Chapter 2

Review of Signal Processing Basics

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Slides to accompany *Intuitive Analog Circuit Design* by Marc T. Thompson

Summary

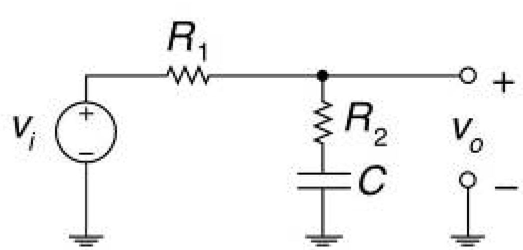
- Review of 1^{st}-order systems
- Relationship between bandwidth and risetime
- 2^{nd} order systems
- Resonance, damping and quality factor
- Energy methods
- Transfer functions, pole/zero plots and Bode plots
- Calculating risetime for systems in cascade
- Comments on PSPICE

Laplace Notation

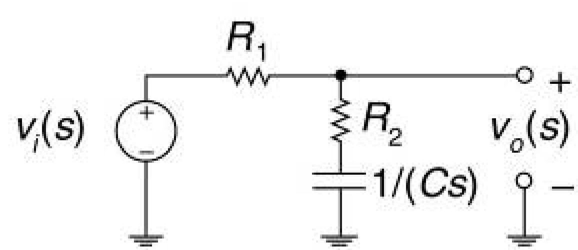
- Basic idea: Laplace transform converts differential equation to algebraic equation $s \Rightarrow \frac{d}{dt}$
- Laplace method is used in sinusoidal steady state after all startup transients have died out

Circuit domain	**Laplace (*s*) domain**
Resistance, R	R
Inductance L	Ls
Capacitance C	$\frac{1}{Cs}$

Circuit

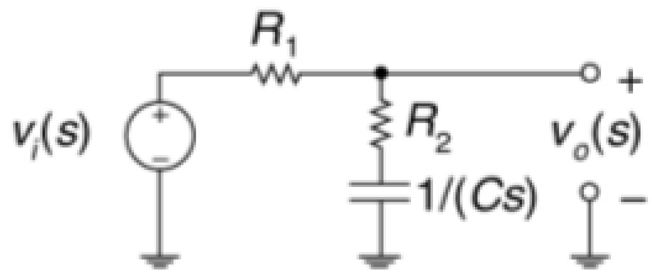


Laplace transformed circuit



System Function H(s)

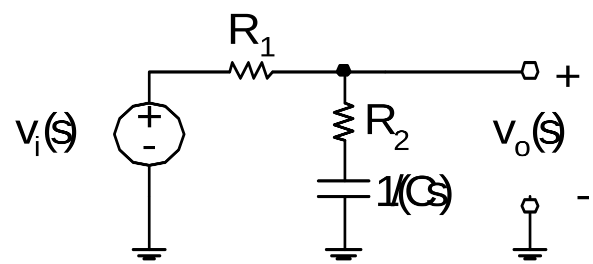
- Find “transfer function” H(s) (also called “transfer function”) by solving Laplace transformed circuit



$$H(s) = \frac{v_o(s)}{v_i(s)} = \frac{R_2 + \frac{1}{Cs}}{R_1 + R_2 + \frac{1}{Cs}} = \frac{R_2 Cs + 1}{(R_1 + R_2)Cs + 1}$$

Does this System Function Make Sense Intuitively?

$$H_1(s) = \frac{v_o(s)}{v_i(s)} = \frac{R_2 + \frac{1}{Cs}}{R_1 + R_2 + \frac{1}{Cs}} = \frac{R_2 Cs + 1}{(R_1 + R_2)Cs + 1}$$



At very low frequencies (s →0), the capacitor is an open-circuit:

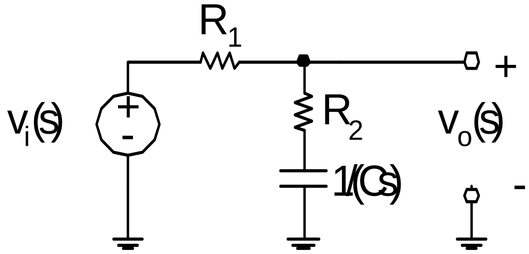
$$H_1(s)\Big|_{s\to 0} \approx 1$$

At very high frequencies (s →∞), the capacitor is an short-circuit:

$$H_1(s)\Big|_{s\to \infty} \approx \frac{R_2}{(R_1 + R_2)}$$

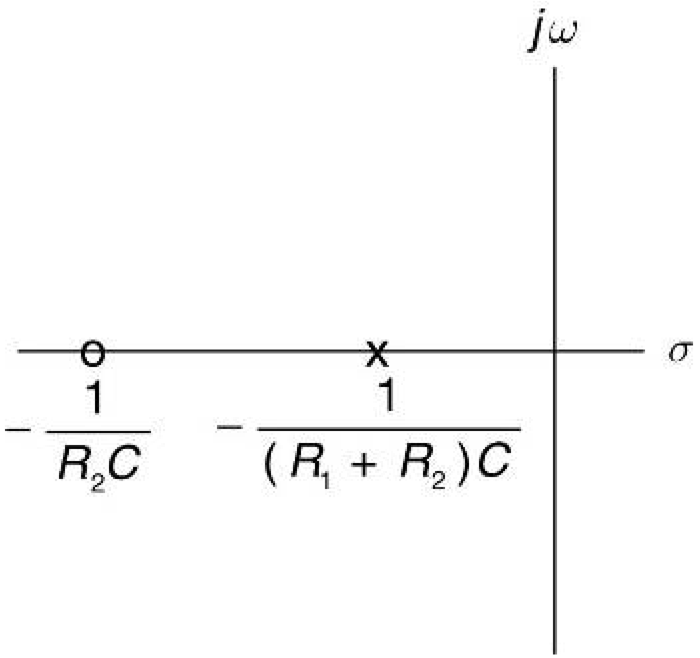
Does this System Function Make Sense Intuitively?

$$H_1(s) = \frac{v_o(s)}{v_i(s)} = \frac{R_2 + \frac{1}{Cs}}{R_1 + R_2 + \frac{1}{Cs}} = \frac{R_2 Cs + 1}{(R_1 + R_2)Cs + 1}$$



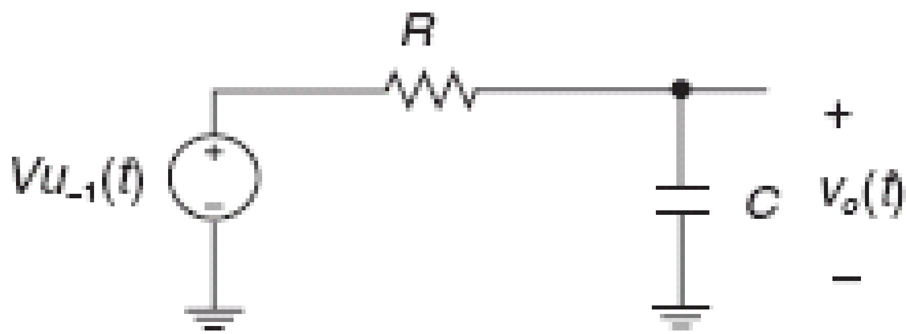
$$s_{pole} = -\frac{1}{(R_1 + R_2)C}$$

$$s_{zero} = -\frac{1}{R_2 C}$$



First-Order System

- Voltage-driven RC lowpass filter



$$v_o(t) = V(1 - e^{\frac{-t}{\tau}})$$

$$i_r(t) = \frac{V}{R} e^{\frac{-t}{\tau}}$$

$\tau = RC$ Time constant

$\omega_h = \dfrac{1}{\tau}$ Bandwidth [Hz]

$\tau_R = 2.2\tau$ Risetime

$f_h = \dfrac{\omega_h}{2\pi}$ Bandwidth [rad/s]

$$\tau_R = \frac{0.35}{f_h}$$

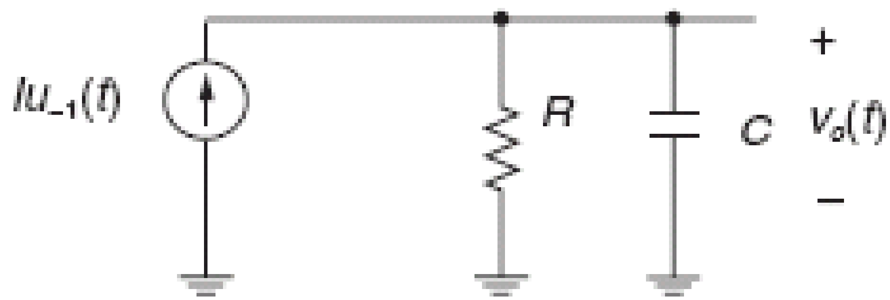
Another First-Order System

- Current-driven RC

$$v_o(t) = IR(1 - e^{\frac{-t}{\tau}})$$

$$i_r(t) = I\ e^{\frac{-t}{\tau}}$$

$$\tau = RC$$



$$\tau_R = 2.2\tau$$

$$\omega_h = \frac{1}{\tau}$$

$$f_h = \frac{\omega_h}{2\pi}$$

$$\tau_R = \frac{0.35}{f_h}$$

First-Order Systems --- Some Details

- Frequency response:

$$H(s) = \frac{1}{\tau s + 1}$$

- Phase response:

$$\angle H(s) = -\tan^{-1}(\omega \tau)$$

- -3 dB bandwidth:

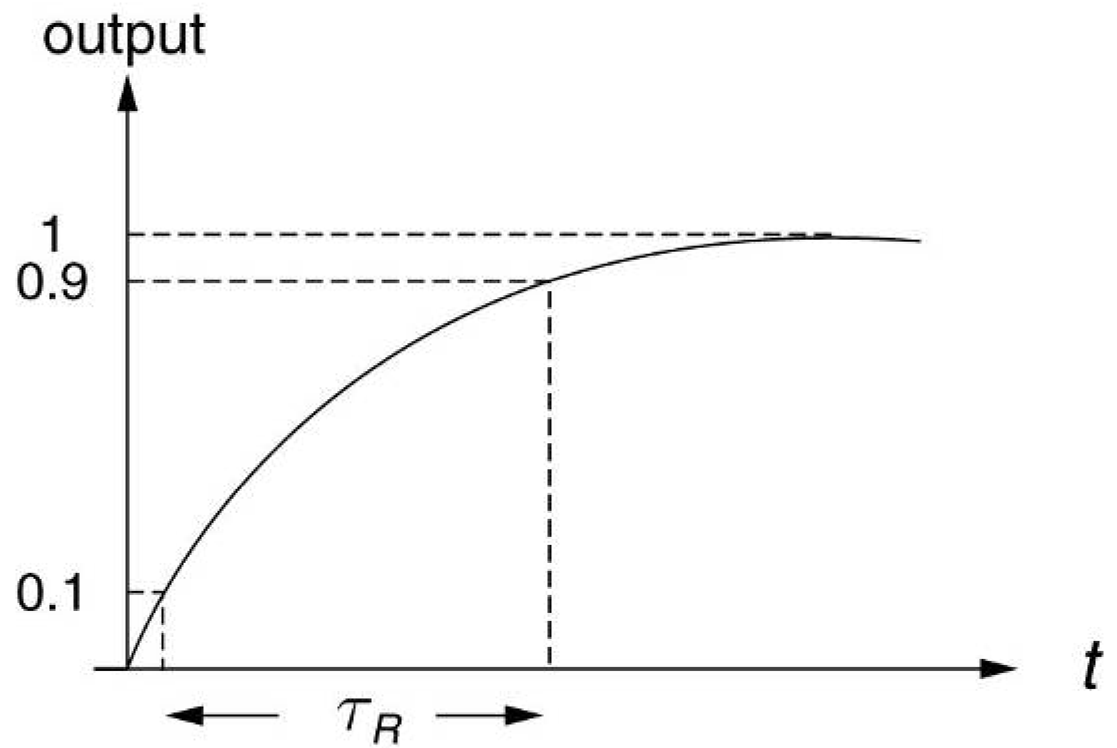
$$\omega_h = \frac{1}{\tau}$$

$$f_h = \frac{\omega_h}{2\pi}$$

10 - 90% Risetime

- Defined as the time it takes a step response to transition from 10% of final value to 90% of final value
- This plot is for a first-order system with no overshoot or ringing

$$\tau_R = 2.2\tau$$



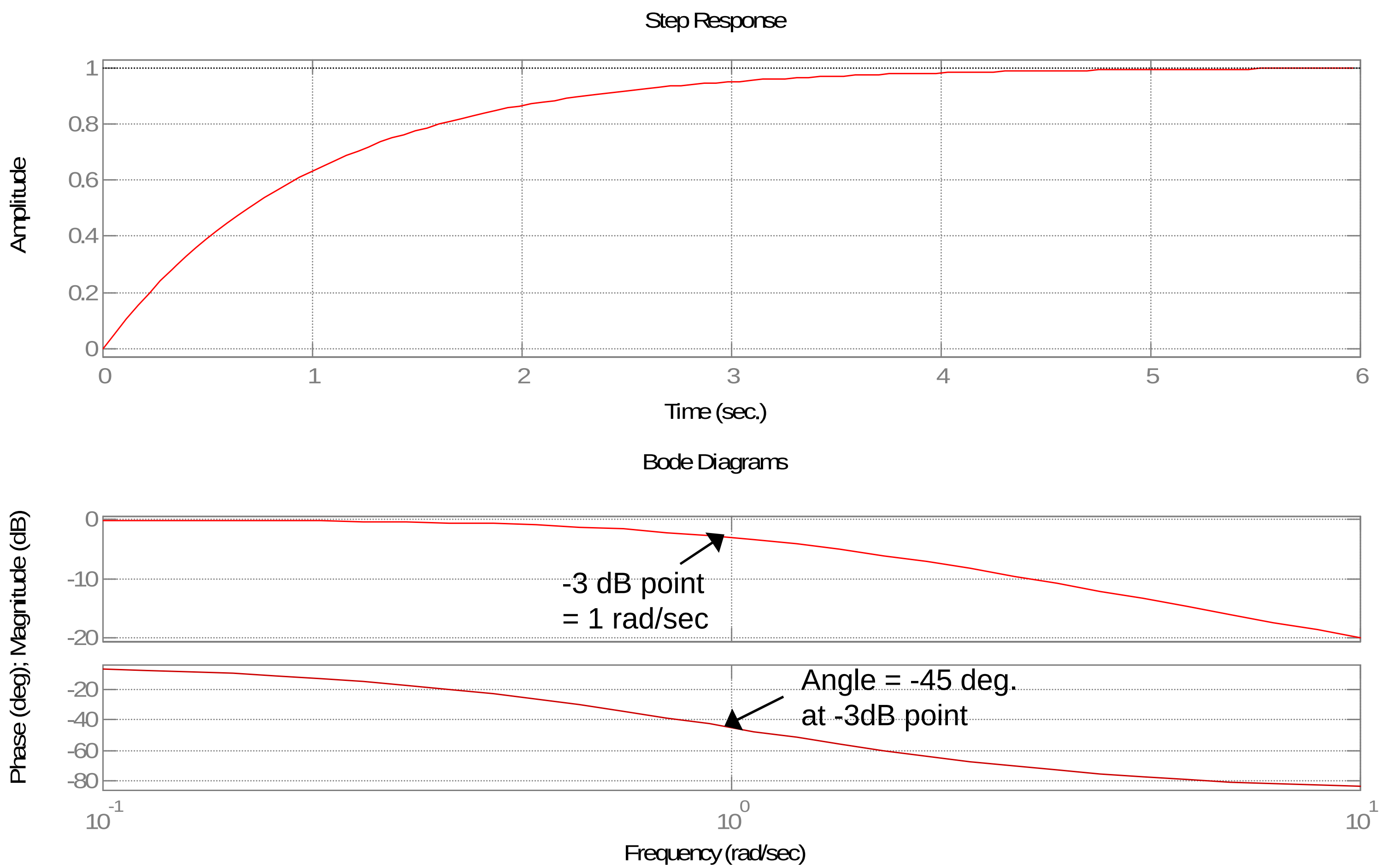
Relationship Between Risetime and Bandwidth

- Exact for a first-order system:

$$\tau_R = \frac{0.35}{f_h}$$

- Approximate for higher-order systems

First-Order System Step and Frequency Response



"Group Delay"

$$H(s) = \frac{1}{\tau s + 1}$$

$$H(j\omega) = \frac{1}{j\omega\tau + 1}$$

$$\left|H(j\omega)\right| = \frac{1}{\sqrt{(\omega\tau)^2 + 1}}$$

$$\angle H(j\omega) = -\tan^{-1}\left(\omega\tau\right)$$

$$G(j\omega) = \frac{-d\angle H(j\omega)}{d\omega} = \frac{\tau}{1 + \left(\omega\tau\right)^2}$$

Group delay is a measure of how much time delay the frequency components in a signal undergo. Mathematically, the group delay of a system is the negative derivative of the phase with respect to omega. To find group delay for the first-order system, we make use of the identity:

$$\frac{d}{dx}\left(\tan^{-1} u\right) = \frac{1}{1 + u^2}\frac{du}{dx}$$

First-Order System --- Low and High Frequency Behavior

- Closed-form solution for frequency response:

$$H(s) = \frac{1}{\tau s + 1}$$

$$H(j\omega) = \frac{1}{j\omega\tau + 1}$$

$$\left|H(j\omega)\right| = \frac{1}{\sqrt{(\omega\tau)^2 + 1}}$$

$$\angle H(j\omega) = -\tan^{-1}\left(\omega\tau\right)$$

- How does this response behave at frequencies well above and well below the pole frequency ?

