</td>
</tr>
<tr>
<td>15</td>
<td>1.45</td>
<td>12.7</td>
<td>14.7</td>
<td>40</td>
</tr>
<tr>
<td>16</td>
<td>1.29</td>
<td>16.0</td>
<td>11.6</td>
<td>51</td>
</tr>
<tr>
<td>17</td>
<td>1.15</td>
<td>20.2</td>
<td>9.23</td>
<td>63</td>
</tr>
<tr>
<td>18</td>
<td>1.02</td>
<td>25.5</td>
<td>7.32</td>
<td>79</td>
</tr>
<tr>
<td>19</td>
<td>0.912</td>
<td>32.1</td>
<td>5.80</td>
<td>98</td>
</tr>
<tr>
<td>20</td>
<td>0.812</td>
<td>40.5</td>
<td>4.60</td>
<td>123</td>
</tr>
<tr>
<td>21</td>
<td>0.723</td>
<td>51.1</td>
<td>3.65</td>
<td>153</td>
</tr>
<tr>
<td>22</td>
<td>0.644</td>
<td>64.4</td>
<td>2.89</td>
<td>192</td>
</tr>
<tr>
<td>23</td>
<td>0.573</td>
<td>81.2</td>
<td>2.30</td>
<td>237</td>
</tr>
<tr>
<td>24</td>
<td>0.511</td>
<td>102</td>
<td>1.82</td>
<td>293</td>
</tr>
<tr>
<td>25</td>
<td>0.455</td>
<td>129</td>
<td>1.44</td>
<td>364</td>
</tr>
<tr>
<td>26</td>
<td>0.405</td>
<td>163</td>
<td>1.15</td>
<td>454</td>
</tr>
<tr>
<td>27</td>
<td>0.361</td>
<td>205</td>
<td>1.10</td>
<td>575</td>
</tr>
<tr>
<td>28</td>
<td>0.321</td>
<td>259</td>
<td>1.39</td>
<td>710</td>
</tr>
<tr>
<td>29</td>
<td>0.286</td>
<td>327</td>
<td>1.75</td>
<td>871</td>
</tr>
<tr>
<td>30</td>
<td>0.235</td>
<td>412</td>
<td>2.21</td>
<td>1090</td>
</tr>
</tbody>
</table>
Copper Wire Data

<table>
<thead>
<tr>
<th>AWG Number</th>
<th>Diameter (mm)</th>
<th>Cross-Sectional Area (mm²)</th>
<th>Resistance, mΩ/m at 25°C</th>
<th>Current Capacity, 500 A/cm²</th>
<th>Current Capacity, 100 A/cm²</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>5.189</td>
<td>21.15</td>
<td>0.8314</td>
<td>105.8</td>
<td>21.15</td>
</tr>
<tr>
<td>6</td>
<td>4.115</td>
<td>13.30</td>
<td>1.322</td>
<td>66.51</td>
<td>13.30</td>
</tr>
<tr>
<td>8</td>
<td>3.264</td>
<td>8.366</td>
<td>2.103</td>
<td>41.83</td>
<td>8.366</td>
</tr>
<tr>
<td>10</td>
<td>2.588</td>
<td>5.261</td>
<td>3.343</td>
<td>26.31</td>
<td>5.261</td>
</tr>
<tr>
<td>12</td>
<td>2.053</td>
<td>3.309</td>
<td>5.315</td>
<td>16.54</td>
<td>3.309</td>
</tr>
<tr>
<td>14</td>
<td>1.628</td>
<td>2.081</td>
<td>8.451</td>
<td>10.40</td>
<td>2.081</td>
</tr>
<tr>
<td>16</td>
<td>1.291</td>
<td>1.309</td>
<td>13.44</td>
<td>6.543</td>
<td>1.309</td>
</tr>
<tr>
<td>18</td>
<td>1.024</td>
<td>0.8230</td>
<td>21.36</td>
<td>4.115</td>
<td>0.8230</td>
</tr>
<tr>
<td>20</td>
<td>0.8118</td>
<td>0.5176</td>
<td>33.96</td>
<td>2.588</td>
<td>0.5176</td>
</tr>
<tr>
<td>22</td>
<td>0.6438</td>
<td>0.3255</td>
<td>54.00</td>
<td>1.628</td>
<td>0.3255</td>
</tr>
<tr>
<td>24</td>
<td>0.5106</td>
<td>0.2047</td>
<td>85.89</td>
<td>1.024</td>
<td>0.2047</td>
</tr>
<tr>
<td>26</td>
<td>0.4049</td>
<td>0.1288</td>
<td>136.5</td>
<td>0.6438</td>
<td>0.1288</td>
</tr>
<tr>
<td>28</td>
<td>0.3211</td>
<td>0.08098</td>
<td>217.1</td>
<td>0.4049</td>
<td>0.08098</td>
</tr>
<tr>
<td>30</td>
<td>0.2546</td>
<td>0.05093</td>
<td>345.1</td>
<td>0.2546</td>
<td>0.05093</td>
</tr>
<tr>
<td>32</td>
<td>0.2019</td>
<td>0.03203</td>
<td>549.3</td>
<td>0.1601</td>
<td>0.03203</td>
</tr>
<tr>
<td>35</td>
<td>0.1601</td>
<td>0.02014</td>
<td>873.3</td>
<td>0.1007</td>
<td>0.02014</td>
</tr>
<tr>
<td>40</td>
<td>0.127000</td>
<td>0.0126677</td>
<td>1389.</td>
<td>0.06334</td>
<td>0.0126677</td>
</tr>
<tr>
<td>50</td>
<td>0.1007</td>
<td>0.007967</td>
<td>2208.</td>
<td>0.03983</td>
<td>0.007967</td>
</tr>
<tr>
<td>100</td>
<td>0.07987</td>
<td>0.005010</td>
<td>3510.</td>
<td>0.02505</td>
<td>0.005010</td>
</tr>
</tbody>
</table>

Causes of Skin Effect

- Self-field of wire causes current to crowd on the surface of the wire, raising the AC resistance
- Method:
  - Current $i(t) \Rightarrow$ changing magnetic flux density $B(t) \Rightarrow$ reaction currents

Effects of Skin Effect

- For high frequencies, current is concentrated in a layer approximately one skin depth $\delta$ thick.
- Skin depth varies with frequency:
  
  $$\delta = \sqrt{\frac{2}{\omega \mu \sigma}} = \sqrt{\frac{1}{\pi f \mu \sigma}}$$

- $\sigma =$ electrical conductivity $= 5.9 \times 10^7 \Omega^{-1} \text{m}^{-1}$ for copper at 300K
- $\mu =$ magnetic permeability of material $= 4\pi \times 10^{-7} \text{H/m}$ in free space

### Skin depth in copper at 300K

<table>
<thead>
<tr>
<th>$f$</th>
<th>skin depth (meter)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6.55E-02</td>
</tr>
<tr>
<td>10</td>
<td>2.07E-02</td>
</tr>
<tr>
<td>100</td>
<td>6.55E-03</td>
</tr>
<tr>
<td>1000</td>
<td>2.07E-03</td>
</tr>
<tr>
<td>1.00E+04</td>
<td>6.55E-04</td>
</tr>
<tr>
<td>1.00E+05</td>
<td>2.07E-04</td>
</tr>
<tr>
<td>1.00E+06</td>
<td>6.55E-05</td>
</tr>
</tbody>
</table>
Skin Effect --- Increase in Wire Resistance

- For high frequencies, resistance of wire increases
- DC resistance of wire: \[ R_{DC} = \frac{l}{\sigma \left( \pi r_w^2 \right)} \]

- For frequencies above critical frequency where skin depth equals wire radius \( r_w \):
  \[
  R_{AC} = \frac{l}{\sigma \left( 2 \pi r_w \delta \right)} = R_{DC} \left( \frac{2\delta}{r_w} \right)
  \]
  \[
  f_{crit} = \frac{1}{\pi r_w^2 \mu \sigma}
  \]
- Result: for high frequency operation, don’t bother making wire radius \( > \delta \)
  - Skin depth in copper at 60 Hz is approximately 8 mm
Proximity Effect

- In multiple-layer windings in inductors and transformers, the proximity effect can also greatly increase the winding resistance.
- Field from one wire affects the current profile in another.

**Currents in opposite direction**

![Diagram showing currents in opposite direction](image1.png)

**Currents in same direction**

![Diagram showing currents in same direction](image2.png)
Cable Inductance

- Inductance is defined by the geometry of the wire loop
- Included in the calculation is wire radius, and loop length and shape

Circular loop of wire

\[ L = \mu_o a \left[ \ln \left( \frac{8a}{R} \right) - 1.75 \right] \]

Parallel-wire line

\[ L = \frac{\mu_o l}{\pi} \ln \left[ \frac{d}{R} + \frac{1}{4} - \frac{d}{l} \right] \]
AC Service with Real-World Impedance

- All electrical services have finite impedance; shown here the impedance is \( R_s + j\omega L_s \).
- These types of power distribution drawings rarely show inductance explicitly; rather the inductive reactance \( jX_s \) is used.
- \( L_s \) may be the sum of wiring inductance, transformer leakage, etc.
Example: Finding Load Voltage and Current

- Find load voltage and load current
- Note that line resistance is 0.1Ω and line reactance is 0.2Ω

\[
I_L = \frac{277}{0.2j + 0.1 + 10} = 27.4 - 0.54j
\]

\[
V_{LOAD} = I_L R_L = 274 - 5.4j
\]

- Note voltage drop across line impedance of a few Volts
National Elec. Code Impedance Estimates