First-Order System --- High Frequency Behavior

For high frequencies, where $\omega\tau >> 1$

$\tan^{-1}(x) = \pi/2 - 1/x + 1/(3x^3) - \ldots$ for $x > 1$

$$\left|H(j\omega)\right|_{\omega\tau>>1} \approx \frac{1}{\omega\tau}$$

$$\angle H(j\omega)_{\omega\tau>>1} \approx -\frac{\pi}{2} + \frac{1}{\omega\tau}$$

First-Order System --- Low Frequency Behavior

<u>For low frequencies, where $\omega\tau << 1$</u>

$\tan^{-1}(x) = x - x^3/3 + x^5/5 - \ldots$ for $x < 1$ and $\frac{1}{\sqrt{1+x}} \approx 1 - \frac{x}{2}$ for $x << 1$

$$\left|H(j\omega)\right|_{\omega\tau<<1} \approx 1 - \frac{1}{2}(\omega\tau)^2 \approx 1$$

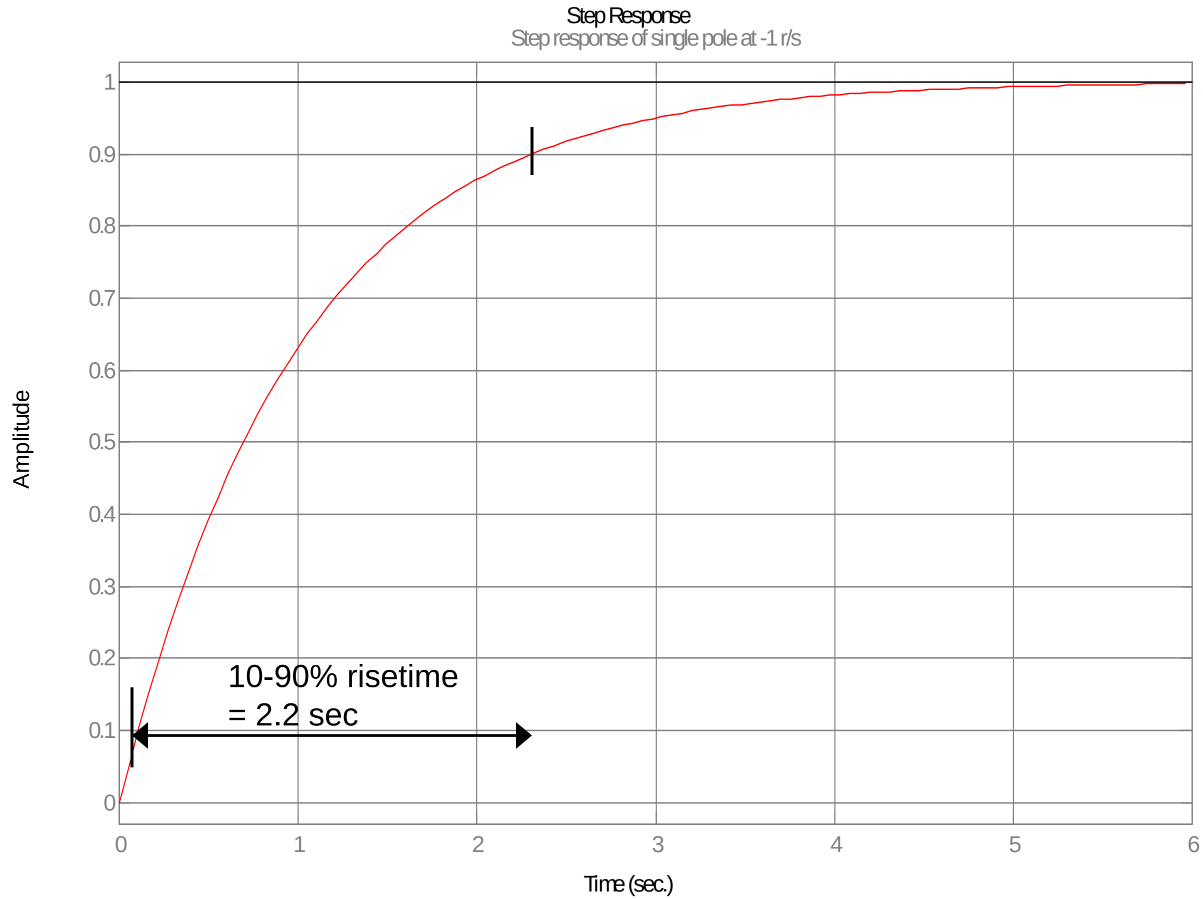
$$\angle H(j\omega)_{\omega\tau<<1} \approx -\omega\tau$$

Well below the pole frequency, the magnitude is approximately 1. The phase is approximately linear phase, behaving like an approximate time delay. "Group delay" is negative derivative of angle with respect to frequency, or:

$$G(j\omega)_{\omega\tau<<1} \approx \tau$$

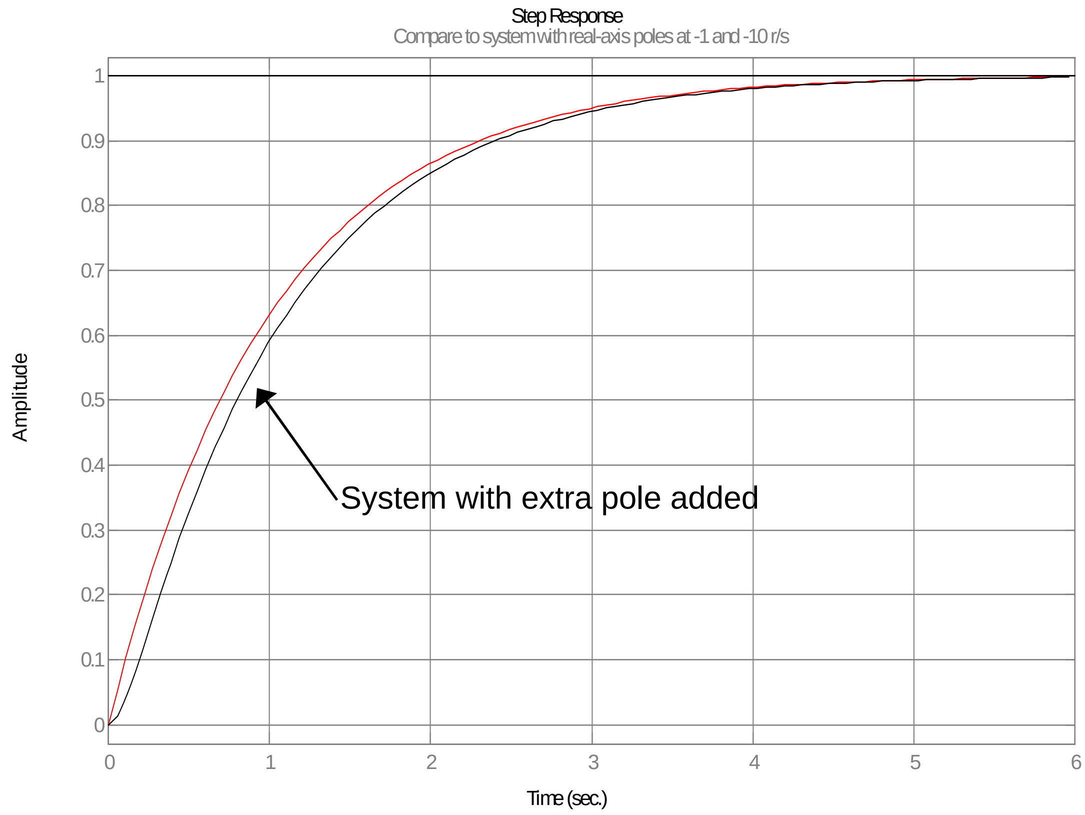
Step Response of Single Pole

- Single pole at -1 rad/sec.
- Note that risetime = 2.2 sec



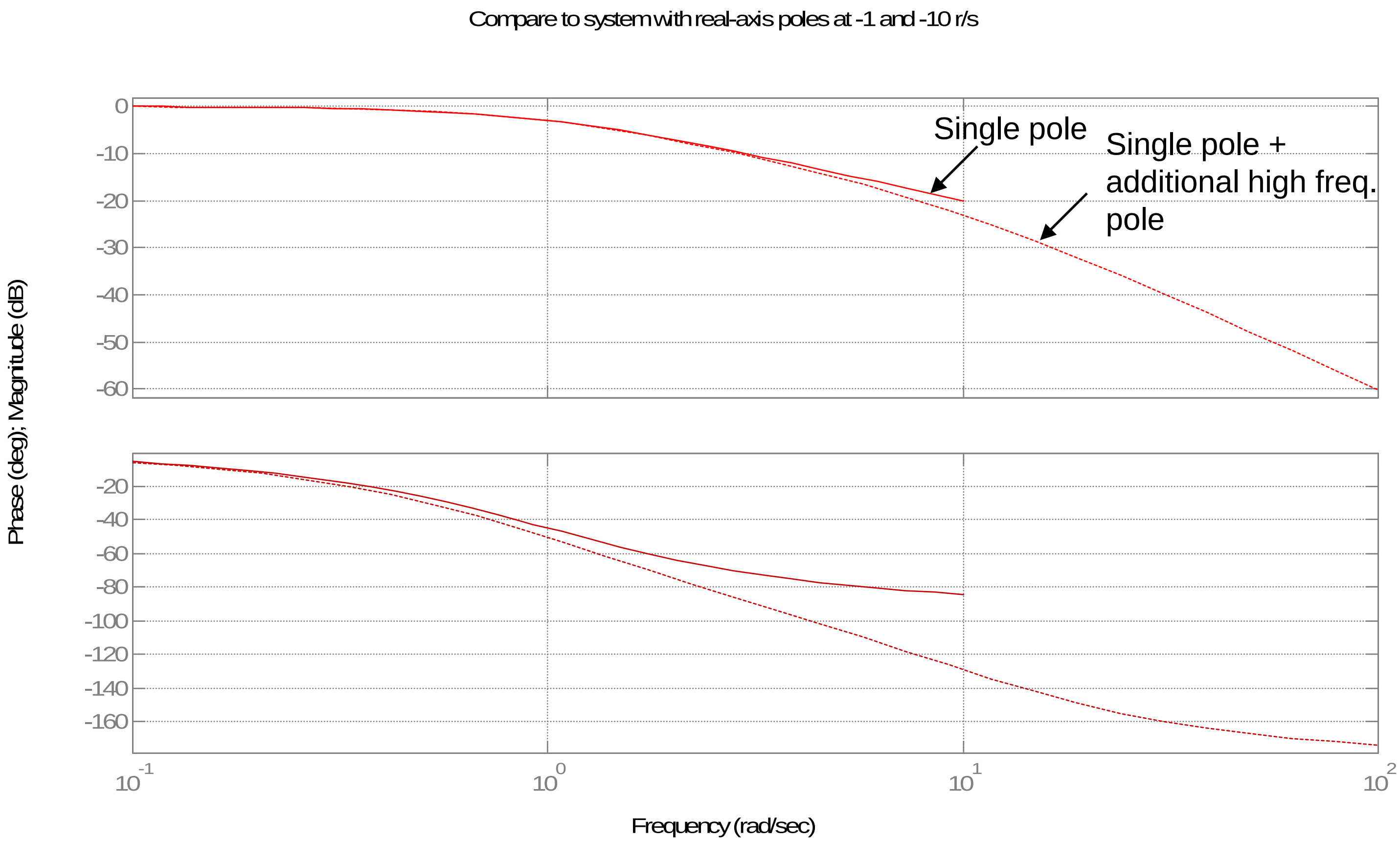
Step Response of Single Pole with High Frequency Pole Added

- Poles at -1 rad/sec. and -10 rad/sec.
- Note the time delay of approximately 0.1 sec.

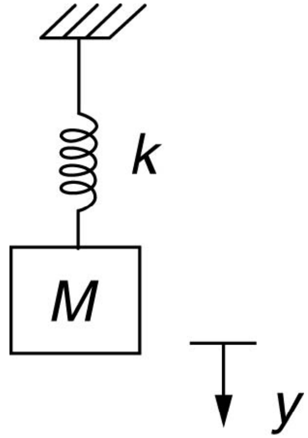


Bode Plot Single Pole with High Frequency Pole Added

- Poles at -1 rad/sec. and -10 rad/sec.



Second-Order Mechanical System



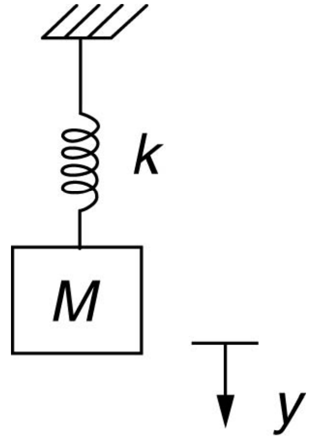
Spring force: $f_y = -ky$

Newton's law for moving mass: $f_y = -ky = M\frac{d^2y}{dt^2}$

Differential equation for mass motion: $M\frac{d^2y}{dt^2} + ky = 0$

Guess a solution of the form: $y(t) = Y_o \sin(\omega t)$

Second-Order Mechanical System



$$y(t) = Y_o \sin(\omega t)$$

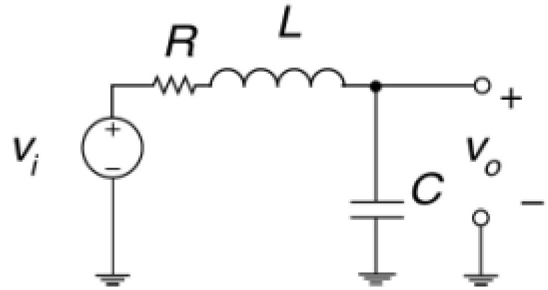
Put this proposed solution into the differential equation:

$$M\left(-\omega^2 Y_o \sin(\omega t)\right) + k\left(Y_o \sin(\omega t)\right) = 0$$

This solution works if:

$$\omega = \sqrt{\frac{k}{M}}$$

Second-Order Electrical System



$$H(s) = \frac{v_o(s)}{v_i(s)} = \frac{\frac{1}{Cs}}{R + Ls + \frac{1}{Cs}} = \frac{1}{LCs^2 + RCs + 1} = \frac{1}{\frac{s^2}{\omega_n^2} + \frac{2\zeta s}{\omega_n} + 1} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

Natural frequency $\omega_n = \frac{1}{\sqrt{LC}}$

Damping ratio $\varsigma = \frac{\omega_n RC}{2} = \frac{1}{2}\frac{R}{\sqrt{\frac{L}{C}}} = \frac{1}{2}\frac{R}{Z_o}$

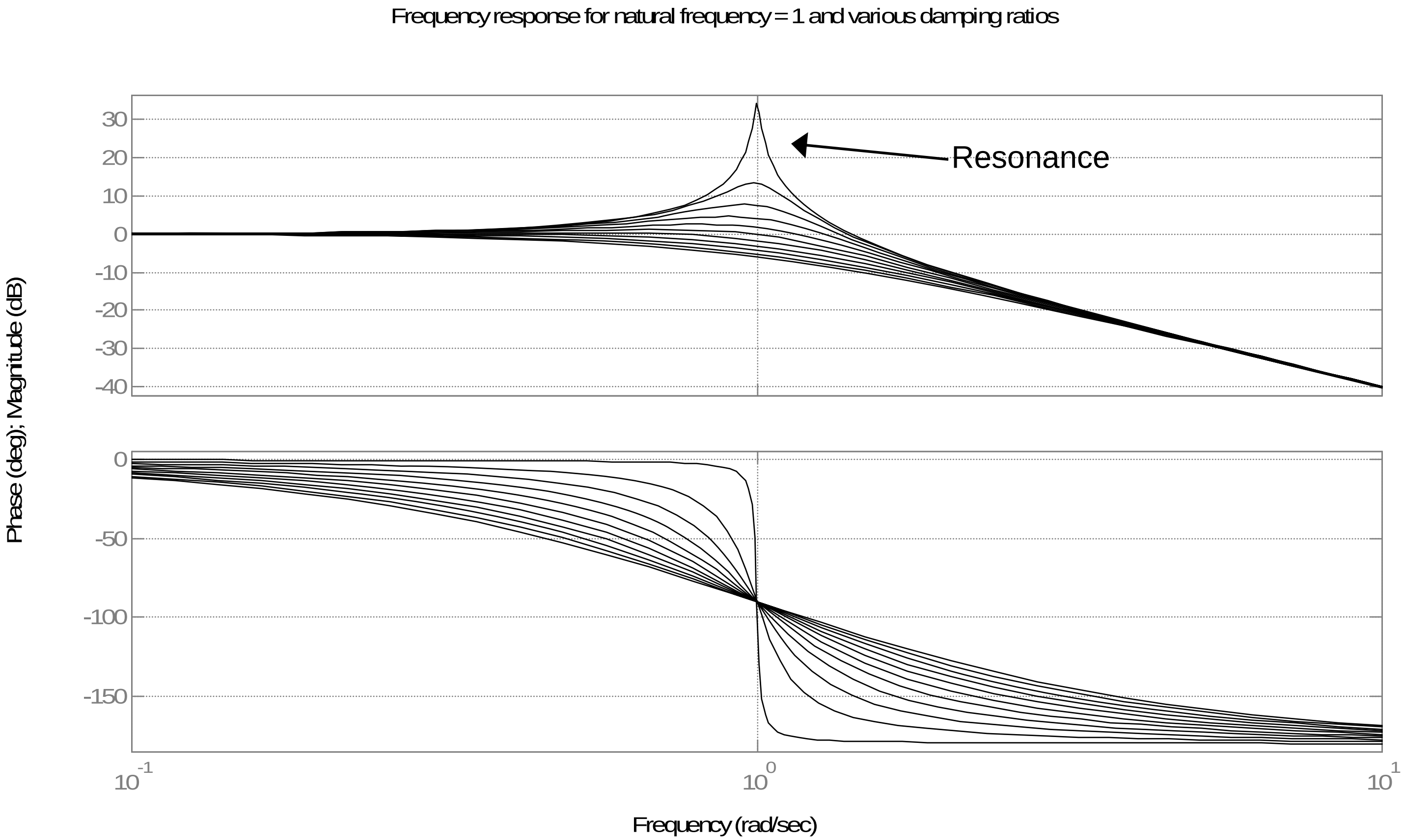
Second-Order System Frequency Response

$$H(j\omega) = \frac{1}{\dfrac{2j\varsigma\omega}{\omega_n} + \left(1 - \dfrac{\omega^2}{\omega_n^2}\right)}$$

$$\left|H(j\omega)\right| = \frac{1}{\sqrt{\left(\dfrac{2\varsigma\omega}{\omega_n}\right)^2 + \left(1 - \dfrac{\omega^2}{\omega_n^2}\right)^2}}$$

$$\angle H(j\omega) = -\tan^{-1}\frac{\left(\dfrac{2\varsigma\omega}{\omega_n}\right)}{\left(1 - \dfrac{\omega^2}{\omega_n^2}\right)} = -\tan^{-1}\left(\frac{2\varsigma\omega\omega_n}{\omega_n^2 - \omega^2}\right)$$

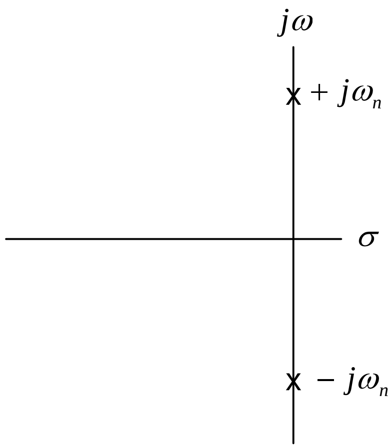
Second-Order System Frequency Response



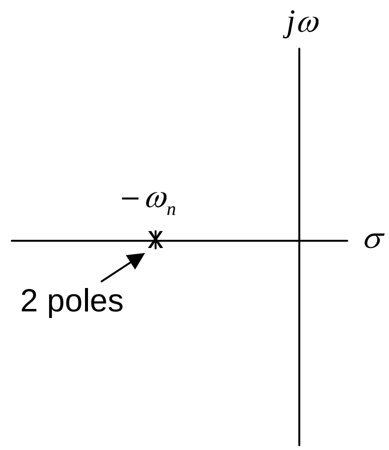
$$\omega_p = \omega_n \sqrt{1-2\varsigma^2} \quad for\ \varsigma < 0.707$$

$$M_p = \frac{1}{2\varsigma\sqrt{1-\varsigma^2}} \quad for\ \varsigma < 0.707$$

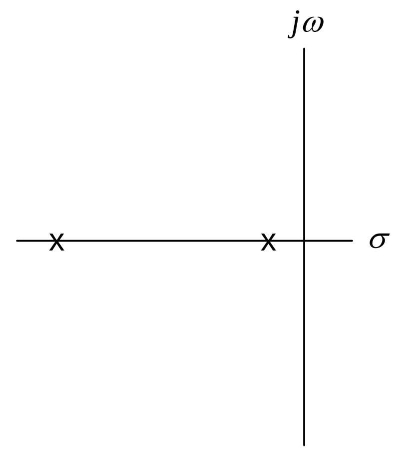
Resonant Circuit --- Pole/Zero Plots



Zero damping
ζ=0



Critical damping
ζ=1



Overdamped
ζ>>1

Second-Order System Frequency Response at Natural Frequency

- Now, what happens if we excite this system exactly at the natural frequency, or $\omega = \omega_n$. ? The response is:

$$\left|H(s)\right|_{\omega=\omega_n} = \frac{1}{2\varsigma}$$

$$\angle H(s)_{\omega=\omega_n} = -\frac{\pi}{2}$$

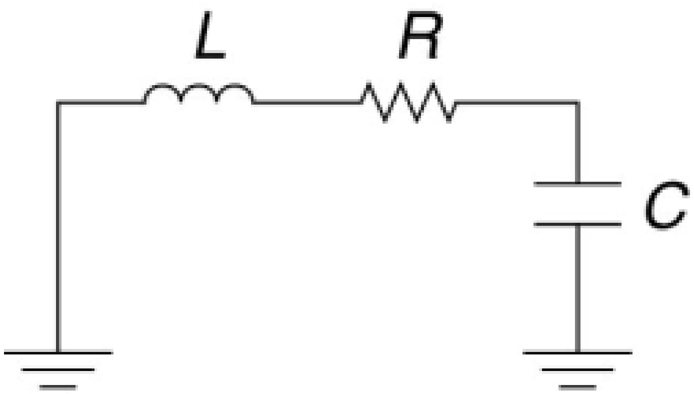
Quality Factor, or “Q”

Quality factor is defined as:

$$\omega \frac{E_{stored}}{P_{diss}}$$

where E_{stored} is the peak stored energy in the system and P_{diss} is the average power dissipation.