- Note conduit type affects resistance and reactance

**TABLE 16-1**

<table>
<thead>
<tr>
<th>Size (AWG or kcmil)</th>
<th>X&lt;sub&gt;L&lt;/sub&gt; (Reactance) for AB Wires</th>
<th>Alternating-Current Resistance for Uncoated Copper Wires</th>
<th>Alternating-Current Resistance for Aluminum Wires</th>
<th>Effective Z at 0.85 PF for Uncoated Copper Wires</th>
<th>Effective Z at 0.85 PF for Aluminum Wires</th>
<th>Size (AWG or kcmil)</th>
</tr>
</thead>
<tbody>
<tr>
<td>14</td>
<td>0.190</td>
<td>10.2</td>
<td>10.2</td>
<td>8.9</td>
<td>8.9</td>
<td>14</td>
</tr>
<tr>
<td></td>
<td>0.059</td>
<td>3.1</td>
<td>3.1</td>
<td>2.7</td>
<td>2.7</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>0.177</td>
<td>6.6</td>
<td>6.6</td>
<td>5.6</td>
<td>5.6</td>
<td>12</td>
</tr>
<tr>
<td></td>
<td>0.054</td>
<td>2.0</td>
<td>2.0</td>
<td>1.7</td>
<td>1.7</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>0.164</td>
<td>3.9</td>
<td>3.9</td>
<td>3.6</td>
<td>3.6</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td>0.056</td>
<td>1.2</td>
<td>1.2</td>
<td>1.1</td>
<td>1.1</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>0.171</td>
<td>2.56</td>
<td>2.56</td>
<td>2.26</td>
<td>2.26</td>
<td>8</td>
</tr>
<tr>
<td></td>
<td>0.052</td>
<td>0.78</td>
<td>0.78</td>
<td>0.69</td>
<td>0.69</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>0.167</td>
<td>1.61</td>
<td>1.61</td>
<td>1.44</td>
<td>1.44</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>0.051</td>
<td>0.49</td>
<td>0.49</td>
<td>0.44</td>
<td>0.44</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>0.157</td>
<td>1.02</td>
<td>1.02</td>
<td>0.95</td>
<td>0.95</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>0.048</td>
<td>0.31</td>
<td>0.31</td>
<td>0.29</td>
<td>0.29</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0.154</td>
<td>0.82</td>
<td>0.82</td>
<td>0.75</td>
<td>0.75</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>0.047</td>
<td>0.25</td>
<td>0.25</td>
<td>0.23</td>
<td>0.23</td>
<td></td>
</tr>
</tbody>
</table>
Power Cable Types

A. Armored Cable (AC)

B. Metal-Clad Cable (MC)

C. Nonmetallic Sheathed Cable (NM)
# Power Cable Types

<table>
<thead>
<tr>
<th>Cable Insulation Types</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>THW-2</td>
<td>Thermoplastic insulation (usually PVC), heat resistant (90°C rating), suitable for wet locations.</td>
</tr>
<tr>
<td>THWN-2</td>
<td>Same as THW except nylon jacket over reduced insulation thickness. Also rated THHN.</td>
</tr>
<tr>
<td>THHN</td>
<td>Thermoplastic insulation (usually PVC), high heat resistant (90°C rating), dry locations only. Nylon jacket. Also rated THWN.</td>
</tr>
<tr>
<td>XHHW-2</td>
<td>Cross-linked polyethylene insulation (X), high heat resistant (90°C rating), for wet and dry locations.</td>
</tr>
<tr>
<td>RHH</td>
<td>Rubber insulation. Most manufacturers use cross-linked polyethylene because it has the same properties as rubber. High heat resistant (90°C rating), for dry locations only.</td>
</tr>
<tr>
<td>RHW-2</td>
<td>Rubber insulation (cross-linked polyethylene), heat resistant (90°C rating), suitable for wet locations.</td>
</tr>
<tr>
<td>USE-2</td>
<td>Underground Service Entrance. Most utilize XLP for 90°C in direct burial applications. Product is usually triple rated: RHH-RHW-USE.</td>
</tr>
<tr>
<td>NM-B</td>
<td>Nonmetallic sheathed cable. The &quot;B&quot; denotes that individual conductor insulation is rated 90°C; however, ampacity is limited to that of a 60°C conductor. Thermoplastic (PVC) conductor insulation, nylon jacketed, with overall PVC cable jacket.</td>
</tr>
<tr>
<td>SEU</td>
<td>Service Entrance Cable, Unarmored. Usually type XHHW insulated conductors with overall PVC jacket. As such, the cable is rated for 90°C dry, 75°C wet locations.</td>
</tr>
<tr>
<td>SER</td>
<td>Service Entrance Cable, Round. Same material construction as SEU, but round construction.</td>
</tr>
</tbody>
</table>

National Elec. Code Ampacity Estimates

Table 310.16 Allowable Ampacities of Insulated Conductors Rated 0 Through 2000 Volts, 60°C Through 90°C (140°F Through 194°F), Not More Than Three Current-Carrying Conductors in Raceway, Cable, or Earth (Directly Buried), Based on Ambient Temperature of 30°C (86°F)

<table>
<thead>
<tr>
<th>Size AWG or kcmil</th>
<th>Copper</th>
<th>Aluminum or Copper-Clad Aluminum</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>18</td>
<td>14</td>
<td></td>
</tr>
<tr>
<td>16</td>
<td>18</td>
<td></td>
</tr>
<tr>
<td>14*</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>12*</td>
<td>25</td>
<td>25</td>
</tr>
<tr>
<td>10*</td>
<td>30</td>
<td>25</td>
</tr>
<tr>
<td>8</td>
<td>40</td>
<td>20</td>
</tr>
<tr>
<td>6</td>
<td>55</td>
<td>20</td>
</tr>
<tr>
<td>4</td>
<td>70</td>
<td>20</td>
</tr>
<tr>
<td>3</td>
<td>85</td>
<td>20</td>
</tr>
<tr>
<td>2</td>
<td>95</td>
<td>20</td>
</tr>
<tr>
<td>1</td>
<td>110</td>
<td>20</td>
</tr>
<tr>
<td>1/0</td>
<td>125</td>
<td>20</td>
</tr>
<tr>
<td>2/0</td>
<td>145</td>
<td>20</td>
</tr>
<tr>
<td>3/0</td>
<td>165</td>
<td>20</td>
</tr>
<tr>
<td>4/0</td>
<td>195</td>
<td>20</td>
</tr>
<tr>
<td>5/0</td>
<td>215</td>
<td>20</td>
</tr>
<tr>
<td>6/0</td>
<td>240</td>
<td>20</td>
</tr>
<tr>
<td>7/0</td>
<td>260</td>
<td>20</td>
</tr>
<tr>
<td>8/0</td>
<td>280</td>
<td>20</td>
</tr>
</tbody>
</table>

(a) Special consideration for 10 AWG.

(b) 85°C rating for 4 AWG and larger.
### National Elec. Code Ampacity Estimates

#### CORRECTION FACTORS

For ambient temperatures other than 30°C (86°F), multiply the allowable ampacities shown above by the appropriate factor shown below.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1.08</td>
<td>1.00</td>
<td>0.91</td>
<td>0.82</td>
<td>0.71</td>
<td>0.58</td>
<td>0.41</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td></td>
<td>1.05</td>
<td>1.00</td>
<td>0.94</td>
<td>0.88</td>
<td>0.82</td>
<td>0.75</td>
<td>0.58</td>
<td>0.41</td>
<td>0.33</td>
<td>0.41</td>
</tr>
<tr>
<td></td>
<td>1.04</td>
<td>1.00</td>
<td>0.96</td>
<td>0.91</td>
<td>0.87</td>
<td>0.58</td>
<td>0.41</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td></td>
<td>1.08</td>
<td>1.00</td>
<td>0.91</td>
<td>0.82</td>
<td>0.71</td>
<td>0.58</td>
<td>0.33</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td></td>
<td>1.05</td>
<td>1.00</td>
<td>0.88</td>
<td>0.82</td>
<td>0.87</td>
<td>0.75</td>
<td>0.33</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td></td>
<td>1.04</td>
<td>1.00</td>
<td>0.91</td>
<td>0.91</td>
<td>0.91</td>
<td>0.82</td>
<td>0.58</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
</tbody>
</table>

*See 240.4(D).*
“Single-Line Drawings”

• Also called “one-line” drawing
• Shorthand to describe power distribution systems
• Symbols include power busses, transmission lines, disconnect switches, air and oil-filled circuit breakers, transformers, motors, generators, etc.
• Shown here is a “radial” power distribution system
Single-Line Drawing Symbols

- There are lots more, but these are some of the important ones

- ROTATING MACHINE
- TWO WINDING POWER TRANSFORMER
- THREE WINDING POWER TRANSFORMER
- FUSE
- CURRENT TRANSFORMER
- POTENTIAL TRANSFORMER
- AIR CIRCUIT BREAKER
- OIL CIRCUIT BREAKER
- THREE-PHASE WYE, NEUTRAL UNGROUNDED
- THREE-PHASE WYE, GROUNDED NEUTRAL
- THREE-PHASE DELTA
Typical Single-Line Drawing
Typical Single-Line Drawing

Power Factor and Power Factor Correction

• Power factor is an important concept in power circuits, power electronics, and electric motors
• Power factor is a measure of how easily you deliver real power to a load
Power Factor

• Ratio of delivered power to the product of RMS voltage and RMS current

\[ PF = \frac{\langle P \rangle}{V_{\text{RMS}} I_{\text{RMS}}} \]