Q of Series Resonant RLC



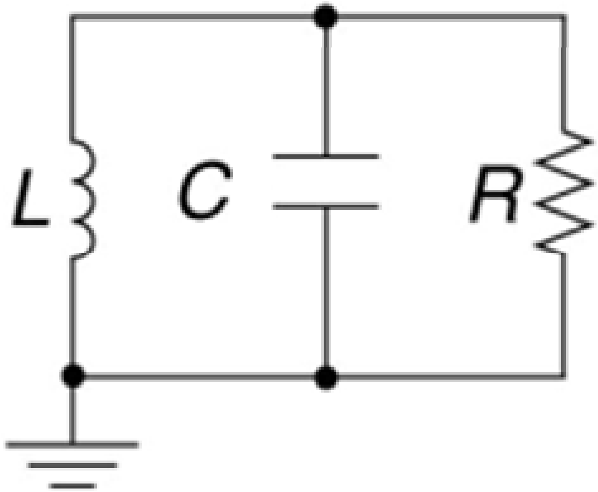
$$\omega = \frac{1}{\sqrt{LC}}$$

$$E_{stored} = \frac{1}{2} L I_{pk}^2$$

$$P_{diss} = \frac{1}{2} I_{pk}^2 R$$

$$Q = \left(\frac{1}{\sqrt{LC}} \right) \left(\frac{\frac{1}{2} L I_{pk}^2}{\frac{1}{2} I_{pk}^2 R} \right) = \frac{\sqrt{\frac{L}{C}}}{R} = \frac{Z_o}{R}$$

Q of Parallel Resonant RLC



$$\omega = \frac{1}{\sqrt{LC}}$$

$$E_{stored} = \frac{1}{2} C V_{pk}^2$$

$$P_{diss} = \frac{1}{2} \frac{V_{pk}^2}{R}$$

$$Q = \left(\frac{1}{\sqrt{LC}} \right) \left(\frac{\frac{1}{2} C V_{pk}^2}{\frac{1}{2} \frac{V_{pk}^2}{R}} \right) = \frac{R}{\sqrt{\frac{L}{C}}} = \frac{R}{Z_o}$$

Relationship Between Damping Ratio and "Quality Factor" Q

- A second order system can also be characterized by it's "Quality Factor" or Q.

$$\left|H(s)\right|_{\omega=\omega_n} = \frac{1}{2\varsigma} = Q$$

- Use Q in transfer function of series resonant circuit:

$$H(s) = \frac{1}{\frac{s^2}{\omega_n^2} + \frac{2\zeta s}{\omega_n} + 1} = \frac{1}{\frac{s^2}{\omega_n^2} + \frac{s}{\omega_n Q} + 1}$$

Series Resonant Circuit at Resonance

The magnitude of this transfer function is:

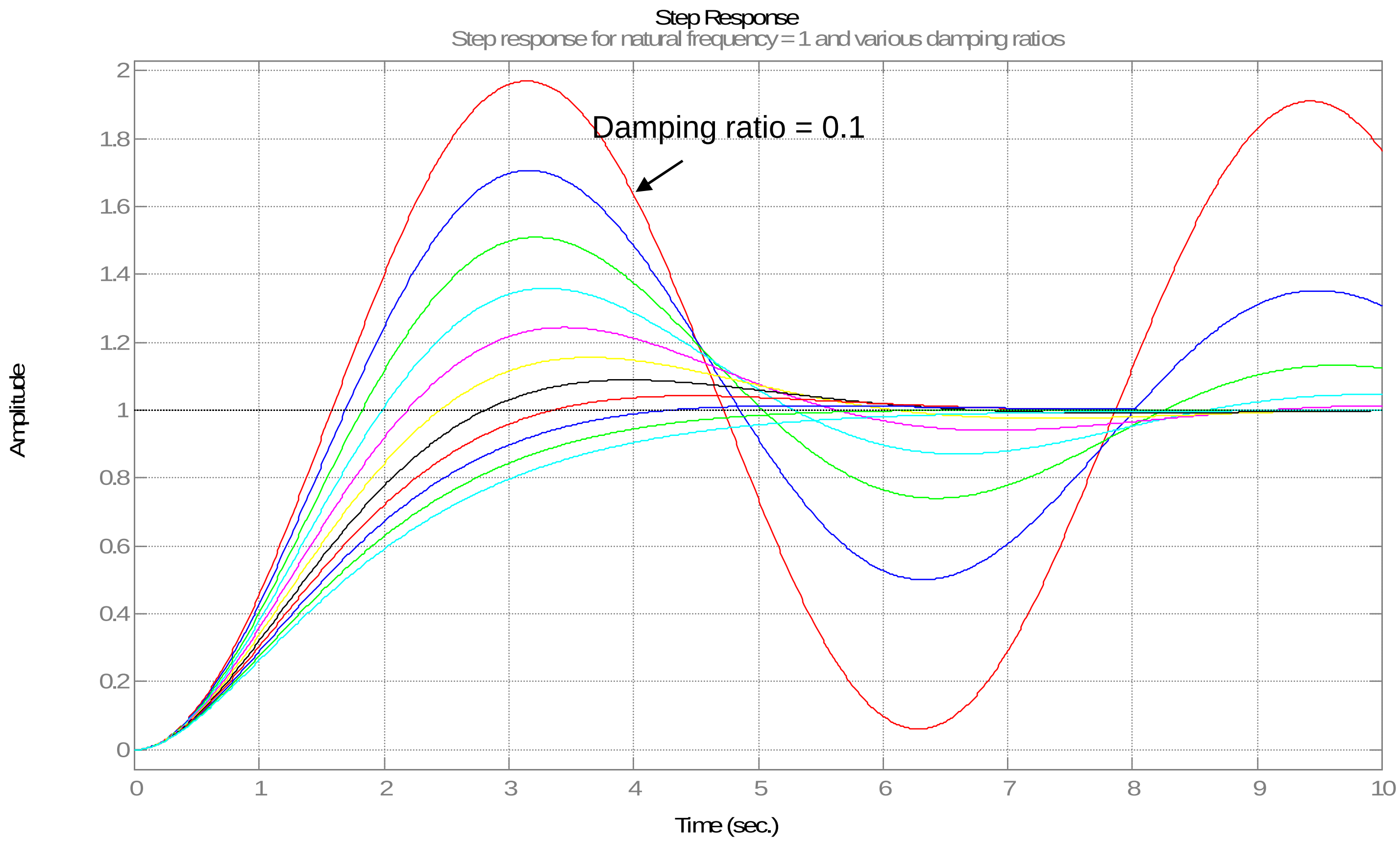
$$\left|H(s)\right| = \frac{1}{\sqrt{\left(1-\frac{\omega^2}{\omega_n^2}\right)^2 + \left(\frac{\omega}{\omega_n Q}\right)^2}}$$

Exactly at resonance ($\omega = \omega_n$), the magnitude of the transfer function is:

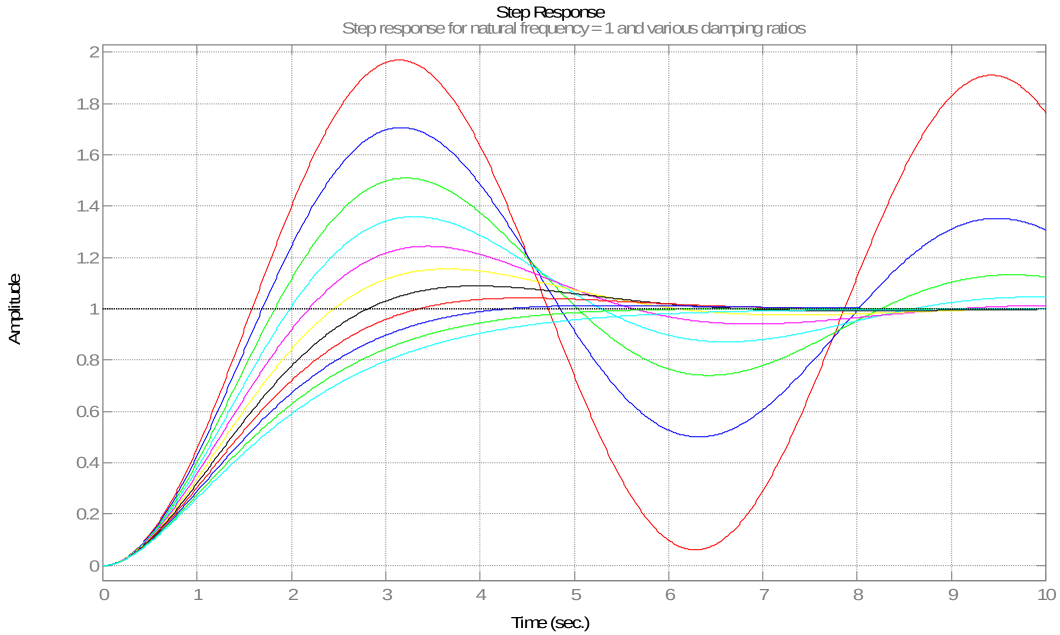
$$\left|H(s)\right|_{\omega=\omega_n} = Q$$

Second-Order System Step Response

- Shown for varying values of damping ratio.



Second-Order System Step Response



$$v_o(t) = 1 - \frac{e^{-\varsigma\omega_n t}}{\sqrt{1-\varsigma^2}} \sin\left(\omega_d t + \tan^{-1}\left(\frac{\sqrt{1-\varsigma^2}}{\varsigma}\right)\right)$$

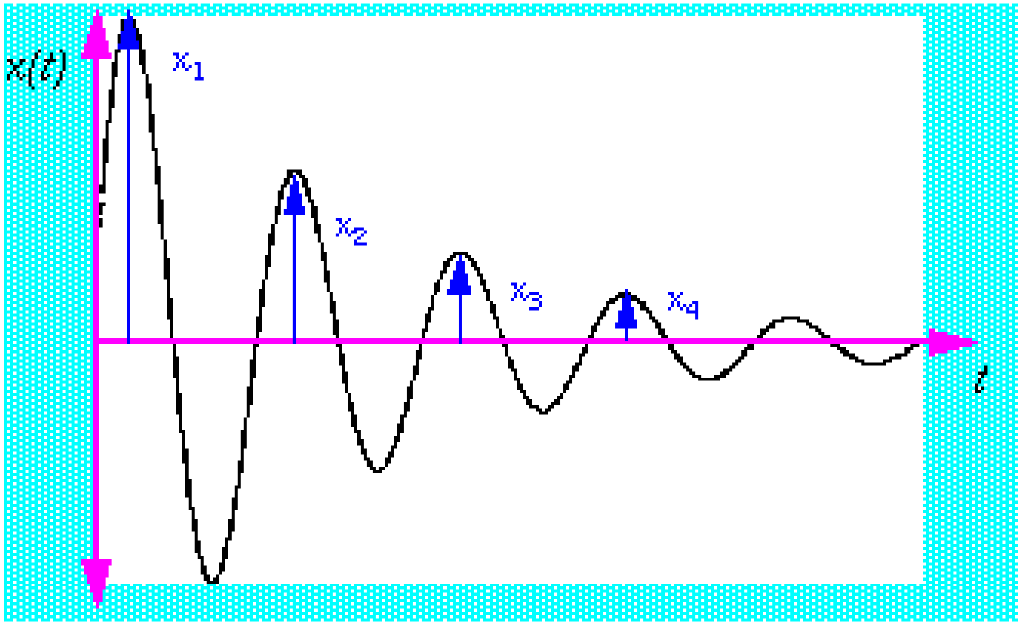
$$\omega_n = \frac{1}{\sqrt{LC}}$$

$$\varsigma = \frac{R}{2\sqrt{\frac{L}{C}}}$$

$$\omega_d = \omega_n\sqrt{1-\varsigma^2}$$ Damped natural frequency

Logarithmic Decrement

- Method of estimating damping ratio based on measurement

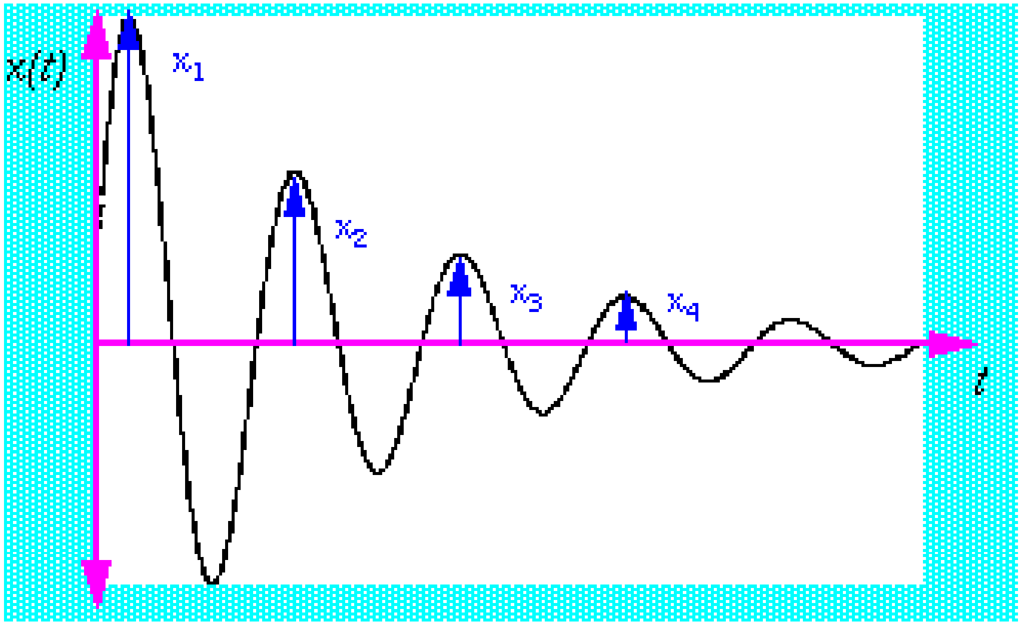


$$\frac{x_1}{x_2} = \frac{Ce^{-\varsigma\omega_n t_1}\sin(\omega_d t_1 + \phi_1)}{Ce^{-\varsigma\omega_n t_2}\sin(\omega_d t_2 + \phi_1)}$$

Logarithmic Decrement (cont.)

- Simplify:

$$\frac{x_1}{x_2} = \frac{e^{-\varsigma\omega_n t_1}}{e^{-\varsigma\omega_n (t_1+T_D)}}$$



- T_D = oscillation period = $(2\pi)/\omega_d$
- Simplify again

$$\frac{x_1}{x_2} = e^{\varsigma\omega_n T_D}$$

- Take log of both sides:

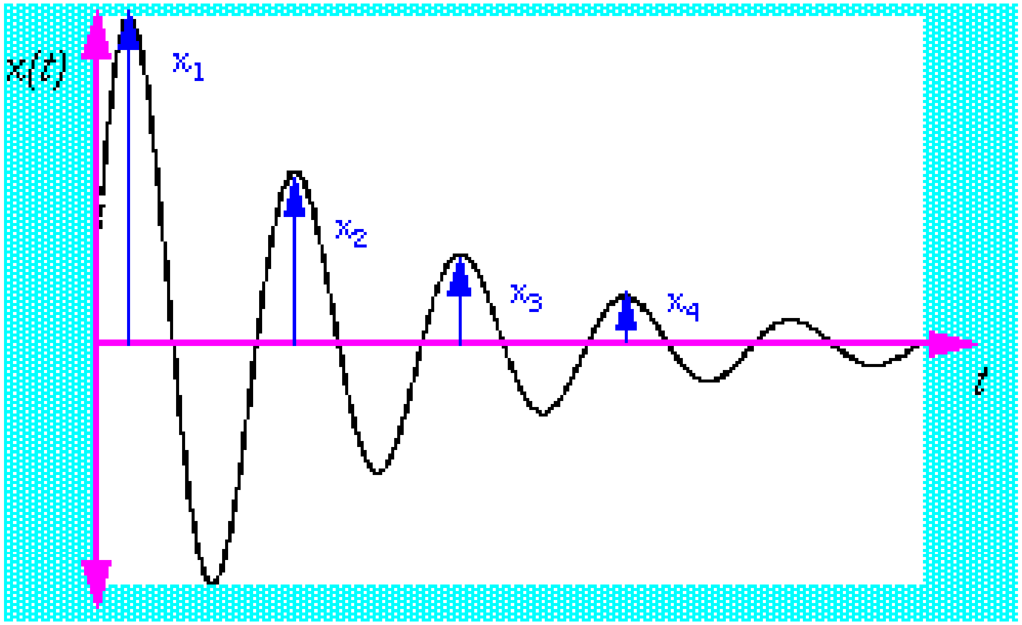
$$\ln\left(\frac{x_1}{x_2}\right) \equiv \delta = \varsigma\omega_n T_D = \varsigma\left(\frac{\omega_d}{\sqrt{1-\varsigma^2}}\right)T_D = \left(\frac{2\pi\varsigma}{\sqrt{1-\varsigma^2}}\right)$$

Logarithmic Decrement (cont.)

- The logarithmic decrement δ is the natural log of the ratio of any two successive oscillation amplitudes.

Logarithmic Decrement (cont.)

- Comment: If we measure how the amplitude decreased cycle-by-cycle, we can find damping ratio
- Example: In a free vibration test, the ratio of amplitudes is 2.5 to 1 on successive oscillations...



$$\delta = \ln\left(\frac{2.5}{1}\right) = 0.916 = \left(\frac{2\pi\varsigma}{\sqrt{1-\varsigma^2}}\right) \Rightarrow \varsigma = 0.145$$

Logarithmic Decrement (cont.)

- If damping ratio is very low,

$$\delta = \ln\left(\frac{x_1}{x_2}\right) = \left(\frac{2\pi\varsigma}{\sqrt{1-\varsigma^2}}\right) \approx 2\pi\varsigma$$

- In the case of low damping, it may not be easy to make measurement of successive oscillations. We can make measurements at time t_1 and at N cycles later. Note that:

$$\frac{x_1}{x_{N+1}} = \left(\frac{x_1}{x_2}\right)\left(\frac{x_2}{x_3}\right)\left(\frac{x_3}{x_4}\right)\bullet\bullet\bullet\left(\frac{x_N}{x_{N+1}}\right)$$

- Take log of both sides

$$\ln\left(\frac{x_1}{x_{N+1}}\right) = \ln\left(\frac{x_1}{x_2}\right) + \ln\left(\frac{x_2}{x_3}\right) + \ln\left(\frac{x_3}{x_4}\right)\bullet\bullet\bullet + \ln\left(\frac{x_N}{x_{N+1}}\right) = N\delta$$

Logarithmic Decrement (cont.)

- Take log of both sides

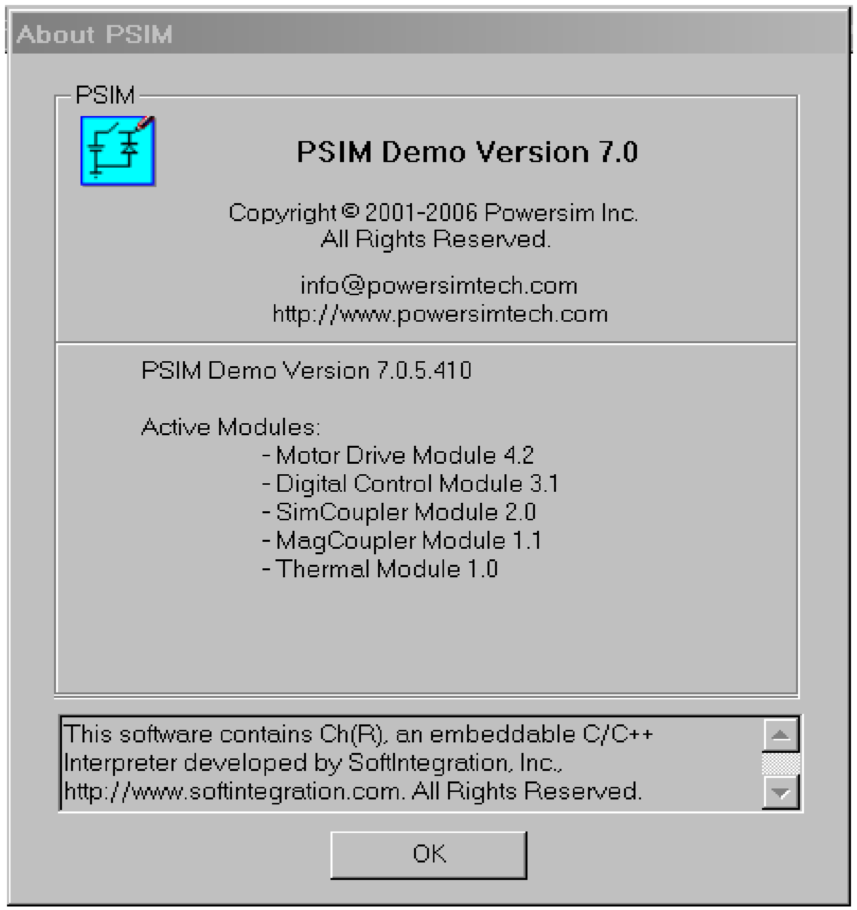
$$\ln\left(\frac{x_1}{x_{N+1}}\right) = \ln\left(\frac{x_1}{x_2}\right) + \ln\left(\frac{x_2}{x_3}\right) + \ln\left(\frac{x_3}{x_4}\right) \bullet\bullet\bullet + \ln\left(\frac{x_N}{x_{N+1}}\right) = N\delta$$

- This means we can find the logarithmic decrement as:

$$\delta = \left(\frac{1}{N}\right)\ln\left(\frac{x_1}{x_{N+1}}\right)$$

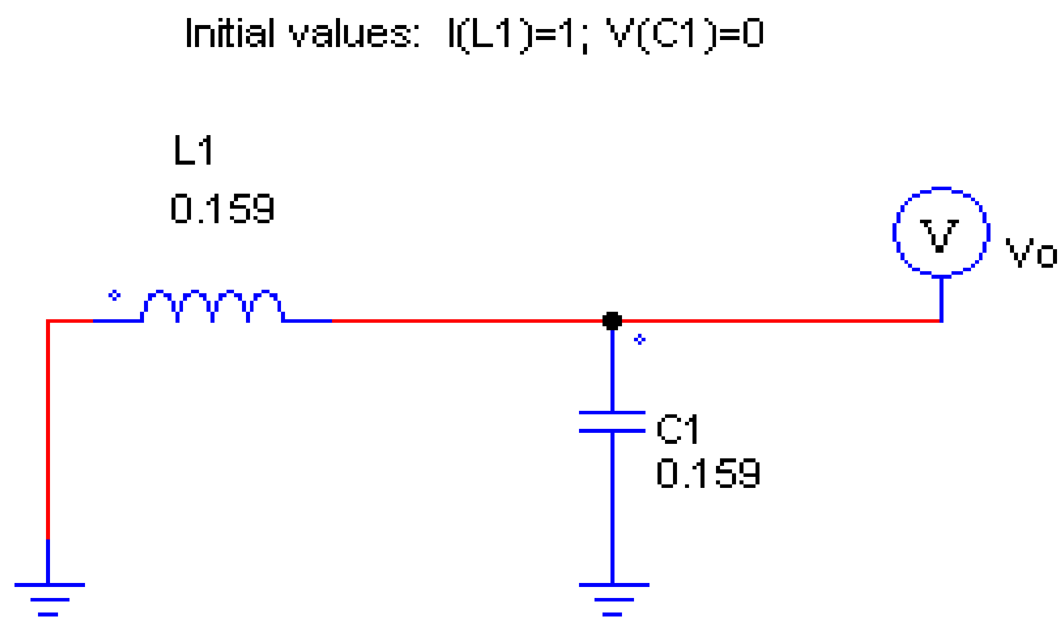
A Potpourri of Resonant Circuits

- Both damped and undamped
- All circuits simulated with PSIM



Example 1: Undamped Resonant Circuit

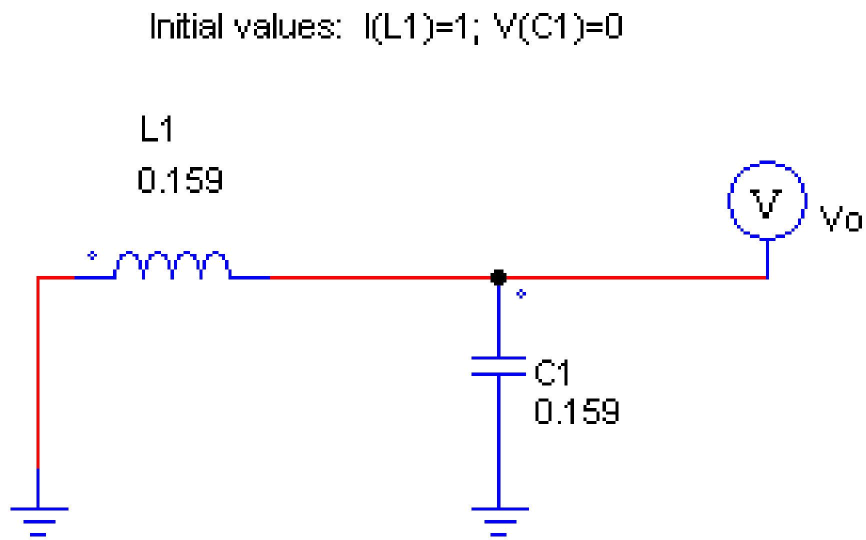
- L = 1/(2π); C = 1/(2π). Initial conditions: capacitor voltage = 0 and inductor current = 1 Amp



PSIM file: Undamped resonant circuit in initial inductor current .sch

Example 1: What Do We Know About This Circuit?

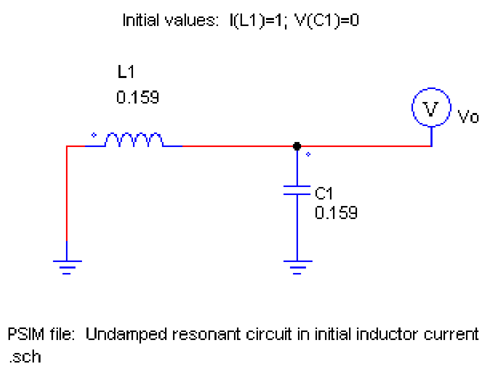
- $L = 1/(2\pi)$; $C = 1/(2\pi)$
- $Z_o = 1$ Ohm
- $\omega_o = 2\pi$ rad/sec; $f_o = 1$ Hz

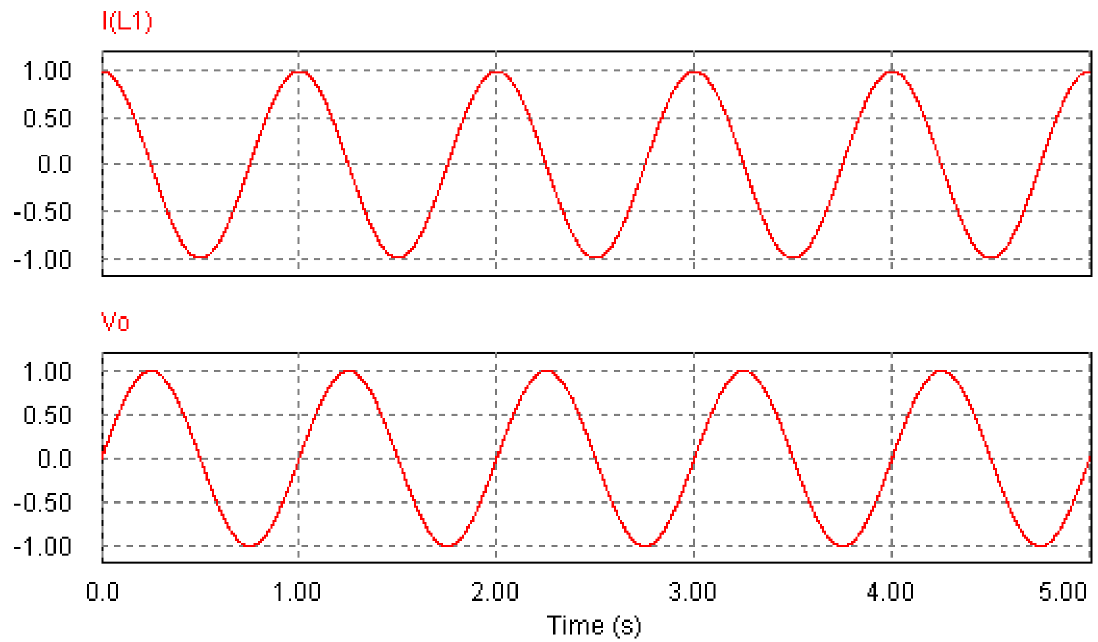


PSIM file: Undamped resonant circuit in initial inductor current .sch

Example 1: Undamped Resonant Circuit Response

- L = 1/(2π); C = 1/(2π)

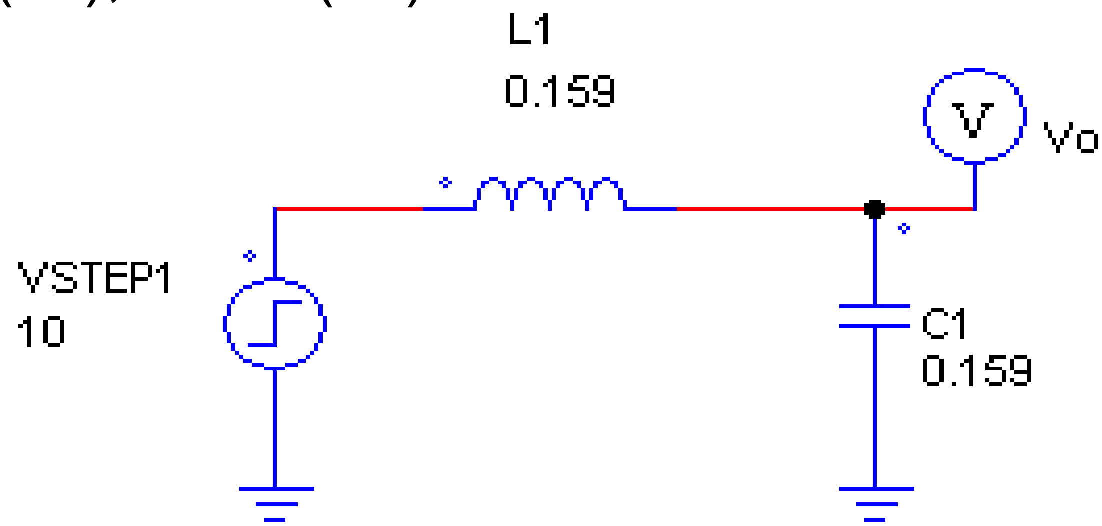




- Note ratio of voltage/current = 1 ohm

Example 1: Series Resonant Step Response

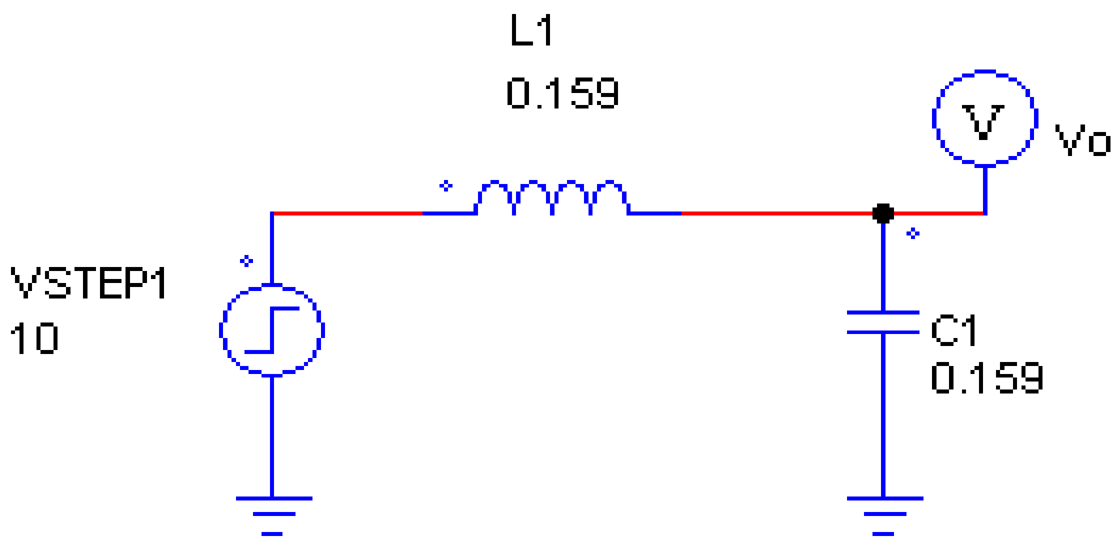
- L = 1/(2π); C = 1/(2π). Assume zero initial conditions



PSIM file: Undamped resonant circuit step response.sch

Example 1: Series Resonant Step Response

- Z_o = 1 Ohm
- Natural frequency: $\omega_o = 2\pi$ rad/sec; f_o = 1 Hz



PSIM file: Undamped resonant circuit step response.sch

Example 1: Series Resonant Step Response

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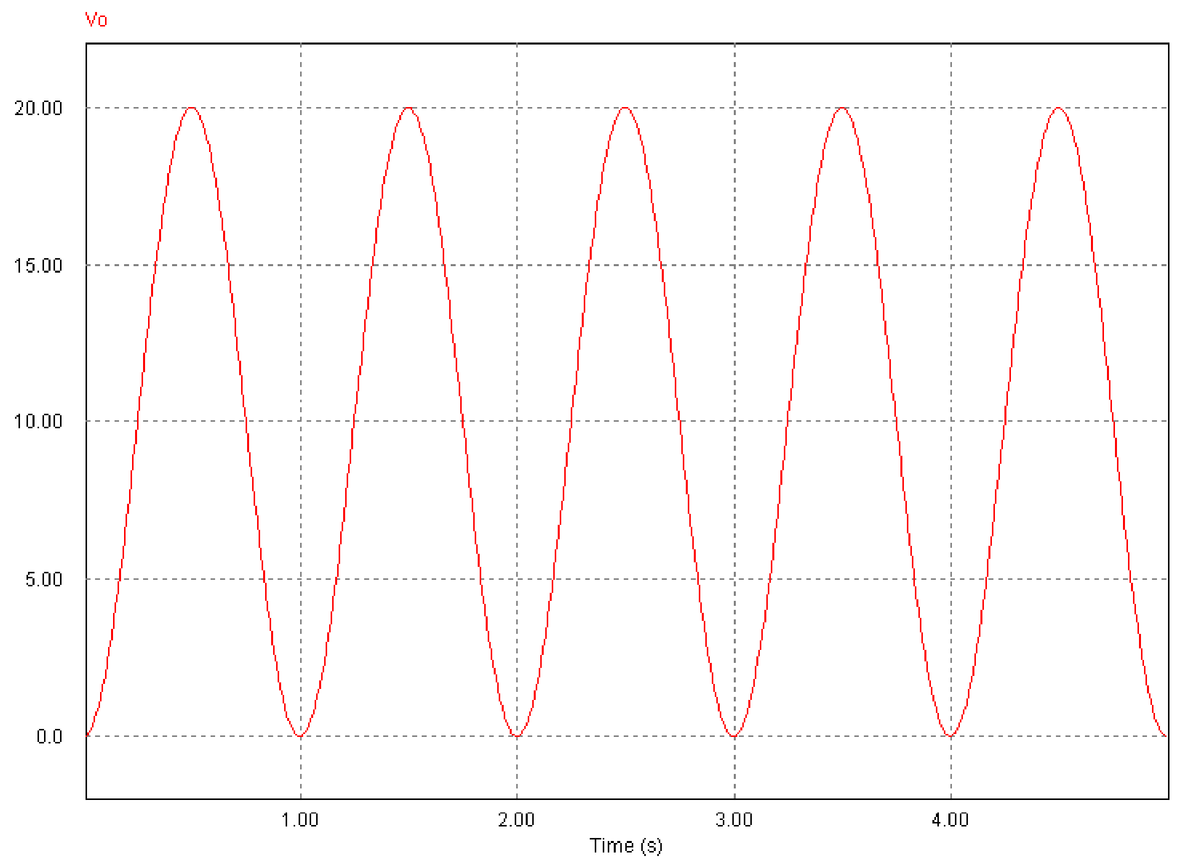
- $\omega_o = 2\pi$ rad/sec; f_o = 1 Hz