• Power factor always <= 1
• With pure sine wave and resistive load, PF = 1
• With pure sine wave and purely reactive load, PF = 0
• Whenever PF < 1 the circuit carries currents or voltages that do not perform useful work
• The more “spikey” a waveform is the worse is its PF
  – Diode rectifiers have poor power factor
• Power factor can be helped by “power factor correction”
Example: Power Factor Calculation

- A 3-phase load consumes 100 kW and 50 kVAr
- Find P, Q, S and power factor PF

\[ P = 100 \text{ kW} = 10^5 \]

\[ Q = 50 \text{ kVAr} \]

\[ S = \sqrt{(10^5)^2 + (5 \times 10^4)^2} = 111.8 \text{ kVA} \]

\[ PF = \frac{P}{S} = \frac{100}{111.8} = 0.894 \]
Example: Another PF Calculation

• A load consumes 500 kW at 0.85 PF lagging. Find apparent power, reactive power, and power factor angle

\[
P = 500 \text{ kW} = 5 \times 10^5
\]

\[
PF = \frac{P}{S} \implies S = \frac{P}{PF} = \frac{5 \times 10^5}{0.85} = 588 \text{ kVA}
\]

\[
P^2 + Q^2 = S^2 \implies Q = \sqrt{S^2 - P^2} = \sqrt{588^2 - 500^2} = 310 \text{ kVAR}
\]

\[
\theta = \cos^{-1}(PF) = \cos^{-1}(0.85) = -31.8^\circ
\]
Causes of Low Power Factor ---
Nonlinear Load

- Nonlinear loads include:
  - Variable-speed drives
  - Frequency converters
  - Uninterruptible power supplies (UPS)
  - Saturated magnetic circuits
  - Dimmer switches
  - Televisions
  - Fluorescent lamps
  - Welding sets
  - Arc furnaces
  - Semiconductors
  - Battery chargers
Further Comment on Power Factor

- Power factor ($k_p$) can be broken up into 2 pieces:
- Displacement power factor $k_\theta$, due to phase shift between 1$^{\text{st}}$ harmonic of current and input voltage
- Distortion power factor ($k_d$) due to harmonics
- Total power factor is product of these 2

\[ k_p = k_d \cdot k_\theta \]
HWR with Resistive Load

- Note that power is delivered in periodic blips; line current has harmonics so PF < 1
HWR with Resistive Load --- Power Factor

• We can calculate P.F. by inspection:

\[ v_{in,RMS} = \frac{V_{pk}}{\sqrt{2}} \]

\[ i_{in,RMS} = \left( \frac{V_{pk}}{\sqrt{2}} \right) \left( \frac{1}{R_L} \right) \left( \frac{1}{\sqrt{2}} \right) = \frac{V_{pk}}{2R_L} \]

\[ < p_L > = i_{in,RMS}^2 R_L = \left( \frac{V_{pk}}{2R_L} \right)^2 R_L = \frac{V_{pk}^2}{4R_L} \]

\[ P.F. = \frac{< p_L >}{v_{in,RMS} i_{in,RMS}} = \frac{\left( \frac{V_{pk}}{4R_L} \right)}{\left( \frac{V_{pk}}{\sqrt{2}} \right) \left( \frac{V_{pk}}{2R_L} \right)} = \frac{\sqrt{2}}{2} = 0.707 \]
FWR with Resistive Load --- PSPICE Simulation

• We need dummy resistor to keep PSPICE happy
FWR with Resistive Load

- Load voltage
- Load current
- Load power

Electromechanics Basics 1-140
FWR with Resistive Load --- Power Factor

\[ v_{in,RMS} = \frac{V_{pk}}{\sqrt{2}} \]

\[ i_{in,RMS} = \left( \frac{V_{pk}}{\sqrt{2}} \right) \left( \frac{1}{R_L} \right) = \frac{V_{pk}}{\sqrt{2}R_L} \]

\[ <p_L> = i_{in,RMS}^2 R_L = \left( \frac{V_{pk}}{\sqrt{2}R_L} \right)^2 R_L = \frac{V_{pk}^2}{2R_L} \]

\[ P.F. = \frac{<p_L>}{v_{in,RMS}i_{in,RMS}} = \frac{\left( \frac{V_{pk}^2}{2R_L} \right)}{\left( \frac{V_{pk}}{\sqrt{2}} \right) \left( \frac{V_{pk}}{\sqrt{2}R_L} \right)} = \frac{2}{2} = 1.0 \]
Half Wave Rectifier with RC Load

• If RC >> 1/f then this operates like a peak detector and the output voltage $<v_{out}>$ is approximately the peak of the input voltage
• Diode is only ON for a short time near the sinewave peaks
Half Wave Rectifier with RC Load

- Note poor power factor due to peaky input line current
Unity Power Factor --- Resistive Load