Example 1: Series Resonant Step Response

- What is inductor current? Remember Z_o = 1 Ohm

Example 1: Series Resonant Step Response

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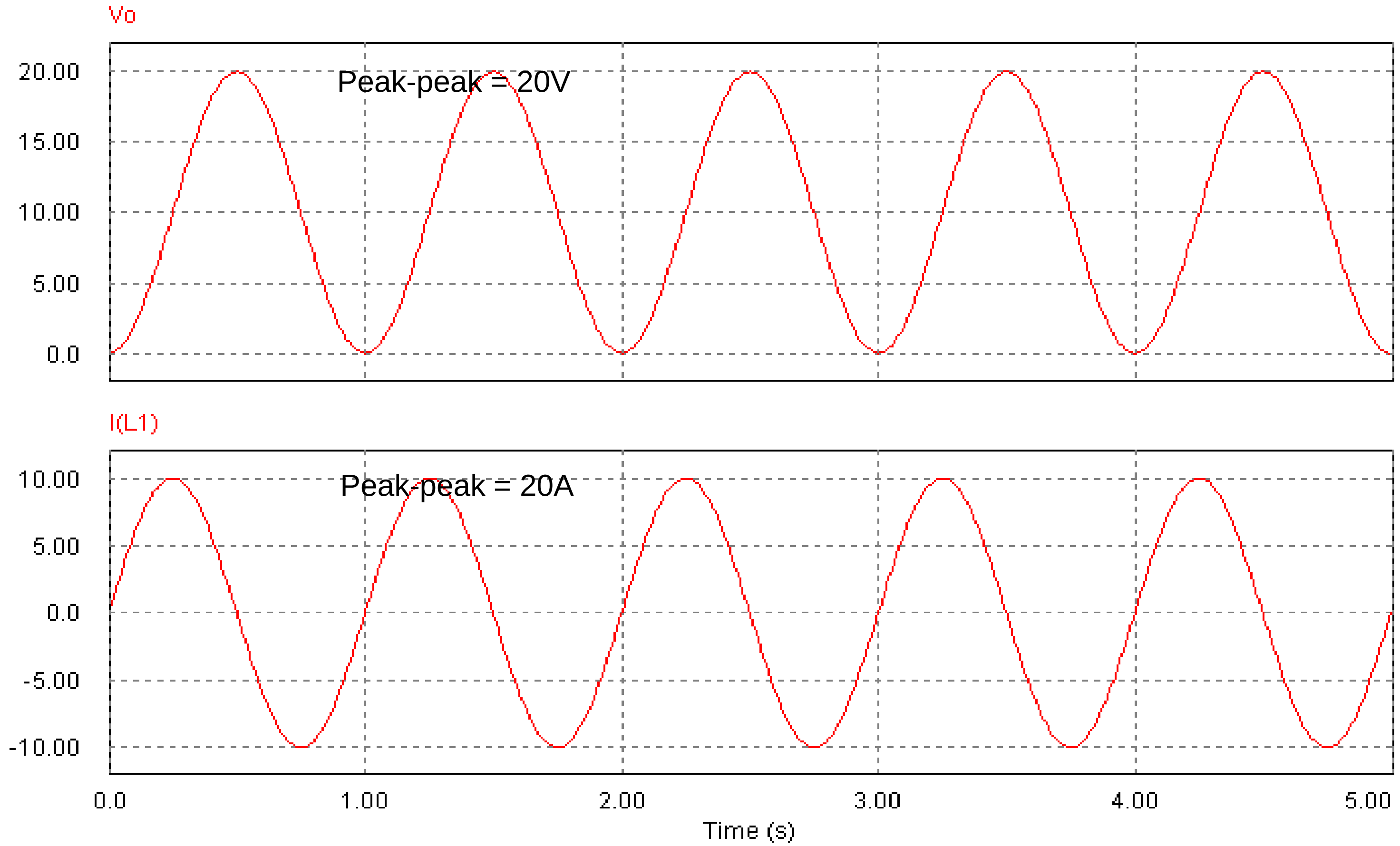
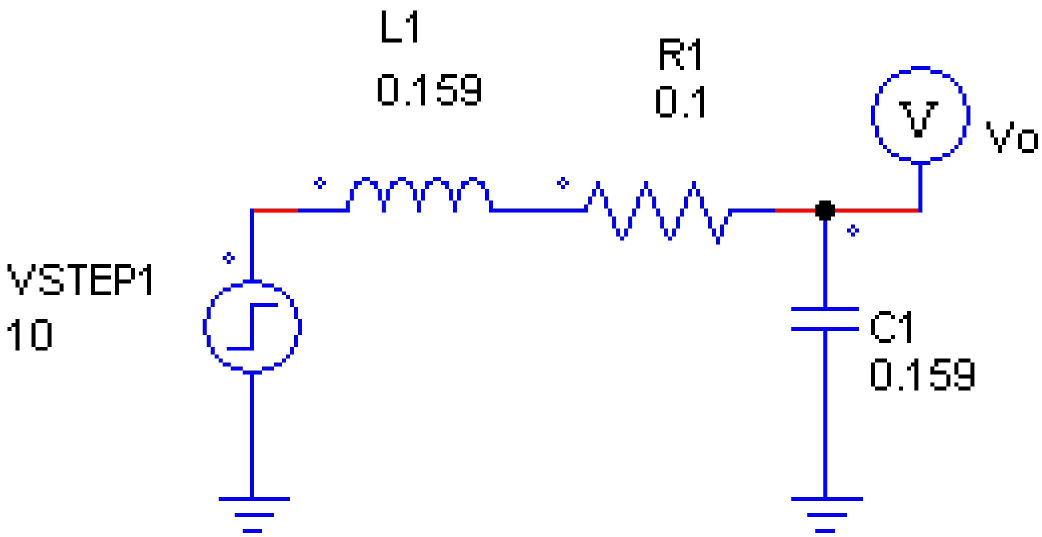


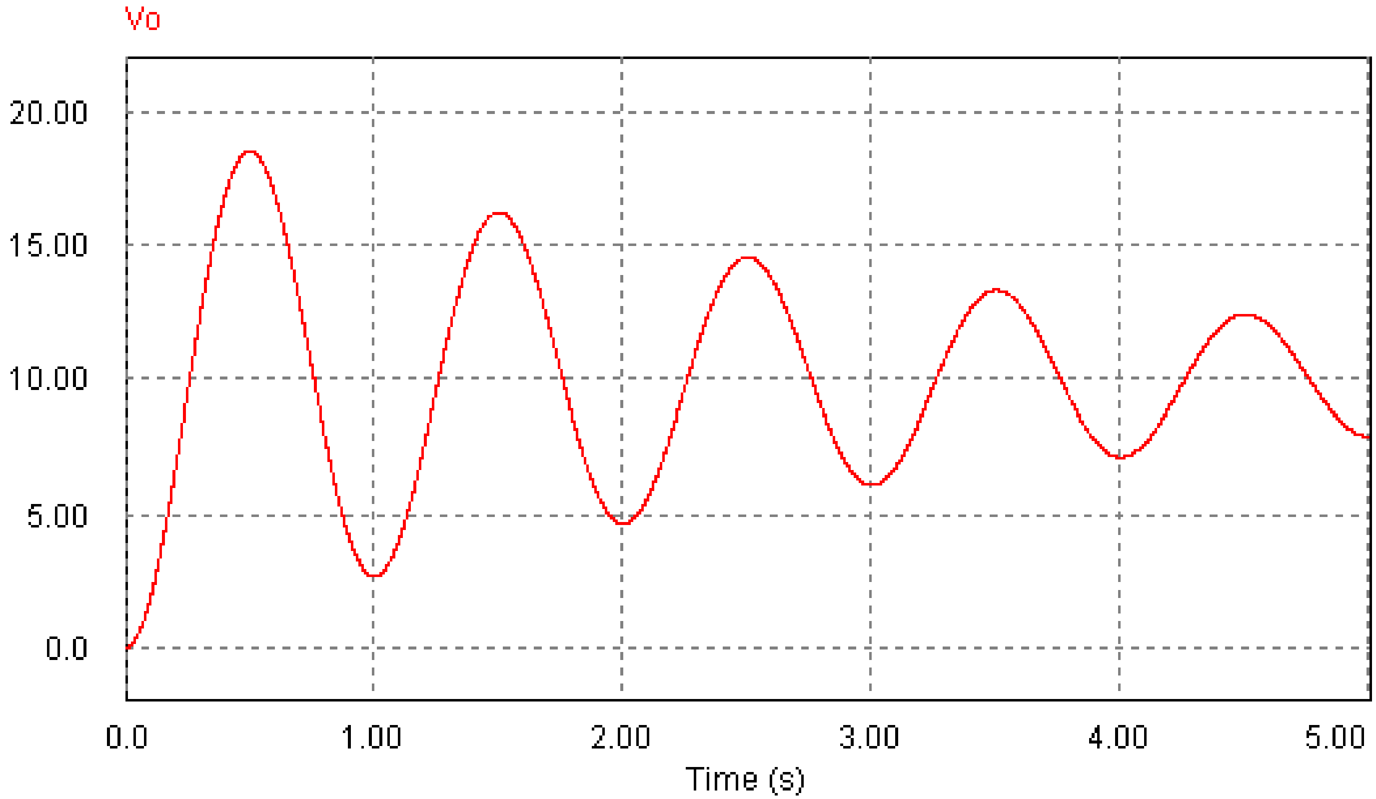
Example 2: Now Add 0.1 Ω Resistor



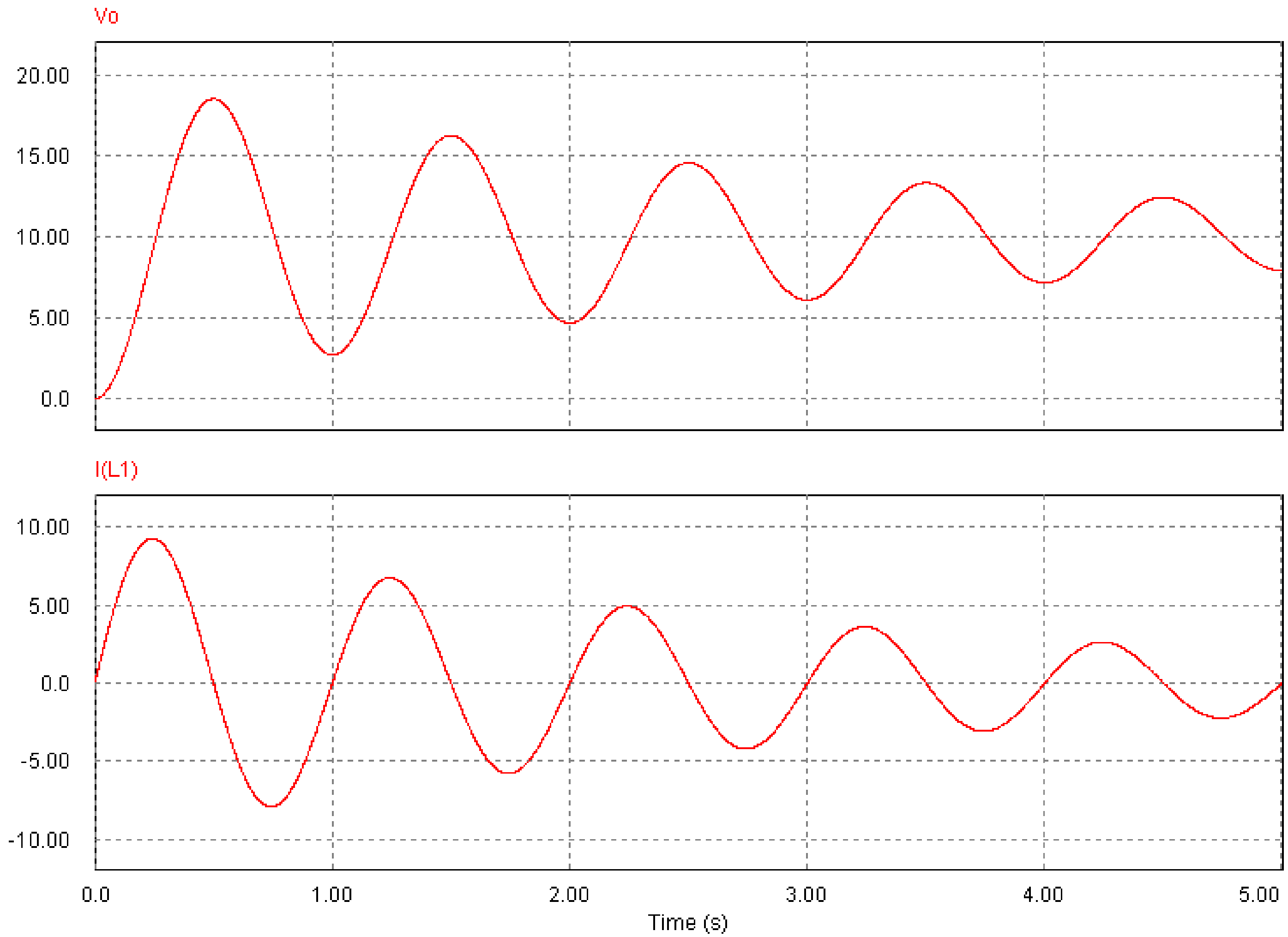
PSIM file: Damped series resonant circuit 1 step response.sch

Example 2: Now Add 0.1 Ω Resistor

- $R << Z_o$, so damping is small

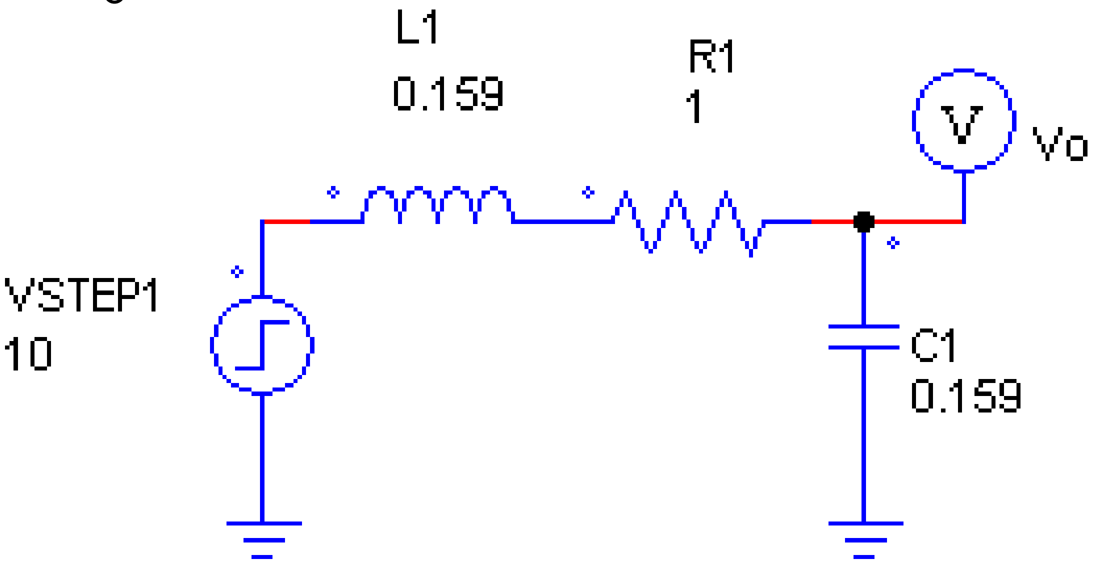


Example 2: Now Add 0.1 Ω Resistor



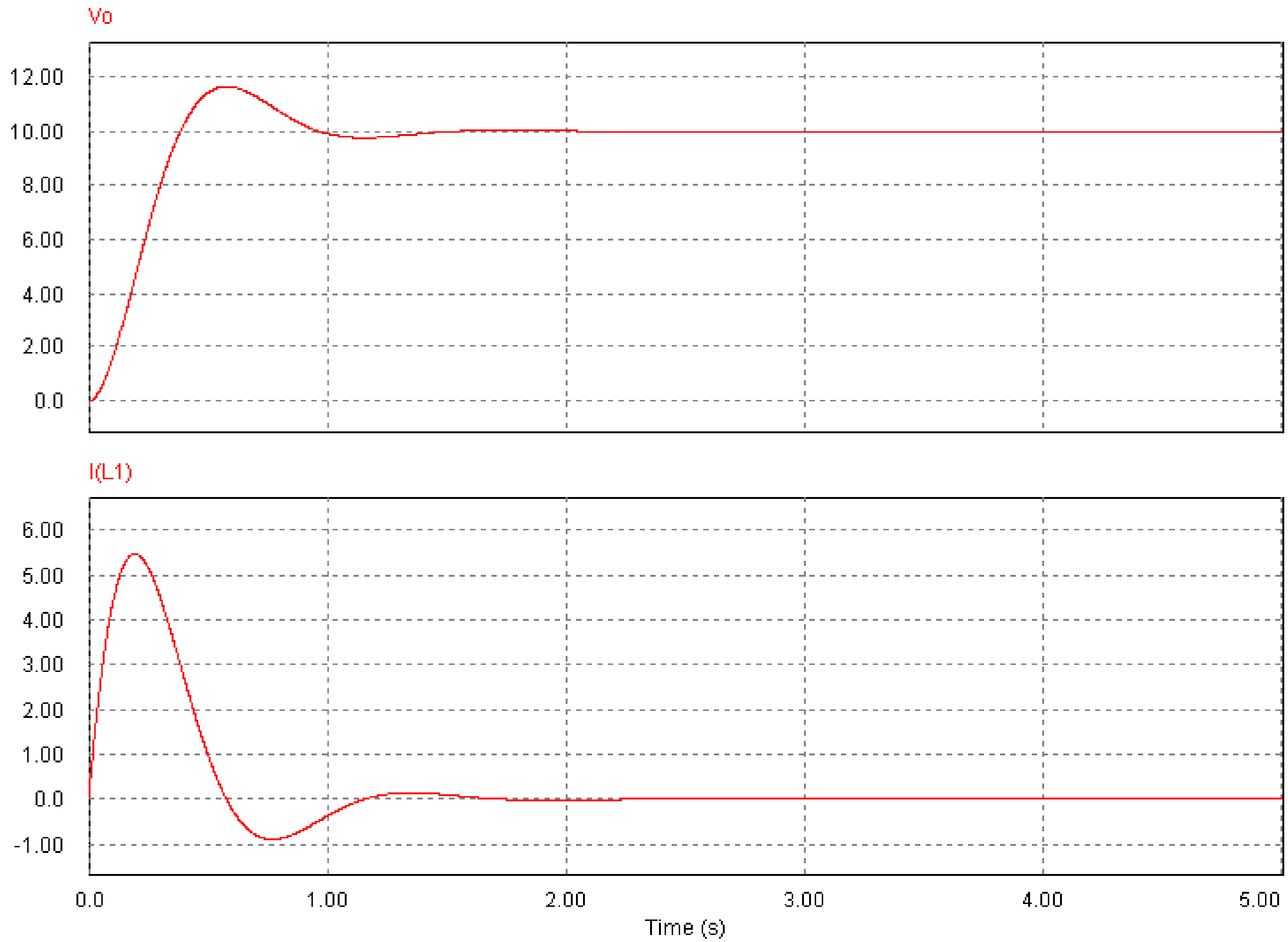
Example 3: Change the 0.1 Ω to 1 Ω

- R = Z_o in this case



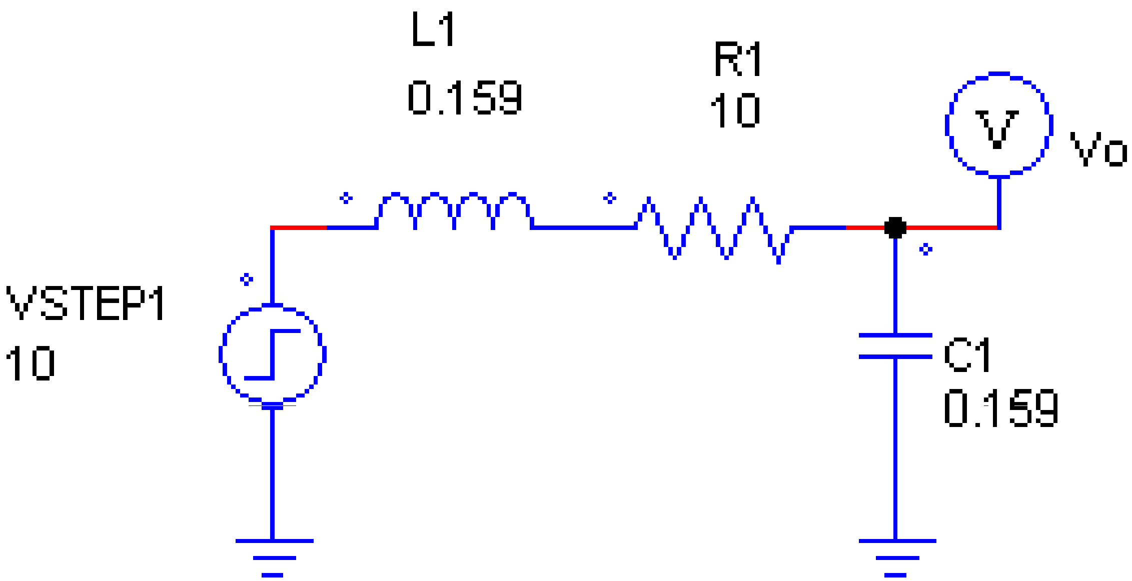
PSIM file: Damped series resonant circuit2 step response.sch

Example 3: Change the 0.1 Ω to 1 Ω



Example 4: Change the 1 Ω to 10 Ω

- R >> Z_o in this case



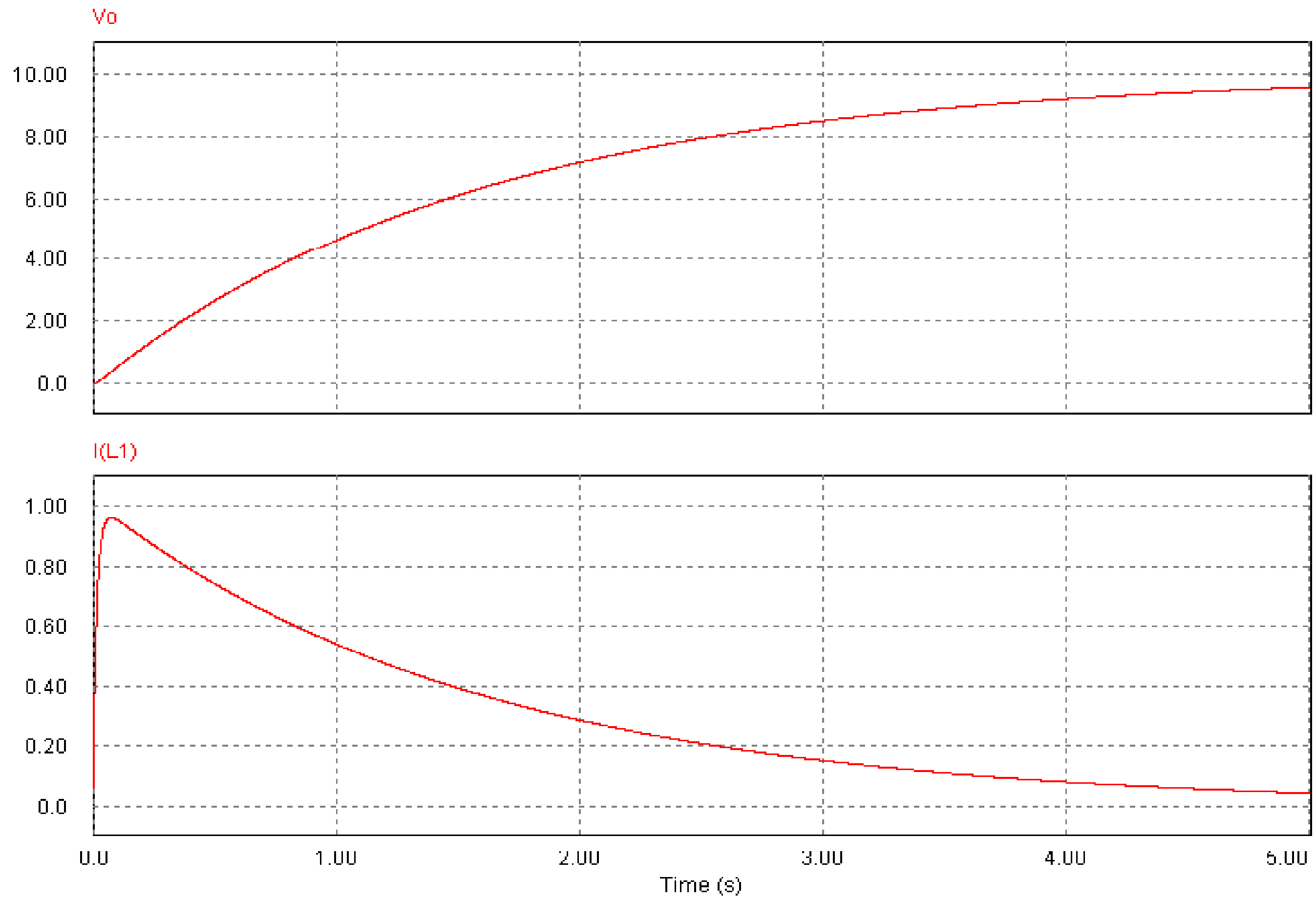
PSIM file: Damped series resonant circuit3 step response.sch

Example 4: Series Resonant Step Response

- What's the initial inductor current, approximately (and why)?

Example 4: Series Resonant Step Response

- What's the initial inductor current, approximately?

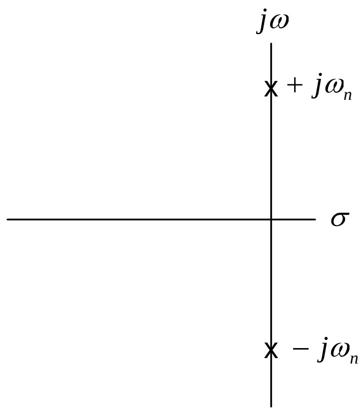


Example 4: Response for R >> Z_o

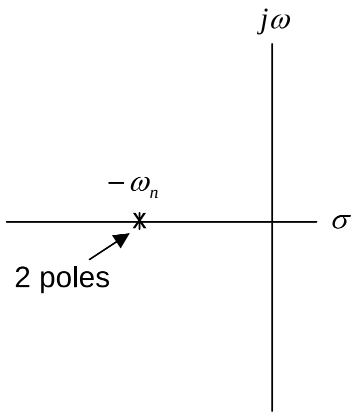
- At low frequencies, this behaves like an RC lowpass filter
- At high frequencies, when capacitor is almost a short, this behaves as a LPF with L/R time constant
- Why is this?

Second Order System --- Pole Location Variation with Damping

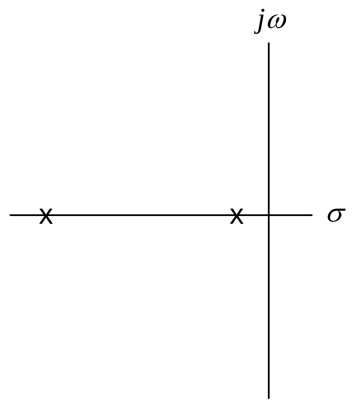
Very underdamped



Critically damped



Overdamped



Overdamped Case

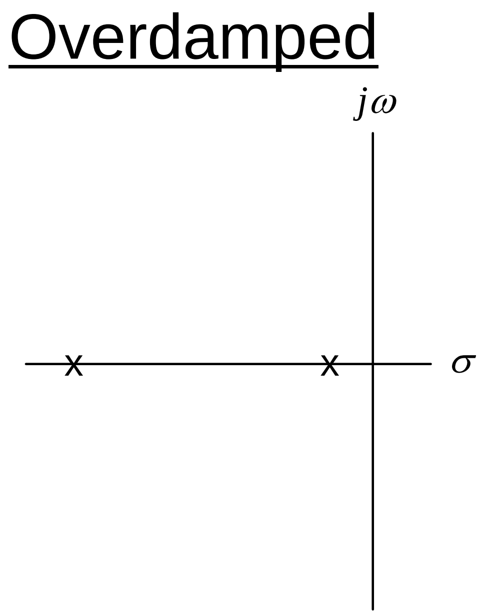
$$H(s) = \frac{1}{LCs^2 + RCs + 1}$$

- At low frequencies, for RCs >> LCs^2, or R >> Ls, then …

$$H(s) \approx \frac{1}{RCs + 1}$$

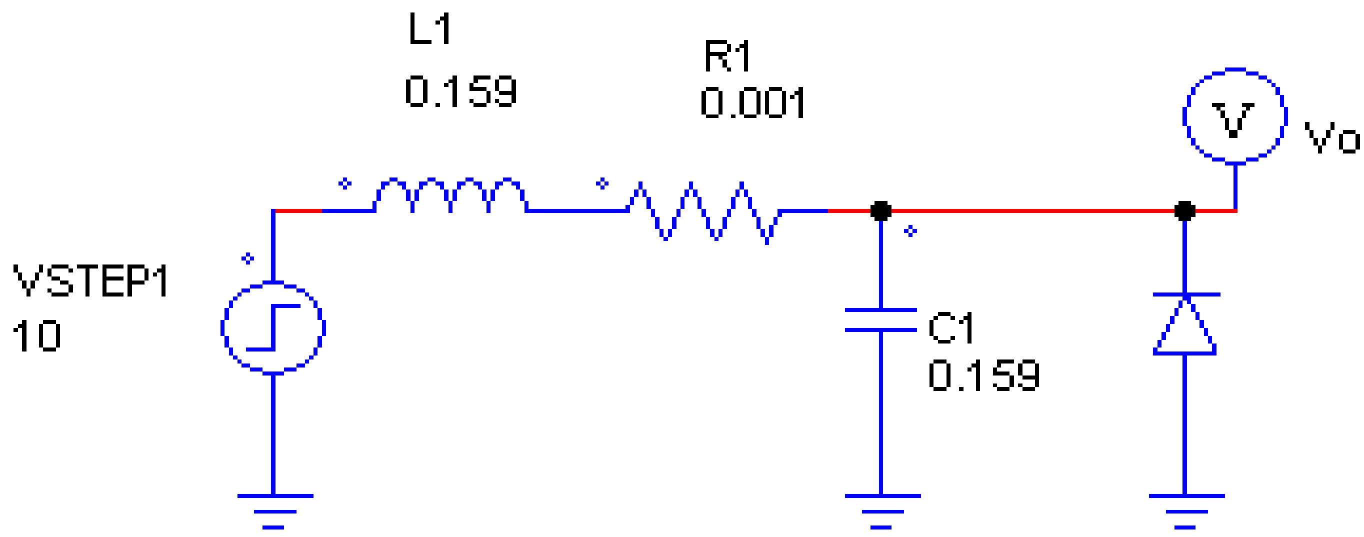
- At high frequencies, with RCs>>1, then

$$H(s) \approx \frac{1}{LCs^2 + RCs} \approx \frac{1}{RCs\left(\frac{L}{R}s + 1\right)}$$



Example 5: Series Resonant Circuit with Diode

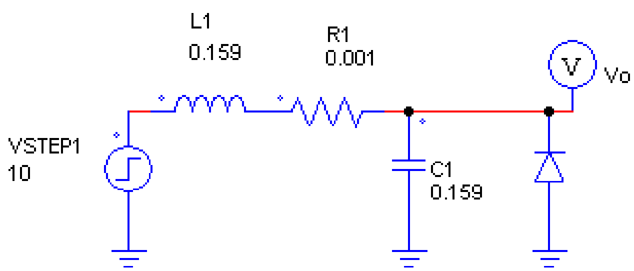
- Q: With a positive voltage step, what does the diode do in this circuit?



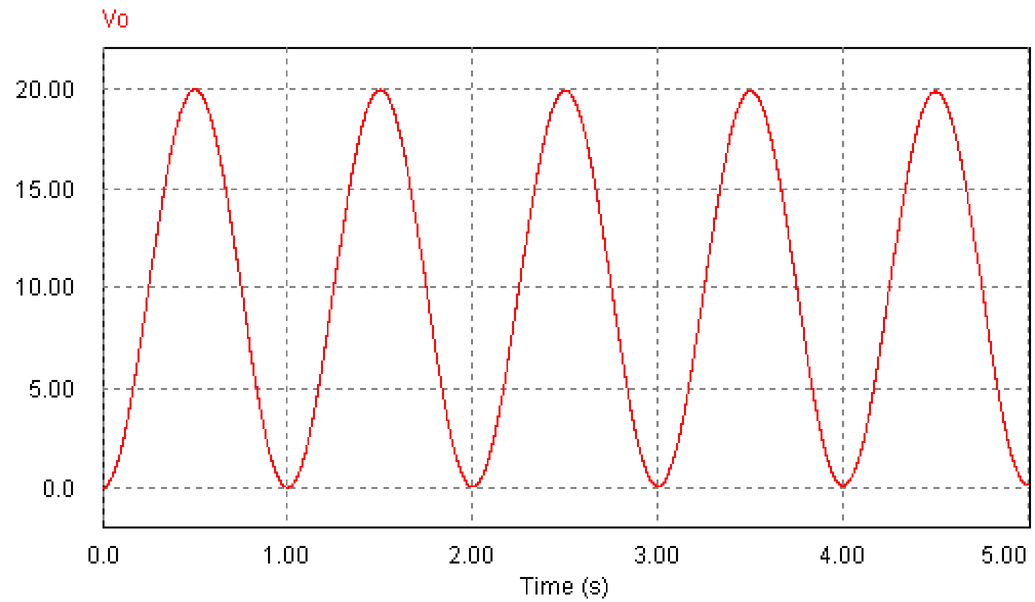
PSIM file: Series resonant circuit with diode1 .sch

Example 5: Series Resonant Circuit with Diode

- A: Nothing…it's always reverse biased

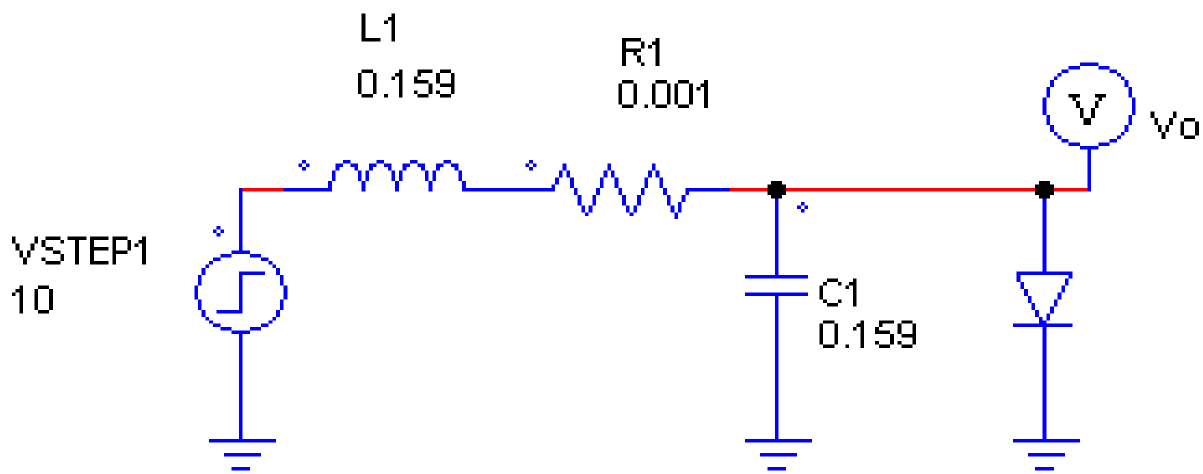


PSIM file: Series resonant circuit with diode1 .sch



Example 6: Now, Flip the Diode

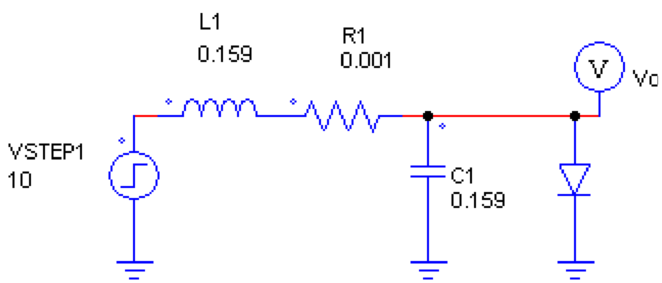
- Q: What does the diode do in this circuit?



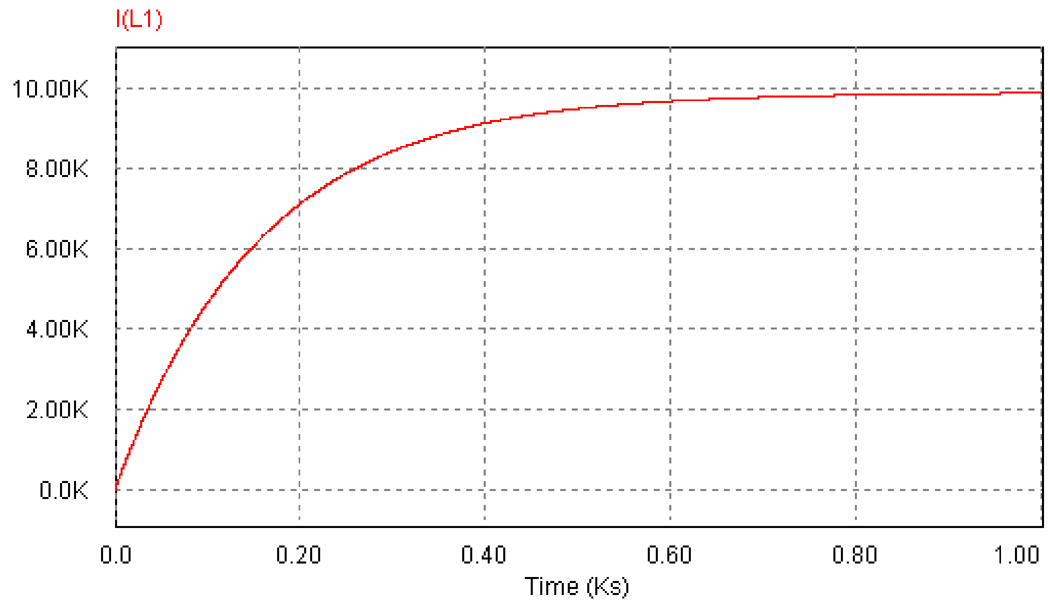
PSIM file: Series resonant circuit with diode2
.sch

Example 6: Series Resonant Circuit with Diode

- A: It shorts out the capacitor; the circuit behaves like a series LR circuit with L/R time constant 159 sec.