- Example: purely resistive load
  - Voltage and currents in phase

\[ v(t) = V \sin \omega t \]
\[ i(t) = \frac{V}{R} \sin \omega t \]
\[ p(t) = v(t)i(t) = \frac{V^2}{R} \sin^2 \omega t \]
\[ \langle p(t) \rangle = \frac{V^2}{2R} \]
\[ V_{\text{RMS}} = \frac{V}{\sqrt{2}} \]
\[ I_{\text{RMS}} = \frac{V}{R\sqrt{2}} \]
\[ PF = \frac{\langle p(t) \rangle}{V_{\text{RMS}}I_{\text{RMS}}} = \frac{V^2}{2R \left( \frac{V}{\sqrt{2}} \right) \left( \frac{V}{R\sqrt{2}} \right)} = 1 \]
Causes of Low Power Factor --- Reactive Load

- Example: purely inductive load
  - Voltage and currents 90° out of phase

\[ v(t) = V \sin \omega t \]
\[ i(t) = \frac{V}{\omega L} \cos \omega t \]
\[ p(t) = v(t)i(t) = \frac{V^2}{\omega L} \sin \omega t \cos \omega t \]
\[ < p(t) >= 0 \]

- For purely reactive load, PF=0
Why is Power Factor Important?

- Consider peak-detector full-wave rectifier

  ![Diagram of peak-detector full-wave rectifier]

- Typical power factor $k_p = 0.6$
- What is maximum power you can deliver to load?
  - $V_{AC} \times \text{current} \times k_p \times \text{rectifier efficiency}$
  - $(120)(15)(0.6)(0.98) = 1058$ Watts

- Assume you replace this simple rectifier by power electronics module with 99% power factor and 93% efficiency:
  - $(120)(15)(0.99)(0.93) = 1657$ Watts
Power Factor Correction

• A toaster can draw 1500W from a 120V/15A line
• Typical offline switching power converter can draw <1000W from the line since it has poor power factor
• High power factor results in: lower utility bills, increased system capacity, better voltage quality, reduced heating losses
• Methods of power factor correction
  – Passive: add capacitors or inductors
  – Active
Power Factor Correction --- Passive

- Switch capacitors in and out as needed as load changes
Power Factor Correction --- Active

• Fluorescent lamp ballast application
Power Factor Corrected Power Supplies

Without PFC

With PFC

Power Supply with Power Factor Correction


Electromechanics Basics 1-151
Harmonics

- Joseph Fourier (1768-1830) worked out that any periodic waveform can be broken up into a fundamental frequency plus harmonics
- The harmonics need to have the proper amplitude and phase relationship to the fundamental
- The result is a “Fourier series” for a periodic waveform
Fourier Series

- Here's the recipe:

\[
f(x) = a_0 + \sum_{n=1}^{\infty} \left( a_n \cos \frac{n\pi x}{L} + b_n \sin \frac{n\pi x}{L} \right)
\]

- Finding the Fourier coefficients:

\[
a_0 = \frac{1}{2L} \int_{-L}^{L} f(x) \, dx
\]

\[
a_n = \frac{1}{L} \int_{-L}^{L} f(x) \cos \frac{n\pi x}{L} \, dx \quad n = 1, 2, ...
\]

\[
b_n = \frac{1}{L} \int_{-L}^{L} f(x) \sin \frac{n\pi x}{L} \, dx \quad n = 1, 2, ...
\]

Harmonics of Square Wave

- A 50% duty cycle square wave:

\[ v(t) = \left( \frac{4}{\pi} \right) \sin(\omega t) + \left( \frac{4}{3\pi} \right) \sin(3\omega t) + \left( \frac{4}{5\pi} \right) \sin(5\omega t) + \cdots \]

\[ \omega = \frac{2\pi}{T} \]
Building Up a Square Wave

- Building up a 50% duty cycle square wave:

Reference: http://mathworld.wolfram.com/FourierSeries.html
Triangle Wave

\[ \nu(t) = \left( \frac{8}{\pi^2} \right)\sin(\omega t) - \left( \frac{8}{3^2 \pi^2} \right)\sin(3\omega t) + \left( \frac{8}{5^2 \pi^2} \right)\sin(5\omega t) + \cdots \]
Harmonics in the Power Line

• Harmonics are created by nonlinear circuits
  – Rectifiers
    • Half-wave rectifier has first harmonic at 60 Hz
    • Full-wave has first harmonic at 120 Hz
  – Switching DC/DC converters
    • DC/DC operating at 100 kHz generally creates harmonics at DC, 100 kHz, 200 kHz, 300 kHz, etc.

• Line harmonics can be treated by line filters
  – Passive
  – Active
Harmonics in the Power Line

• The issue of harmonics has become much more important in recent years in power systems

• Harmonic sources include
  – Switching power supplies
  – Variable speed drives (VSDs)
  – Arc furnaces
  – Welders
  – Fluorescent lamp ballasts
Half-Wave Rectifier, Resistive Load

- Simplest, cheapest rectifier
- Line current has DC component; this current appears in neutral
- High harmonic content, Power factor = 0.7

\[ P.F. = \frac{P_{\text{avg}}}{V_{\text{RMS}} I_{\text{RMS}}} \]

Figure 5-2 Basic rectifier with a load resistance.