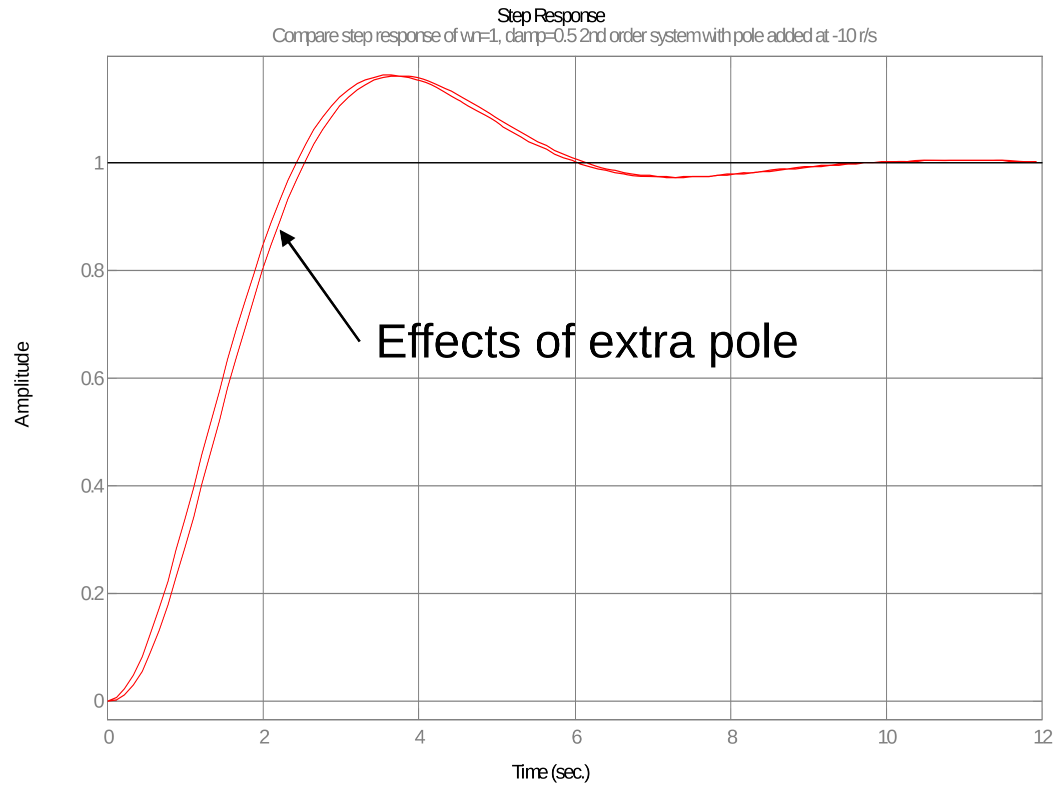


PSIM file: Series resonant circuit with diode2 .sch



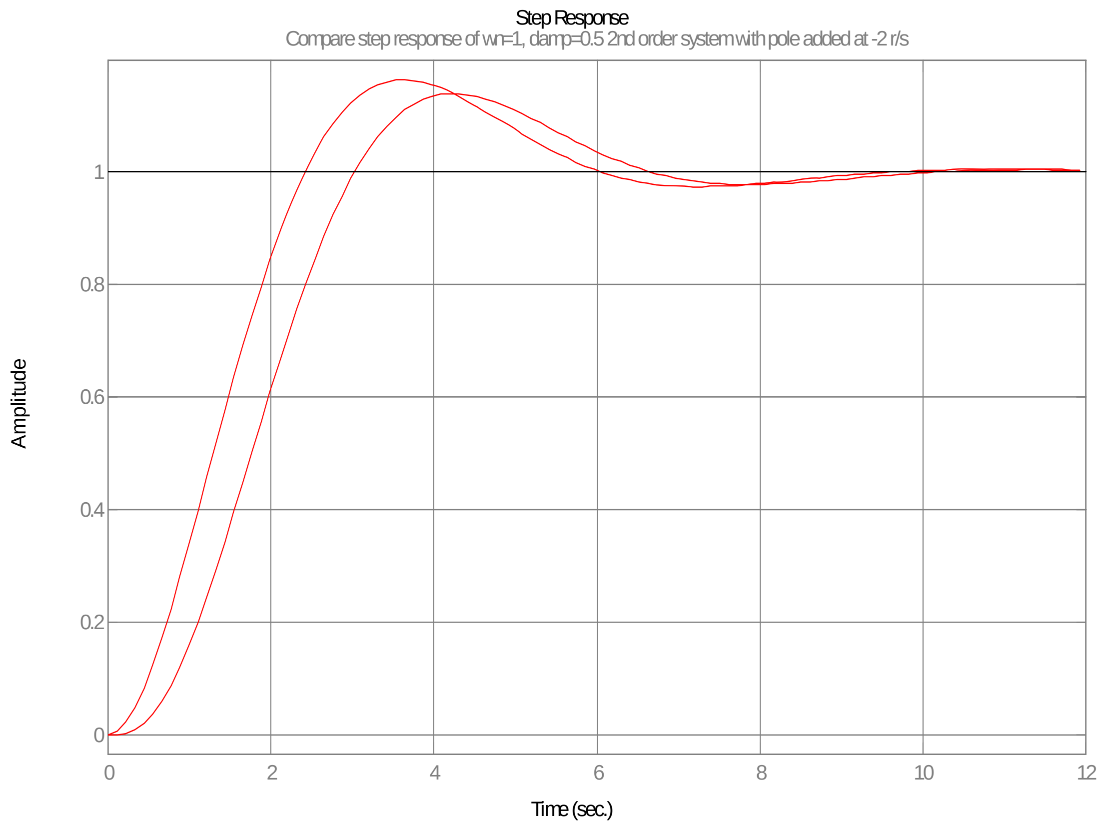
Example 7: Second-Order System Step Response, with Extra Pole

- Second order system with ω_n = 1 r/s, damping ratio = 0.5
- Extra pole added at -10 r/s



Example 8: Second-Order System Step Response, with Extra Pole

- Second order system with ω_n = 1 r/s, damping ratio = 0.5
- Extra pole added at -2 r/s



Widely-Spaced Pole Approximation

For a transfer function of the form:

$$\frac{1}{s^2 + As + B}$$

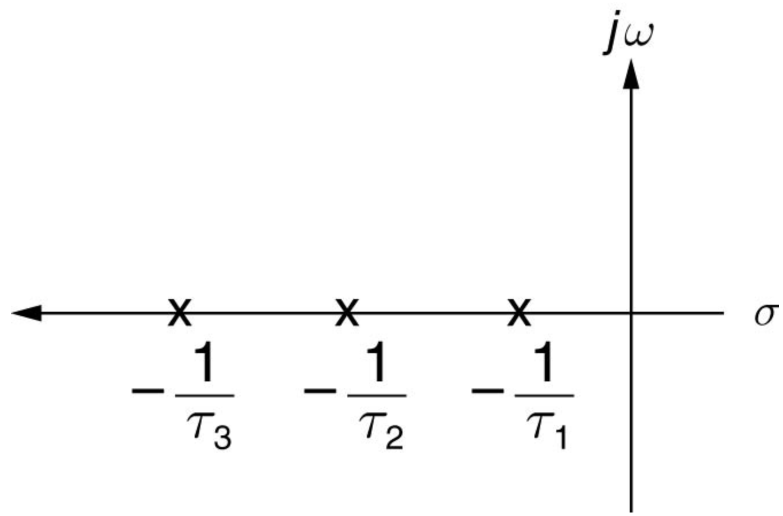
If the poles are on the real axis and are widely spaced, we can approximate them by:

$$s_{fast} \approx -A$$

$$s_{slow} \approx -B / A$$

Widely-Spaced Pole Approximation

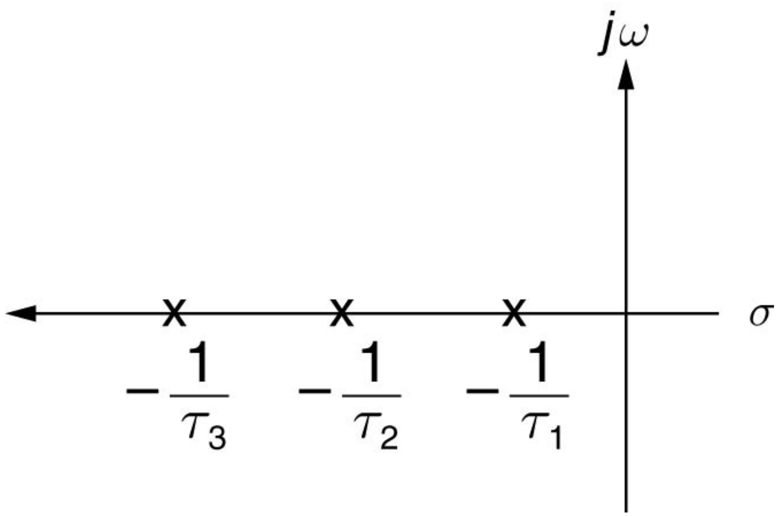
- Assume that we have 3 real-axis poles



$$H(s) = \frac{1}{(\tau_1 s + 1)(\tau_2 s + 1)(\tau_3 s + 1)} = \frac{1}{\tau_1 \tau_2 \tau_3 s^3 + \left(\tau_1 \tau_2 + \tau_1 \tau_3 + \tau_2 \tau_3\right) s^2 + \left(\tau_1 + \tau_2 + \tau_3\right) s + 1}$$

Widely-Spaced Pole Approximation

- We can re-write this transfer function:



$$H(s) = \frac{1}{a_3 s^3 + a_2 s^2 + a_1 s + 1}$$

$$a_3 = \tau_1 \tau_2 \tau_3$$

$$a_2 = \tau_1 \tau_2 + \tau_1 \tau_3 + \tau_2 \tau_3$$

$$a_1 = \tau_1 + \tau_2 + \tau_3$$

Widely-Spaced Pole Approximation

- Pole locations can be approximated if the poles are widely spaced:

$$p_{low} \approx -\frac{1}{a_1} \approx -\frac{1}{\tau_1 + \tau_2 + \tau_3} \approx -\frac{1}{\tau_1}$$

$$p_{high} \approx -\frac{a_2}{a_3} \approx -\frac{\tau_1\tau_2 + \tau_1\tau_3 + \tau_2\tau_3}{\tau_1\tau_2\tau_3} \approx -\frac{\tau_1\tau_2}{\tau_1\tau_2\tau_3} \approx -\frac{1}{\tau_3}$$

$$p_{medium} \approx \frac{a_1}{a_2} \approx \frac{\tau_1 + \tau_2 + \tau_3}{\tau_1\tau_2 + \tau_1\tau_3 + \tau_2\tau_3} \approx -\frac{\tau_1}{\tau_1\tau_2} \approx -\frac{1}{\tau_2}$$

Widely-Spaced Pole Approximation

- In the general case with k poles, if they are widely spaced and on the real axis:

$$p_k \approx -\frac{a_{k-1}}{a_k}$$

$$a_o = 1$$

Widely-Spaced Pole Approximation --- Sanity Check

- Examine system with poles at -1, -10, -100, -1000 and -10000 r/s

$$H(s) = \frac{1}{10^{-10}s^5 + 1.111\times10^{-6}s^4 + 1.122\times10^{-3}s^3 + 1.122\times10^{-1}s^2 + 1.111s + 1}$$

$$p_5 \approx -\frac{1.111\times10^{-6}}{10^{-10}} \approx -11{,}110\ r/s$$

$$p_4 \approx -\frac{1.122\times10^{-3}}{1.111\times10^{-6}} \approx -1{,}010\ \ r/s$$

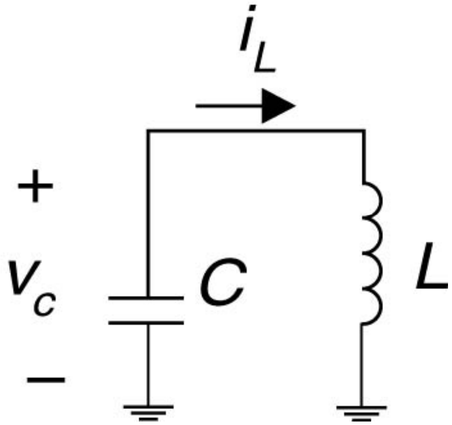
$$p_3 \approx -\frac{1.122\times10^{-1}}{1.122\times10^{-3}} \approx -100\ \ r/s$$

$$p_2 \approx -\frac{1.111}{1.122\times10^{-1}} \approx -9.9\ \ r/s$$

$$p_2 \approx -\frac{1}{1.111} \approx -0.9\ r/s$$

- The approximation does a decent job

Undamped Resonant Circuit



$$\frac{di_L}{dt} = \frac{v_c}{L} \qquad \frac{dv_c}{dt} = \frac{-i_L}{C}$$

$$\frac{d^2 v_c}{dt^2} = -\frac{1}{C}\frac{di_L}{dt} = -\frac{v_c}{LC}$$

Guess that the voltage *v(t)* is sinusoidal with $v(t) = V_o \sin \omega t$. Putting this into the equation for capacitor voltage results in:

$$-\omega^2 \sin(\omega t) = -\frac{1}{LC}\sin(\omega t)$$

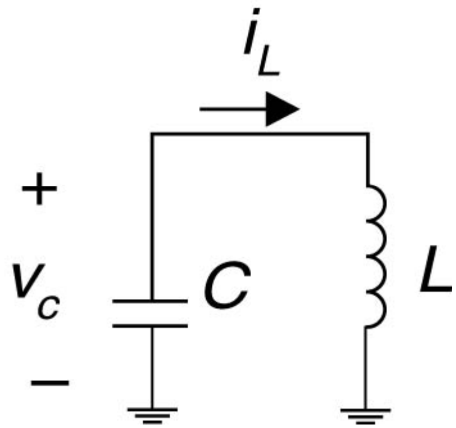
This means that the resonant frequency is the standard (as expected) resonance:

$$\omega_r^2 = \frac{1}{LC}$$

Energy Methods

Storage Mode	**Relationship**	**Comments**
Capacitor/electric field storage	$E_{elec} = \frac{1}{2}CV^2$	
Inductor/magnetic field storage	$E_{mag} = \frac{1}{2}LI^2 = \int \frac{B^2}{2\mu_o}dV$	
Kinetic energy	$E_k = \frac{1}{2}Mv^2$	
Rotary energy	$E_r = \frac{1}{2}I\omega^2$	I ≡ mass moment of inertia (kg- m^2)
Spring	$E_{spring} = \frac{1}{2}kx^2$	k ≡ spring constant (N/m)
Potential energy	$\Delta E_p = Mg\Delta h$	Δh ≡ height change
Thermal energy	$\Delta E_T = C_{TH}\Delta T$	C_{TH} ≡ thermal capacitance (J/K)

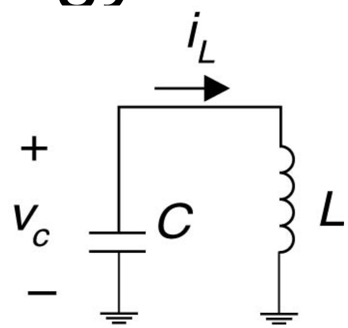
Energy Methods



By using energy methods we can find the ratio of maximum capacitor voltage to maximum inductor current. Assuming that the capacitor is initially charged to V_o volts, and remembering that capacitor stored energy $E_c = ½CV^2$ and inductor stored energy is $E_L = ½LI^2$, we can write the following:

$$\frac{1}{2}CV_o^2 = \frac{1}{2}LI_o^2$$

Energy Methods

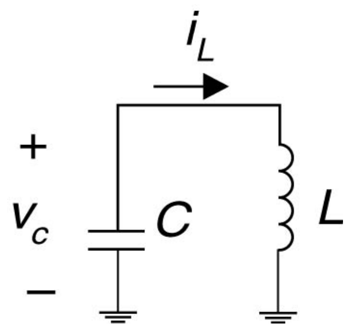


What does this mean about the magnitude of the inductor current ? Well, we can solve for the ratio of V_o/I_o resulting in:

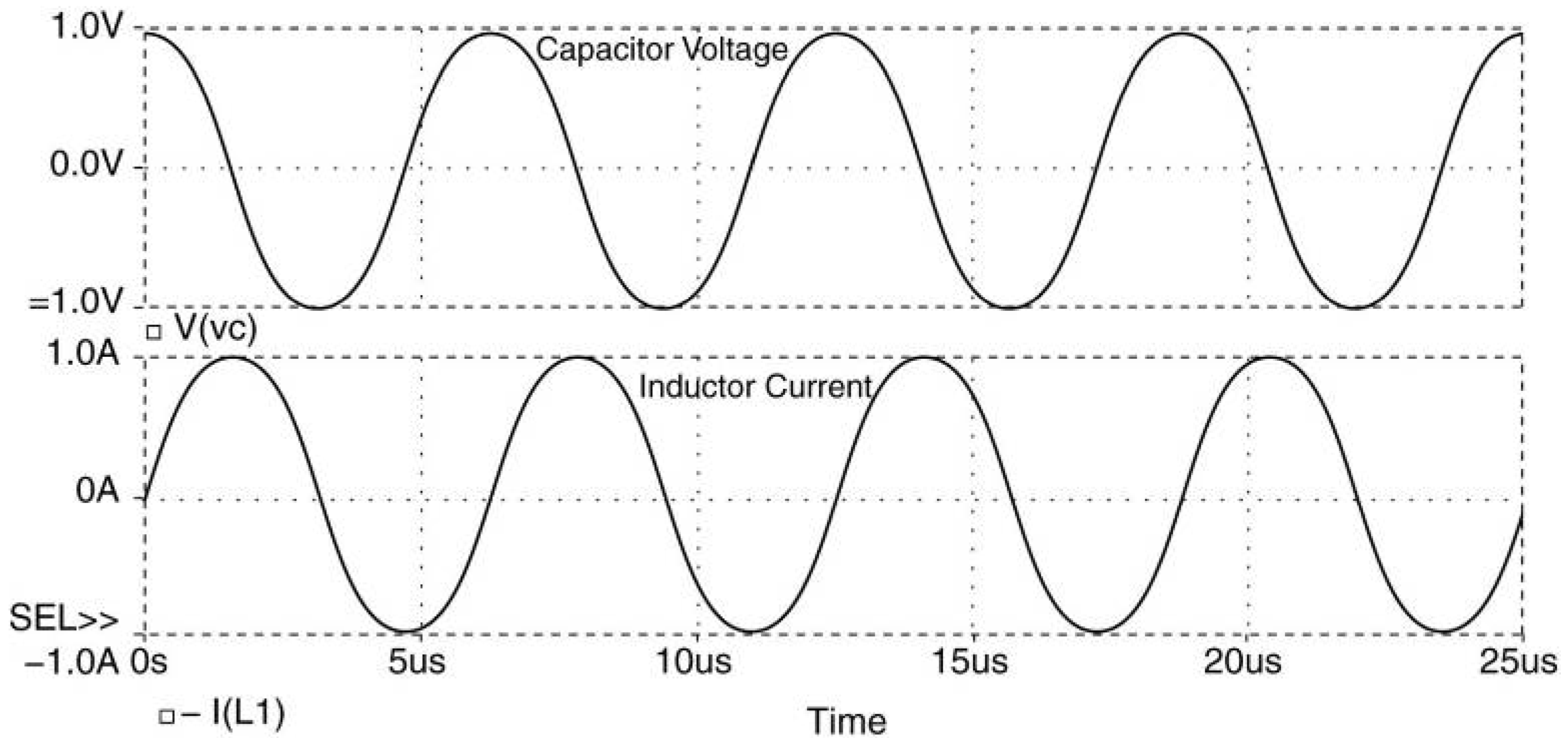
$$\frac{V_o}{I_o} = \sqrt{\frac{L}{C}} \equiv Z_o$$

The term "Z_o" is defined as the characteristic impedance of a resonant circuit. Let's assume that we have an inductor-capacitor circuit with C = 1 microFarad and L = 1 microHenry. This means that the resonant frequency is 10^6 radians/second (or 166.7 kHz) and that the characteristic impedance is 1 Ohm.

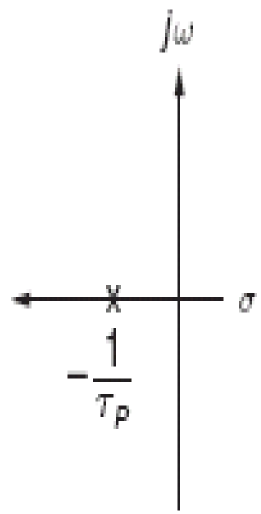
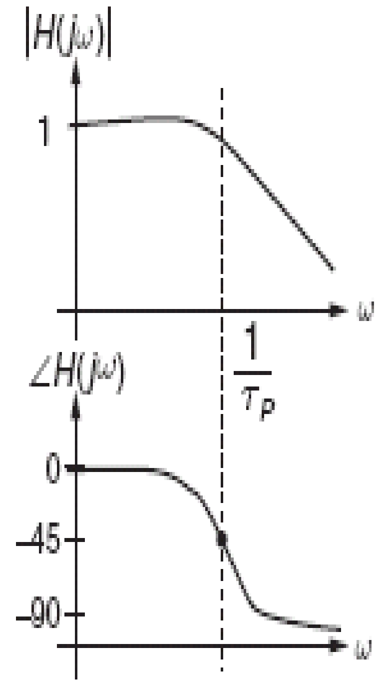
Example 9: Simulation



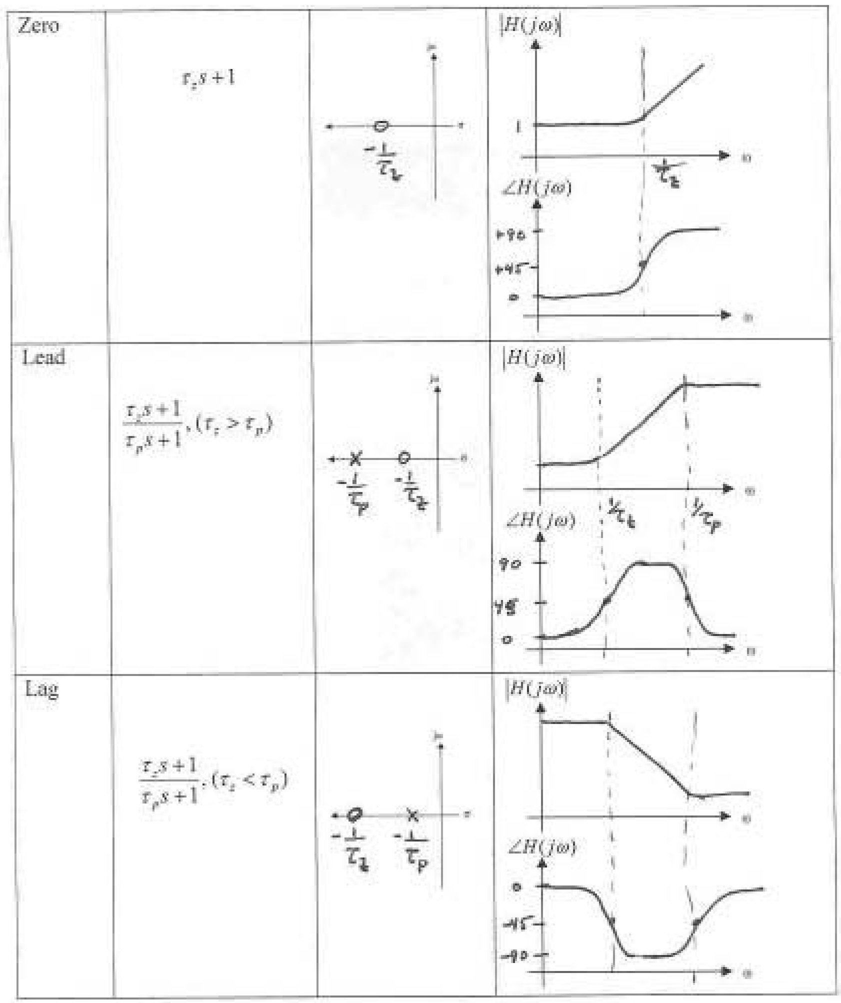
• Initial conditions at t = 0: capacitor voltage = 1; inductor current = 0.



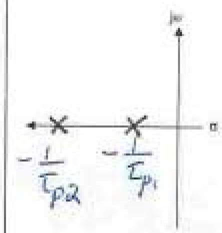
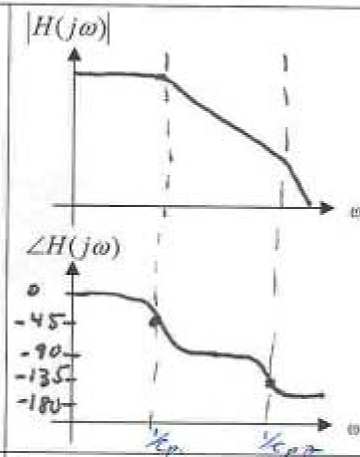
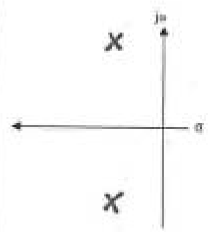
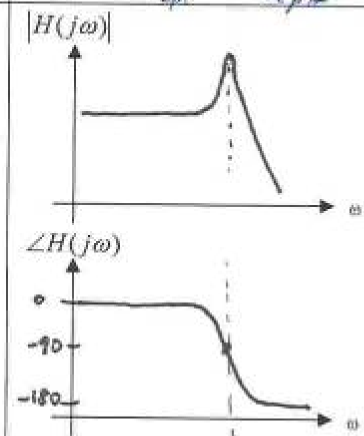
Transfer Functions, Pole-Zero and Bode Plots

System Type	Transfer Function H(s)	Pole/Zero Plot	Bode Plot
Single Pole	$\frac{1}{\tau_p s+1}$		

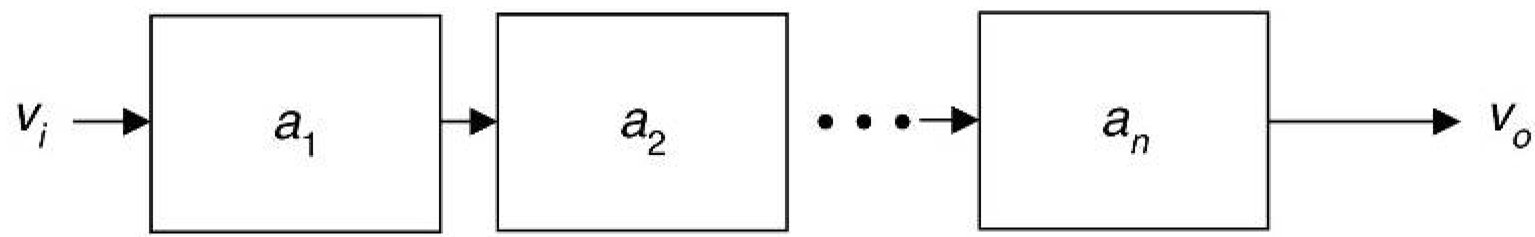
Transfer Functions, Pole-Zero and Bode Plots



Transfer Functions, Pole-Zero and Bode Plots

2 Pole	$\left(\frac{1}{\tau_{p1}s+1}\right)\left(\frac{1}{\tau_{p2}s+1}\right)$		
2nd order, low damping	$\frac{1}{\left(\frac{s}{\omega_n}\right)^2+2\varsigma\left(\frac{s}{\omega_n}\right)+1}$		

Risetime for Systems in Cascade

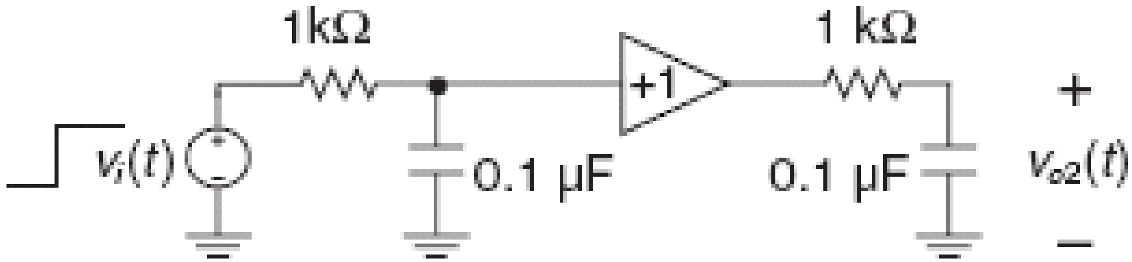


For multiple systems in cascade, the risetimes do not simply add; for instance, for N systems wired in series, each with its own risetime τ_{R1}, τ_{R2}, … τ_{RN}, the overall risetime of the cascade τ_R is:

$$\tau_R \approx \sqrt{\tau_{R1}^2 + \tau_{R2}^2 + \tau_{R3}^2 + \cdots \tau_{RN}^2}$$

Note that this equation works if each system is buffered/isolated from the next sytem; i.e. the systems don't load each other down.

Example 10: Risetime for Systems in Cascade



For v_{o1}:

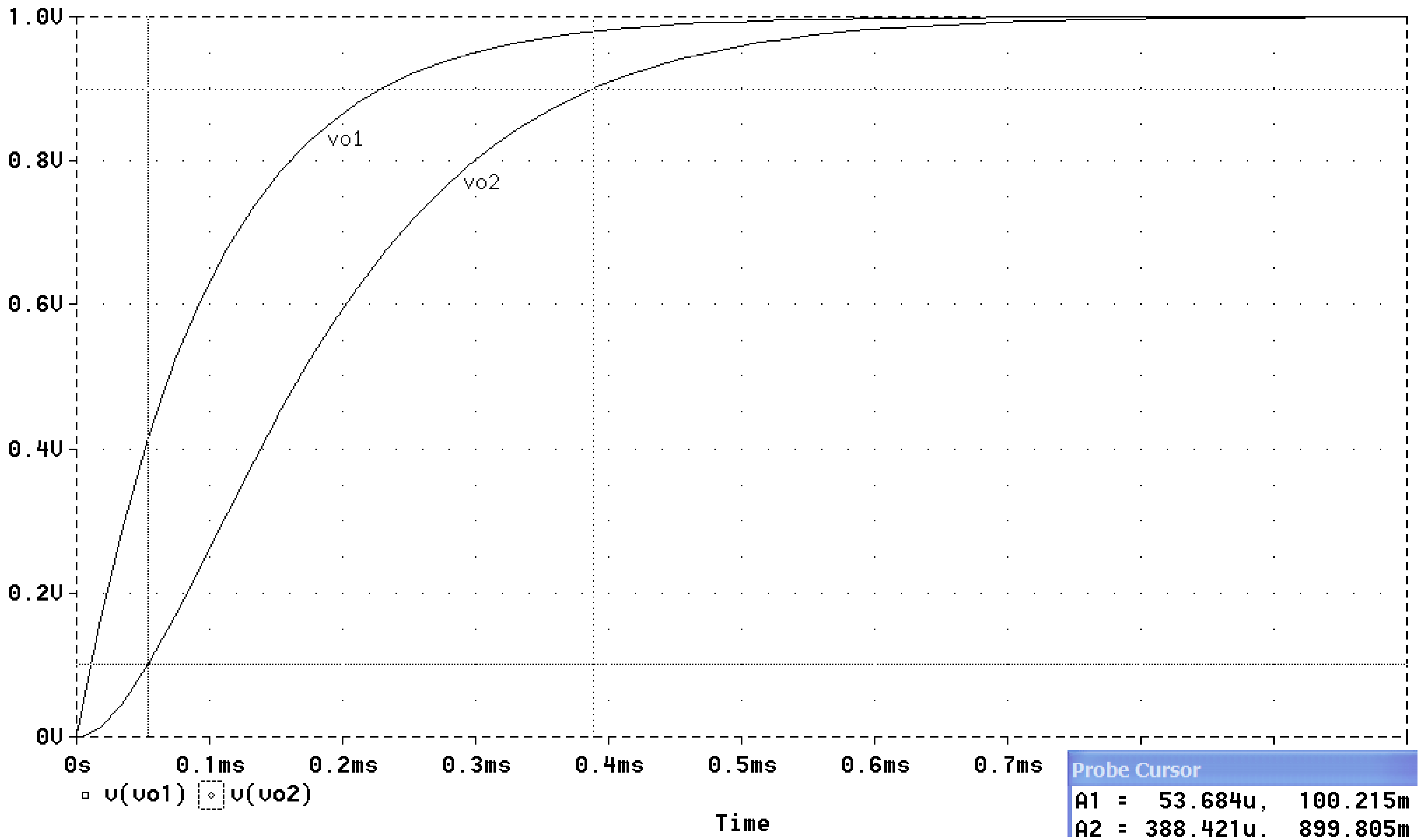
$$\tau_{R1} = 2.2RC = (2.2)(1000)(10^{-7}) = 220\mu\sec$$

For v_{o2}:
Consider TR2, which is risetime of 2nd RC circuit by itself:

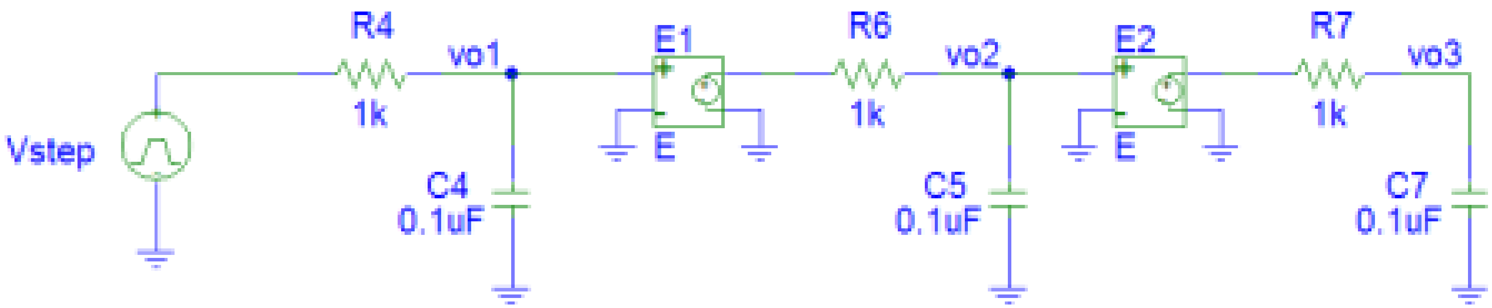
$$\tau_{R2} = 2.2RC = (2.2)(1000)(10^{-7}) = 220\mu\sec$$

$$\tau_{R,system} \approx \sqrt{\tau_{R1}^2 + \tau_{R2}^2} \approx \left(\sqrt{2}\right)\left(220\mu\sec\right) \approx 311\mu\sec$$

Example 10: Risetime for Systems in Cascade



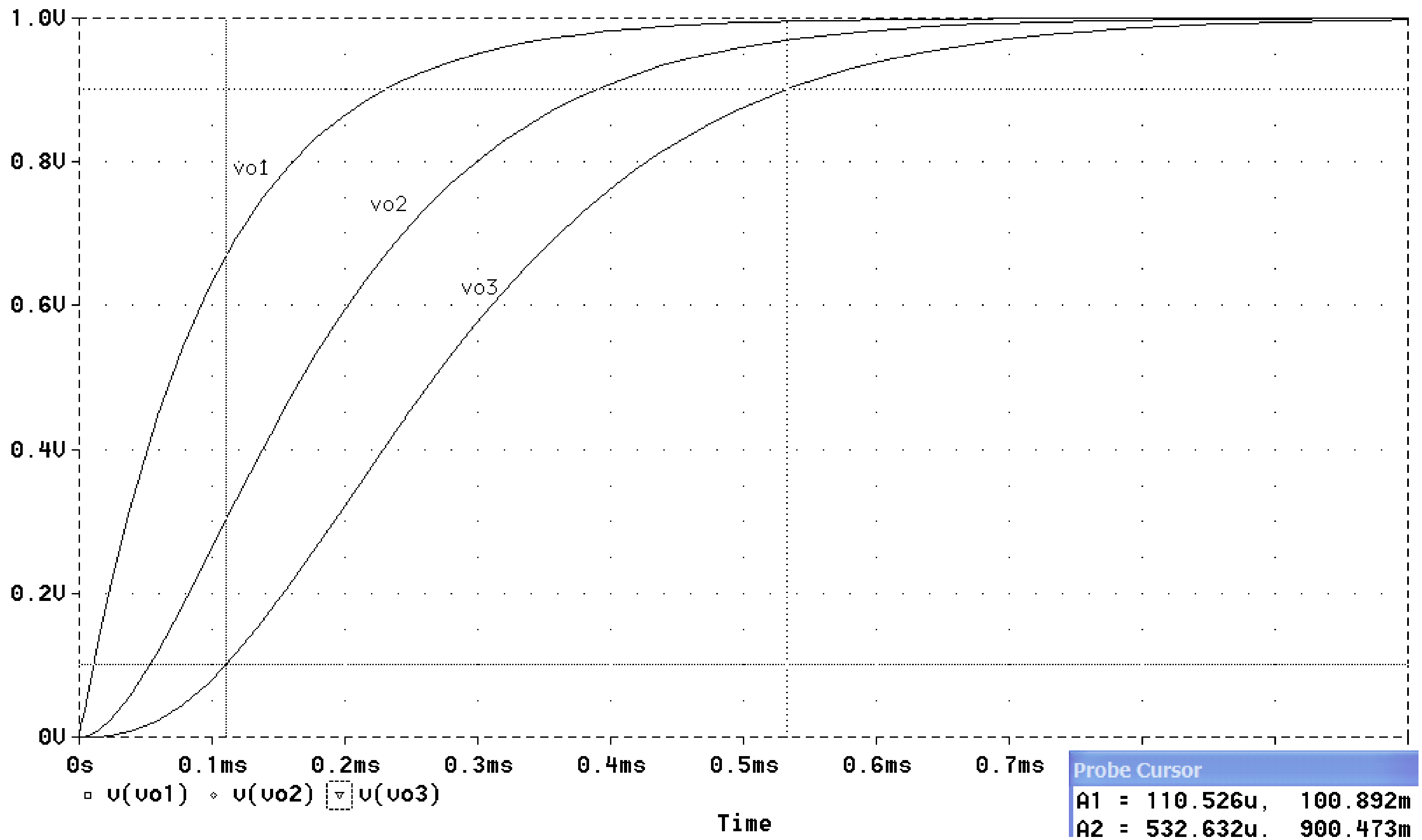
Example 13: Risetime for 3 Systems in Cascade



For v_{o3}:

$$\tau_{R,system} \approx \sqrt{\tau_{R1}^2 + \tau_{R2}^2 + \tau_{R3}^2} = \left(\sqrt{3}\right)\left(220\mu\sec\right) = 381\mu\sec$$

Example 13: Risetime for 3 Systems in Cascade



Comments on PSPICE

- PSPICE is a useful tool, but beware of some pitfalls
  - GIGO problem
  - Models
    - Sometimes the manufacturers' models are incorrect
  - Tolerances
    - Set tolerances sufficiently small. However, this will increase simulation time
  - Convergence issues
    - Transient response is not guaranteed to converge
  - Maximum time step
    - Choose a time step small enough to capture the detail that you need. If you make the time step too small, simulation time can be very large
  - Parasitics

Comments on PSPICE (cont.)

- Continued ...
  - Parasitic elements can cause your PSPICE simulation not to match protoboard results. Parasitic capacitance and inductance; resistive shunt paths, etc.
  - You should always do a sanity check to verify that your PSPICE result is reasonable
  - PSPICE simulation is not a replacement for good design and hand calculations.

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