Half Wave Rectifier with Resistive Load --- Power Factor and Average Output Voltage

Average output voltage:

\[
< v_d > = \frac{1}{2\pi} \int_0^\pi V_{pk} \sin(\omega t) d(\omega t) = \frac{V_{pk}}{2\pi} [\cos(\omega t)]_{\omega t=0}^{\omega t=\pi} = \frac{V_{pk}}{\pi}
\]

Power factor calculation:

\[
< P > = \frac{1}{2} \left( \frac{I_{pk}}{\sqrt{2}} \right)^2 R = \frac{I_{pk}^2}{4} R
\]

\[
V_{RMS} = \frac{V_{pk}}{\sqrt{2}} = \frac{I_{pk}}{R\sqrt{2}}
\]

\[
I_{RMS} = \frac{I_{pk}}{\sqrt{2}} \frac{1}{\sqrt{2}}
\]

\[
P_F = \frac{< P >}{V_{RMS}I_{RMS}} = \frac{\frac{I_{pk}^2}{4} R}{\left( \frac{I_{pk}}{R\sqrt{2}} \right) \left( \frac{I_{pk}}{\sqrt{2}} \frac{1}{\sqrt{2}} \right)} = 0.707
\]
Half-Wave Rectifier, Resistive Load --- Spectrum of Load Voltage
Half Wave Rectifier with RC Load

- More practical rectifier
- For large RC, this behaves like a peak detector
Half Wave Rectifier with RC Load

- Note poor power factor due to peaky line current
- Note DC component of line current

![Graph showing diode current and output voltage over time](image-url)
Half Wave Rectifier with RC Load --- Spectrum of Line Current

![Graph of Half Wave Rectifier with RC Load]
Total Harmonic Distortion

- Total harmonic distortion (THD)
  - Ratio of the RMS value of all the nonfundamental frequency terms to the RMS value of the fundamental

\[
THD = \sqrt{\frac{\sum_{n \neq 1} I_{n,\text{RMS}}^2}{(I_{1,\text{RMS}})^2}} = \sqrt{\frac{I_{\text{RMS}}^2 - I_{1,\text{RMS}}^2}{I_{1,\text{RMS}}^2}}
\]

- Symmetrical square wave: THD = 48.3%
- Symmetrical triangle wave: THD = 12.1%
Crest Factor

• Another term sometimes used in power engineering
• Ratio of peak value to RMS value
• For a sinewave, crest factor = 1.4
  – Peak = 1; RMS = 0.707
• For a square wave, crest factor = 1
  – Peak = 1; RMS = 1
Harmonics and THD --- Pure Sinewave

- THD = 0%
Harmonics and THD - Sinewave + 3rd Harmonic

- THD = 33.3%
Harmonics and THD --- Sinewave + 3rd + 5th Harmonic

- THD = 38.9%
Harmonics --- Up to N = 103

- THD = 48%
Single Phase, Full-Wave Rectifier

- Draws significant harmonics from the line
Full-Wave Rectifier with RC Filter --- PSPICE
Three-Phase Circuits and Three-Phase Power

- Three phase is used extensively in high power applications
- For high power, it has multiple advantages over single phase

2-pole

4-pole

Y-connection of windings
A Three-Phase, Four-Wire System

- A common neutral wire is assumed
- If there are 3 equal linear loads, $i_n = 0$
- However, if there are nonlinear loads, there can be a neutral current

![Diagram of a three-phase, four-wire system](image.png)

**Figure 5-28** Three-phase, four-wire system.
Balanced, Three-Phase, Four-Wire System

• Result shows that for balanced load, $i_n = 0$
Three-Phase, Four-Wire System, Unbalanced Load
Three-Phase, Full-Bridge Rectifier

- Commonly used in high power applications
- Also called "6-pulse rectifier"
- This circuit generates line harmonics

![Three-phase, full-bridge rectifier diagram](image)

**Figure 5-30** Three-phase, full-bridge rectifier.
6-Pulse Rectifier with Current Source Load

- 3-phase source; draws 5\textsuperscript{th}, 7\textsuperscript{th}, 11\textsuperscript{th}, 13\textsuperscript{th}, … harmonics
6-Pulse Rectifier: Redrawn

- Two groups with three diodes each

![Diagram of 6-Pulse Rectifier](image)

*Figure 5-31* Three-phase rectifier with a constant dc current.
6-Pulse Rectifier Waveforms

- Shown for output DC current source load

![Waveform Diagram]

Figure 5-32 Waveforms in the circuit of Fig. 5-31.
6-Pulse Rectifier: Line Current

- Assuming output current to be purely dc and zero ac-side inductance

**Figure 5-33** Line current in a three-phase rectifier in the idealized case with $L_s = 0$ and a constant dc current.
6-Pulse Rectifier with Current Source Load Simulation

- Simplified with $L_s = 0$ (no line inductance); and current source load
6-Pulse Rectifier with Current Source Load
6-Pulse Rectifier with Resistive Load Simulation
  • Simplified with $L_s = 0$ (no line inductance)
6-Pulse Rectifier with Resistive Load --- Output

- Note that fundamental of ripple frequency = 360 Hz
- Note that average output voltage is higher than single line voltage. Peak value is $\sqrt{3} \times$ peak of line = 294V
6-Pulse Rectifier with Resistive Load and Capacitor Filter

• Let’s add filter capacitor
• Note that a smaller capacitor can be used for the 3 phase rectifier compared to single phase rectifier, because (1) Ripple is smaller and (2) Ripple frequency is higher
6-Pulse Rectifier with Resistive Load and Capacitor Filter
12-Pulse Rectifier

- Add 2 phase-shifted 6-pulse rectifiers; 5\textsuperscript{th} and 7\textsuperscript{th} are eliminated
- Only harmonics are the 11\textsuperscript{th}, 13\textsuperscript{th}, 23\textsuperscript{rd}, 25\textsuperscript{th} ...

Three-Phase, Four-Wire System with Nonlinear Load

- With single-phase nonlinear load, there can be a neutral current

![Diagram of three-phase, four-wire system with nonlinear load](image)

**Figure 5-28** Three-phase, four-wire system.

Neutral Current in a 3-Phase, Four-Wire System

- The neutral current can be very high if driving nonlinear loads line to neutral which generate 3\textsuperscript{rd} or other harmonics
- If line currents are highly discontinuous, the neutral current can be as large as 1.73x line current 3rd harmonic
- Note 3rd harmonic here


**Figure 5-29**  Neutral-wire current $i_n$.  

Simulation of Simple Case
Simulation of Simple Case --- Neutral Current vs. Time
Simulation of Simple Case --- Spectrum of Neutral Current

- Note high 3\textsuperscript{rd} harmonic
Example: Three-Phase Line Current Calculation

- Assume 60 Hz, 3-phase 4-wire, 480 V phase to phase (277V phase-neutral). Loads on phases A and B are 48 Ohms to neutral. Load on phase C is 24 Ohms to neutral. We’ll find phase currents and neutral current.
Example: Three-Phase Line Current Calculation

Let’s find phase currents:

\[
\begin{align*}
v_{an} &= \frac{480}{\sqrt{3}} \angle 0^\circ = 277 \angle 0^\circ \\
v_{bn} &= \frac{480}{\sqrt{3}} \angle -120^\circ = 277 \angle -120^\circ \\
v_{cn} &= \frac{480}{\sqrt{3}} \angle -240^\circ = 277 \angle -240^\circ \\
i_a &= \frac{v_{an}}{48 \Omega} = 5.77 A \angle 0^\circ \\
i_b &= \frac{v_{bn}}{48 \Omega} = 5.77 A \angle -120^\circ \\
i_c &= \frac{v_{cn}}{24 \Omega} = 11.54 A \angle -240^\circ
\end{align*}
\]
Example: Three-Phase Line Current Calculation

- Phase currents
Example: Neutral Current Calculation

• Calculation of neutral current: remember that in a balanced, 3-phase system with linear loads the neutral current sums to zero.
• In this case, we have an imbalance because the amplitude of phase C current is higher than the others.
• The “leftover” current equals the neutral current

\[
i_n = i_a + i_b + i_c \\
= 5.77 \angle 0^\circ + 5.77 \angle -120^\circ + \left( 5.77 \angle -240^\circ + 5.77 \angle -240^\circ \right) \\
= 5.77 \angle -240^\circ
\]
Example: Neutral Current Calculation

\[ i_n = i_a + i_b + i_c \]
\[ = 5.77 \angle 0^\circ + 5.77 \angle -120^\circ + \left(5.77 \angle -240^\circ + 5.77 \angle -240^\circ\right) \]
\[ = 5.77 \angle -240^\circ \]
Thyristor Converter --- Single Phase with Resistive Load

• Angle $\alpha$ is called “firing angle”

$$< v_d > = \frac{V_{pk}}{2\pi} (1 + \cos \alpha)$$
Thyristor Converter --- Single Phase with Resistive Load

- Control characteristic: average output voltage vs. firing angle

\[
< v_d > = \frac{V_{pk}}{2\pi} \left(1 + \cos \alpha \right)
\]
Three-Phase Thyristor Converter

• AC-side inductance is included

Figure 6-24 Three-phase converter with $L_s$ and a constant dc current.
Today’s Summary

• Today we’ve covered:
  • Signal basics, step response, resonance, etc.
  • Power in its various forms
  • Root-mean square (RMS)
  • Power cables and cable impedance
  • Power factor and power factor correction (PFC)
  • Fourier series and harmonics
  • Basic three-phase circuits